Microwave:-

→ $\boxed{\text{Micro Wave}}$

Micro → Small
Wave → EM wave

→ Introduced by Nellocarne in 1932.

→ Microwaves are Electromagnetic waves whose Wavelength ranges from 1mm (millimeter) to 1m (meter) and frequency ranges from 300MHz to 300GHz.

Electromagnetic Spectrum:-

Violet   Red

Cosmic rays	Gamma rays	X-rays	UV rays	Visible Region	Infrared Region	Microwaves	Radio waves

← f↑, λ↓

Infrared rays --→ Used in remotes

Microwaves ---→ Point to Point Communication links, wireless networks.

Radiowaves ---→ broadcasting, communication satellites.

UV rays ---→ mineral water purifiers.

X-rays ---→ to identify the structure of an atom, to find out the mass of e^-. Also used in bone fracture treatment.

Γ-rays ----→ medicine, industry & nuclear industry.

Cosmic rays ----→ Satellite and space probes.

S.no	Band	frequency range	Wavelength	Application
01.	Extremely Low frequency	(30-300) HZ	(10-1) Megameter	Communication with submarine.
02.	Very Low frequency	(3-30) KHZ	(100-10) Km	Long distance Point to Point communication.
03.	Low frequency	(30-300) KHz	(10-1) Km	Point to Point marine communication
04.	Medium frequency	(300-3000) KHZ	(1000-100) m	broadcasting & marine Communication.
05.	High frequency	(3-30) MHZ	(100-10) m	moderate & long distance communication
06.	very high frequency	(30-300) MHZ	(100-10) cm	Short-distance communication
07.	Ultra high frequency	(300-3000) MHZ	(10-1) m	Television & FM series
08.	Super high frequency	(3-30) GHZ	(10-1) cm	Radar, microwave & space communication
09.	Extremely high frequency	(30-300) GHZ	mm	Radar, microwave & space communication

Microwave band designation:-

Application

L-band → (1-2) GHz ---> Radar Communication, GPS, aircraft surveilliance.

S-band → (2-4) GHz ---> Weather forecasting, Surface ship Radar communication, WiFi and bluetooth.

C-band → (4-8) GHz ---> Satellite communication

X-band → (8-12) GHz ---> Radar Communication

Ku-band → (12-18) GHz ---> Airtraffic control

K-band → (18-27) GHz ---> Astronomy, Satellite communication

Ka-band → (27-40) GHz ---> Low Range Radar Applications

millimeter → (40-300) GHz

Sub-millimeter → (> 300 GHz)

Due to these many applications, We need to study the subject of Microwave Engineering.

Advantages of MWE:-

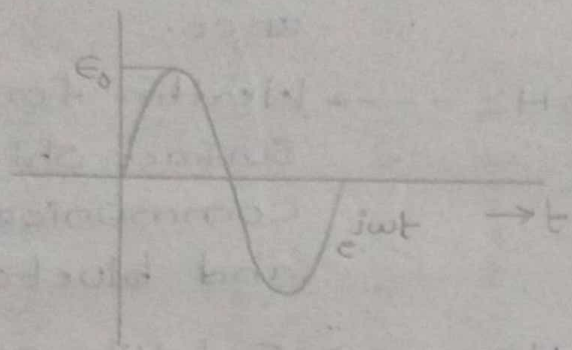
→ Increased bandwidth, so that You can transmit more information.

→ Better directivity (Frequency ↑, λ ↓, beamwidth ↓ and hence directivity ↑).

→ Fading effect is less as the signal frequency is high.

→ Power requirement is Less.

Wave:- A wave is defined as, a Physical quantity whose amplitude changes at every instant of time.



$$\epsilon = \epsilon_0 e^{j\omega t} \rightarrow ①$$

Partial differentiating on both sides we get,

$$\frac{\partial \epsilon}{\partial t} = \epsilon_0 e^{j\omega t} \cdot j\omega$$

$$\Rightarrow \frac{\partial \epsilon}{\partial t} = j\omega\epsilon \quad (\text{from } ①)$$

$$\Rightarrow \boxed{\frac{\partial}{\partial t} = j\omega}$$

Again Partial differentiate on both sides we get

$$\frac{\partial^2 \epsilon}{\partial t^2} = \epsilon_0 j\omega e^{j\omega t} j\omega$$

$$\Rightarrow \frac{\partial^2 \epsilon}{\partial t^2} = -\omega^2 \epsilon$$

$$\Rightarrow \frac{\partial^2 \epsilon}{\partial t^2} = -\omega^2 \epsilon$$

$$\Rightarrow \boxed{\frac{\partial^2}{\partial t^2} = -\omega^2}$$

Electromagnetic waves:-

The waves which satisfy Maxwell's equations are referred to as Electromagnetic waves (EM waves). Maxwell is the Person who Proved that there exists a relation between electric and magnetic fields.

Maxwell's equations for time-varying fields:-

1. $\nabla \cdot \bar{D} = \rho_v$

2. $\nabla \times \bar{E} = -\dot{B} = -\frac{\partial B}{\partial t}$

3. $\nabla \times \bar{H} = J + \dot{D} = J + \frac{\partial D}{\partial t}$

4. $\nabla \cdot \bar{B} = 0$

Maxwell's equations for freespace:-

for freespace, $\rho_v = 0$ and $\sigma = 0$ (from ohm's law $J = \sigma E$ when $\sigma = 0$ $J = 0$)

$$\nabla \cdot \bar{D} = 0 \longrightarrow ①$$

$$\nabla \times \bar{E} = -\dot{B} = -\frac{\partial B}{\partial t} \longrightarrow ②$$

$$\nabla \times \bar{H} = \dot{D} = \frac{\partial D}{\partial t} \longrightarrow ③$$

$$\nabla \cdot \bar{B} = 0 \longrightarrow ④$$

We know that

$$\boxed{\begin{aligned} \bar{D} &= \varepsilon \bar{E} \\ \bar{B} &= \mu \bar{H} \end{aligned}}$$

from this we can write,

$$\left. \begin{aligned} \nabla \cdot \bar{D} &= 0 \\ \nabla \cdot \bar{E} &= 0 \end{aligned} \right\} \longrightarrow ①$$

$$\nabla \times \bar{E} = -j\omega\mu \bar{H} \longrightarrow ②$$

$$\nabla \times \bar{H} = j\omega\varepsilon \bar{E} \longrightarrow ③$$

$$\nabla \cdot \bar{B} = 0 \longrightarrow ④$$

Now, Let us derive the wave equation for Microwave (EM wave).

Wave equation for Microwave:-

Consider, Maxwell's second equation and third equation

$\nabla \times \bar{E} = -j\omega\mu\bar{H}$

Taking 'curl' on b.s

$\nabla \times (\nabla \times \bar{E}) = -j\omega\mu(\nabla \times \bar{H})$

$\Rightarrow \nabla \times (\nabla \times \bar{E}) = -j\omega\mu(j\omega\varepsilon\bar{E})$

$\Rightarrow \nabla \times (\nabla \times \bar{E}) = \omega^2\mu\varepsilon\bar{E}$

$\Rightarrow \nabla\cdot(\nabla\cdot\bar{E}) - \bar{E}(\nabla\cdot\nabla) = \omega^2\mu\varepsilon\bar{E}$

$\Rightarrow 0 - \nabla^2\bar{E} = \omega^2\mu\varepsilon\bar{E}$

$\Rightarrow \boxed{\nabla^2\bar{E} = -\omega^2\mu\varepsilon\bar{E}}$

$\nabla \times \bar{H} = j\omega\varepsilon\bar{E}$

Taking 'curl' on b.s

$\nabla \times (\nabla \times \bar{H}) = j\omega\varepsilon(\nabla \times \bar{E})$

$\Rightarrow \nabla \times (\nabla \times \bar{H}) = j\omega\varepsilon(-j\omega\mu\bar{H})$

$\Rightarrow \nabla \times (\nabla \times \bar{H}) = \omega^2\mu\varepsilon\bar{H}$

$\Rightarrow \nabla\cdot(\nabla\cdot\bar{H}) - \bar{H}(\nabla\cdot\nabla) = \omega^2\mu\varepsilon\bar{H}$

$\Rightarrow 0 - \nabla^2\bar{H} = \omega^2\mu\varepsilon\bar{H}$

$\Rightarrow \boxed{\nabla^2\bar{H} = -\omega^2\mu\varepsilon\bar{H}}$

Vector identity:-

$\bar{A}\times(\bar{B}\times\bar{C}) = \bar{A}\cdot(\bar{B}\cdot\bar{C}) - \bar{C}\cdot(\bar{B}\cdot\bar{A})$

$$\boxed{\begin{aligned}\nabla^2\bar{E} &= -\omega^2\mu\varepsilon\bar{E}\\ \nabla^2\bar{H} &= -\omega^2\mu\varepsilon\bar{H}\end{aligned}} \rightarrow \text{(Helmholtz' wave equations)}$$

$\beta = \omega\sqrt{\mu\varepsilon} \rightarrow$ Phaseshift constant

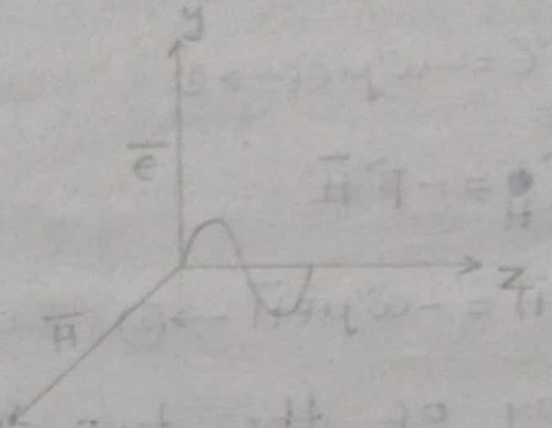
$\alpha \rightarrow$ Attenuation constant

$\gamma \rightarrow$ Propagation constant

$\boxed{\gamma = \alpha + j\beta}$

TEM wave :-

A wave consisting of both electric field and magnetic field which are ⊥er to eachother and are Perpendicular to the direction of Wave Propagation is referred to as "Transverse Electromagnetic wave".



$\boxed{E_z = 0 \\ H_z = 0}$

TE Wave :- A Wave Whose electric field component is Zero in the direction of Propagation but with non-zero magnetic field component is referred to as "Transverse electric wave".

i.e., $\boxed{E_z = 0; H_z \neq 0}$

TM wave :- A Wave whose magnetic field component is Zero in the direction of Propagation but with non-zero electric field component is referred to as "Transverse magnetic wave.

i.e., $\boxed{H_z = 0; E_z \neq 0}$

Hybrid wave :- A Wave whose electric field and magnetic field components are non-zero in the direction of Propagation is referred to as Hybrid wave. On the other hand, the wave consists of electric & magnetic field components in the direction of Propagation.

i.e., $\boxed{E_z \neq 0 \text{ and } H_z \neq 0}$

To findout the field components:-

$(E_x, H_y, E_y \& H_x)$

We know that

$$\nabla^2 \bar{E} = -\beta^2 \bar{E}$$

$$\Rightarrow \nabla^2 \bar{E} = -\omega^2 \mu \bar{E}\varepsilon \rightarrow ① \text{ (TM mode)}$$

$$\nabla^2 \bar{H} = -\beta^2 \bar{H}$$

$$\Rightarrow \nabla^2 \bar{H} = -\omega^2 \mu \varepsilon \bar{H} \rightarrow ② \text{ (TE mode)}$$

Consider any of the two equations. Let's consider, eqn-②

$$\nabla^2 \bar{H} = -\omega^2 \mu \varepsilon \bar{H}$$

$$\Rightarrow \frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \frac{\partial^2 H_z}{\partial z^2} = -\omega^2 \mu \varepsilon \bar{H}_z \rightarrow ③$$

$\left(\because \nabla = \frac{\partial}{\partial x} a_x + \frac{\partial}{\partial y} a_y + \frac{\partial}{\partial z} a_z\right.$

$\left. H_z \rightarrow \text{Wave Propagating in z-direction}\right)$

Replace $\frac{\partial}{\partial z} = -\gamma$ in eqn-③

indicates that wave is in forward z-direction (Propagation constant)

$$\Rightarrow \frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \gamma^2 H_z = -\omega^2 \mu \varepsilon \bar{H}_z$$

$$\Rightarrow \frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \gamma^2 H_z + \omega^2 \mu \varepsilon H_z = 0$$

$$\Rightarrow \frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + (\gamma^2 + \omega^2 \mu \varepsilon) H_z = 0$$

Let $\boxed{h^2 = \gamma^2 + \omega^2 \mu \varepsilon} \rightarrow ④$

$$\Rightarrow \frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + h^2 H_z = 0 \rightarrow ⑤$$

Similarly,

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + h^2 E_z = 0 \rightarrow ⑥$$

From Maxwell's Second equation,

$$\nabla \times \overline{E} = -j\omega\mu \overline{H}$$

$$\Rightarrow \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = -j\omega\mu \overline{H}$$

$$\Rightarrow \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & -\gamma \\ E_x & E_y & E_z \end{vmatrix} = -j\omega\mu (H_x a_x + H_y a_y + H_z a_z)$$

$$\Rightarrow a_x\left(\frac{\partial E_z}{\partial y} + \gamma E_y\right) - a_y\left(\frac{\partial E_z}{\partial x} + \gamma E_x\right) + a_z\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right) =$$

$$-j\omega\mu (H_x a_x + H_y a_y + H_z a_z)$$

Comparing the coefficients of a_x, a_y, a_z on b.s

$$\left(\frac{\partial E_z}{\partial y} + \gamma E_y\right) = -j\omega\mu H_x \rightarrow ⑦$$

$$-\left(\frac{\partial E_z}{\partial x} + \gamma E_x\right) = -j\omega\mu H_y \rightarrow ⑧$$

$$\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right) = -j\omega\mu H_z \rightarrow ⑨$$

Consider Maxwell's third equation,

$$\nabla \times \overline{H} = j\omega\varepsilon\overline{E}$$

$$\Rightarrow \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} = j\omega\varepsilon\overline{E}$$

$$\Rightarrow \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} = j\omega\varepsilon(E_x a_x + E_y a_y + E_z a_z)$$

$$\Rightarrow \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & -\gamma \\ H_x & H_y & H_z \end{vmatrix} = j\omega\varepsilon(E_x a_x + E_y a_y + E_z a_z)$$

$$\Rightarrow a_x\left(\frac{\partial H_z}{\partial y} + \gamma H_y\right) - a_y\left(\frac{\partial H_z}{\partial x} + \gamma H_x\right) + a_z\left(\frac{\partial H_z}{\partial x} - \frac{\partial H_z}{\partial y}\right)$$
$$= j\omega\varepsilon(E_x a_x + E_y a_y + E_z a_z)$$

Comparing the coefficients of a_x, a_y, a_z on bothsides.

$$\left(\frac{\partial H_z}{\partial y} + \gamma H_y\right) = j\omega\varepsilon E_x \rightarrow (10)$$

$$-\left(\frac{\partial H_z}{\partial x} + \gamma H_x\right) = j\omega\varepsilon E_y \rightarrow (11)$$

$$\left(\frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial y}\right) = j\omega\varepsilon E_z \rightarrow (12)$$

from these Parameters find E_x, H_y, E_y and H_x

from eqⁿ-(10) We have,

$$\frac{\partial H_z}{\partial y} + \gamma H_y = j\omega\varepsilon E_x$$

$$\Rightarrow E_x = \frac{1}{j\omega\varepsilon}\left(\frac{\partial H_z}{\partial y} + \gamma H_y\right) \rightarrow (13)$$

from eqⁿ-(8) we have,

$$\cancel{-}\left(\frac{\partial E_z}{\partial x} + \gamma E_x\right) = \cancel{-}j\omega\mu H_y$$

$$\Rightarrow \frac{\partial E_z}{\partial x} + \gamma E_x = j\omega\mu H_y$$

$$\Rightarrow H_y = \frac{1}{j\omega\mu}\left(\frac{\partial E_z}{\partial x} + \gamma E_x\right) \rightarrow (14)$$

from eqⁿ-(13) and eqⁿ-(14)

$$E_x = \frac{1}{j\omega\varepsilon}\left\{\frac{\partial H_z}{\partial y} + \gamma\left[\frac{1}{j\omega\mu}\left(\frac{\partial E_z}{\partial x} + \gamma E_x\right)\right]\right\}$$

$$\Rightarrow E_x = \frac{1}{j\omega\varepsilon}\frac{\partial H_z}{\partial y} - \frac{\gamma}{\omega^2\mu\varepsilon}\frac{\partial E_z}{\partial x} - \frac{\gamma^2}{\omega^2\mu\varepsilon}E_x$$

$$\Rightarrow E_x\left(1 + \frac{\gamma^2}{\omega^2\mu\varepsilon}\right) = \frac{1}{j\omega\varepsilon}\frac{\partial H_z}{\partial y} - \frac{\gamma}{\omega^2\mu\varepsilon}\frac{\partial E_z}{\partial x}$$

$$\Rightarrow E_x\left(\frac{\omega^2\mu\varepsilon + \gamma^2}{\omega^2\mu\varepsilon}\right) = \frac{1}{j\omega\varepsilon}\frac{\partial H_z}{\partial y} - \frac{\gamma}{\omega^2\mu\varepsilon}\frac{\partial E_z}{\partial x}$$

from eqⁿ-(4), we have $h^2 = \gamma^2 + \omega^2\mu\varepsilon$

$$\Rightarrow E_x\left(\frac{h^2}{\omega^2\mu\varepsilon}\right) = \frac{1}{j\omega\varepsilon}\frac{\partial H_z}{\partial y} - \frac{\gamma}{\omega^2\mu\varepsilon}\frac{\partial E_z}{\partial x}$$

$$\Rightarrow E_x = \frac{\omega^2\mu\cancel{\varepsilon}}{h^2}\cdot\frac{1}{j\cancel{\omega\varepsilon}}\frac{\partial H_z}{\partial y} - \frac{\cancel{\omega^2\mu\varepsilon}}{h^2}\cdot\frac{\gamma}{\cancel{\omega^2\mu\varepsilon}}\frac{\partial E_z}{\partial x}$$

$$\Rightarrow E_x = \frac{\omega\mu}{h^2}\cdot\frac{1}{j}\frac{\partial H_z}{\partial y} - \frac{\gamma}{h^2}\frac{\partial E_z}{\partial x}$$

$$\Rightarrow \boxed{E_x = -\frac{\gamma}{h^2}\frac{\partial E_z}{\partial x} - j\frac{\omega\mu}{h^2}\frac{\partial H_z}{\partial y}}$$

<u>To find E_y:-</u>

from eqn-11 we have

$$-\left(\frac{\partial H_z}{\partial x} + \gamma H_x\right) = j\omega\varepsilon E_y$$

$$\Rightarrow E_y = -\frac{1}{j\omega\varepsilon}\left(\frac{\partial H_z}{\partial x} + \gamma H_x\right) \rightarrow 15$$

from eqn-7 we have

$$\left(\frac{\partial E_z}{\partial y} + \gamma E_y\right) = -j\omega\mu H_x$$

$$\Rightarrow H_x = \frac{-1}{j\omega\mu}\left(\frac{\partial E_z}{\partial y} + \gamma E_y\right) \rightarrow 16$$

From eqn's 15 and 16

$$E_y = -\frac{1}{j\omega\varepsilon}\left\{\frac{\partial H_z}{\partial x} + \gamma\left[\frac{-1}{j\omega\mu}\left(\frac{\partial E_z}{\partial y} + \gamma E_y\right)\right]\right\}$$

$$\Rightarrow E_y = -\frac{1}{j\omega\varepsilon}\frac{\partial H_z}{\partial x} - \frac{\gamma}{\omega^2\mu\varepsilon}\frac{\partial E_z}{\partial y} - \frac{1}{\omega^2\mu\varepsilon}\gamma^2 E_y$$

$$\Rightarrow E_y\left(1 + \frac{\gamma^2}{\omega^2\mu\varepsilon}\right) = -\frac{1}{j\omega\varepsilon}\frac{\partial H_z}{\partial x} - \frac{\gamma}{\omega^2\mu\varepsilon}\frac{\partial E_z}{\partial y}$$

$$\Rightarrow E_y\left(\frac{\omega^2\mu\varepsilon + \gamma^2}{\omega^2\mu\varepsilon}\right) = -\frac{1}{j\omega\varepsilon}\frac{\partial H_z}{\partial x} - \frac{\gamma}{\omega^2\mu\varepsilon}\frac{\partial E_z}{\partial y}$$

$$\Rightarrow E_y\left(\frac{h^2}{\omega^2\mu\varepsilon}\right) = -\frac{1}{j\omega\varepsilon}\frac{\partial H_z}{\partial x} - \frac{\gamma}{\omega^2\mu\varepsilon}\frac{\partial E_z}{\partial y}$$

$$\Rightarrow E_y = \frac{\omega^2\mu\cancel{\varepsilon}}{h^2}\cdot -\frac{1}{j\omega\cancel{\varepsilon}}\frac{\partial H_z}{\partial x} - \frac{\omega^2\cancel{\mu\varepsilon}}{h^2}\cdot\frac{\gamma}{\omega^2\cancel{\mu\varepsilon}}\frac{\partial E_z}{\partial y}$$

$$\Rightarrow E_y = -\frac{\gamma}{h^2}\frac{\partial E_z}{\partial y} - \frac{1}{j}\frac{\omega\mu}{h^2}\frac{\partial H_z}{\partial x}$$

$$\Rightarrow \boxed{E_y = -\frac{\gamma}{h^2}\frac{\partial E_z}{\partial y} + \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x}}$$

To find H_x:-

from eqn-7 we have,

$$\frac{\partial E_z}{\partial y} + \gamma E_y = -j\omega\mu H_x$$

$$\Rightarrow H_x = -\frac{1}{j\omega\mu}\left(\frac{\partial E_z}{\partial y} + \gamma E_y\right) \rightarrow (17)$$

from eqn-11 We have,

$$-\left(\frac{\partial H_z}{\partial x} + \gamma H_x\right) = j\omega\varepsilon E_y$$

$$\Rightarrow E_y = -\frac{1}{j\omega\varepsilon}\left(\frac{\partial H_z}{\partial x} + \gamma H_x\right) \rightarrow (18)$$

from eqn's - 17 and 18

$$H_x = -\frac{1}{j\omega\mu}\left\{\frac{\partial E_z}{\partial y} + \gamma\left[-\frac{1}{j\omega\varepsilon}\left(\frac{\partial H_z}{\partial x} + \gamma H_x\right)\right]\right\}$$

$$\Rightarrow H_x = -\frac{1}{j\omega\mu}\frac{\partial E_z}{\partial y} - \frac{\gamma}{\omega^2\mu\varepsilon}\frac{\partial H_z}{\partial x} - \frac{1}{\omega^2\mu\varepsilon}\gamma^2 H_x$$

$$\Rightarrow H_x\left(1 + \frac{\gamma^2}{\omega^2\mu\varepsilon}\right) = -\frac{1}{j\omega\mu}\frac{\partial E_z}{\partial y} - \frac{\gamma}{\omega^2\mu\varepsilon}\frac{\partial H_z}{\partial x}$$

$$\Rightarrow H_x\left(\frac{\omega^2\mu\varepsilon + \gamma^2}{\omega^2\mu\varepsilon}\right) = -\frac{1}{j\omega\mu}\frac{\partial E_z}{\partial y} - \frac{\gamma}{\omega^2\mu\varepsilon}\frac{\partial H_z}{\partial x}$$

$$\Rightarrow H_x\left(\frac{h^2}{\omega^2\mu\varepsilon}\right) = -\frac{1}{j\omega\mu}\frac{\partial E_z}{\partial y} - \frac{\gamma}{\omega^2\mu\varepsilon}\frac{\partial H_z}{\partial x}$$

$$\Rightarrow H_x = \frac{\omega^{\cancel{2}}\cancel{\mu}\varepsilon}{h^2}\cdot -\frac{1}{j\cancel{\omega}\cancel{\mu}}\frac{\partial E_z}{\partial y} - \frac{\cancel{\omega^2\mu\varepsilon}}{h^2}\cdot\frac{\gamma}{\cancel{\omega^2\mu\varepsilon}}\frac{\partial H_z}{\partial x}$$

$$\Rightarrow H_x = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial x} - \frac{1}{j}\frac{\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial y}$$

$$\Rightarrow \boxed{H_x = -\frac{\gamma}{h^2}\frac{\partial H_z}{\partial x} + \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial y}}$$

To find Hy:-

from eq^n-⑧ we have,

$$\cancel{-}\left(\frac{\partial E_z}{\partial x} + \gamma E_x\right) = \cancel{-}j\omega\mu H_y$$

$$\Rightarrow H_y = \frac{1}{j\omega\mu}\left(\frac{\partial E_z}{\partial x} + \gamma E_x\right) \longrightarrow (19)$$

from eq^n-⑩ we have;

$$\frac{\partial H_z}{\partial y} + \gamma H_y = j\omega\varepsilon E_x$$

$$\Rightarrow E_x = \frac{1}{j\omega\varepsilon}\left(\frac{\partial H_z}{\partial y} + \gamma H_y\right) \longrightarrow (20)$$

from eq^n's - (19) and (20)

$$H_y = \frac{1}{j\omega\mu}\left\{\frac{\partial E_z}{\partial x} + \gamma\left[\frac{1}{j\omega\varepsilon}\left(\frac{\partial H_z}{\partial y} + \gamma H_y\right)\right]\right\}$$

$$\Rightarrow H_y = \frac{1}{j\omega\mu}\frac{\partial E_z}{\partial x} - \frac{\gamma}{\omega^2\mu\varepsilon}\frac{\partial H_z}{\partial y} - \frac{1}{\omega^2\mu\varepsilon}\gamma^2 H_y$$

$$\Rightarrow H_y\left(1 + \frac{\gamma^2}{\omega^2\mu\varepsilon}\right) = \frac{1}{j\omega\mu}\frac{\partial E_z}{\partial x} - \frac{\gamma}{\omega^2\mu\varepsilon}\frac{\partial H_z}{\partial y}$$

$$\Rightarrow H_y\left(\frac{\omega^2\mu\varepsilon + \gamma^2}{\omega^2\mu\varepsilon}\right) = \frac{1}{j\omega\mu}\frac{\partial E_z}{\partial x} - \frac{\gamma}{\omega^2\mu\varepsilon}\frac{\partial H_z}{\partial y}$$

$$\Rightarrow H_y\left(\frac{h^2}{\omega^2\mu\varepsilon}\right) = \frac{1}{j\omega\mu}\frac{\partial E_z}{\partial x} - \frac{\gamma}{\omega^2\mu\varepsilon}\frac{\partial H_z}{\partial y}$$

$$\Rightarrow H_y = \frac{\omega^{\cancel{2}}\cancel{\mu}\varepsilon}{h^2}\cdot\frac{1}{j\cancel{\omega\mu}}\frac{\partial E_z}{\partial x} - \frac{\cancel{\omega^2\mu\varepsilon}}{h^2}\cdot\frac{\gamma}{\cancel{\omega^2\mu\varepsilon}}\frac{\partial H_z}{\partial y}$$

$$\Rightarrow H_y = -\frac{\gamma}{h^2}\frac{\partial H_z}{\partial y} + \frac{1}{j}\frac{\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x}.$$

$$\Rightarrow \boxed{H_y = -\frac{\gamma}{h^2}\frac{\partial H_z}{\partial y} - j\frac{\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x}}$$

$$E_x = -\frac{\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}$$

$$E_y = -\frac{\gamma}{h^2}\frac{\partial E_z}{\partial y} + \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x}$$

$$H_x = -\frac{\gamma}{h^2}\frac{\partial H_z}{\partial x} + \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial y}$$

$$H_y = -\frac{\gamma}{h^2}\frac{\partial H_z}{\partial y} - \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x}$$

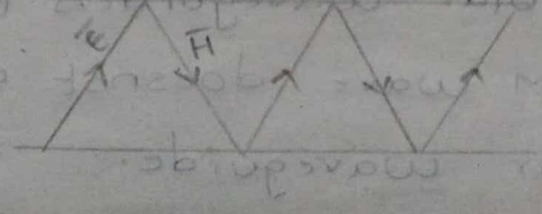
How to transmit Microwaves:-

→ Microwave frequency range is "300MHz to 300GHz"

To transmit such high frequency signals from one point to another point, various types of Transmission lines are used as listed below:
1. Openwire
2. Twinlead
3. Twisted Pair
4. co-axial
5. Optical fibre cables
6. Simply a copper wire

→ However, the most widely used Tx line for Microwave Propagation is "Waveguides".

→ Waveguides are hollow metallic tubes, in which the electric field and magnetic field of the wave propagating, are perpendicular to the direction of propagation.



Rectangular waveguides:-

Analysis of TEM mode :-

→ Consider a rectangular waveguide, in which a Transverse Electromagnetic wave is Propagating.

→ We know that, for a TEM wave

$E_z = 0$ and $H_z = 0$ if the wave is Propagating along z-direction.

→ Substituting $E_z = 0$ and $H_z = 0$ in below equations,

$$E_x = -\frac{\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}$$

$$E_y = -\frac{\gamma}{h^2}\frac{\partial E_z}{\partial y} + \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x}$$

$$H_x = -\frac{\gamma}{h^2}\frac{\partial H_z}{\partial x} + \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial y}$$

$$H_y = -\frac{\gamma}{h^2}\frac{\partial H_z}{\partial y} - \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x}$$

We get, $E_x = 0$

$E_y = 0$

$H_x = 0$ and

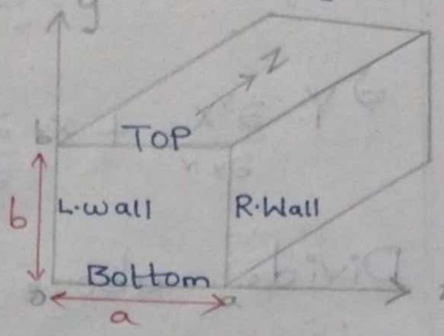
$H_y = 0$

→ From this we can conclude that one cannot Propagate a TEM wave in rectangular waveguides (or) on the other hand, TEM wave doesnot exist in a rectangular waveguide.

Analysis of TM mode:-

→ consider a rectangular waveguide with width 'a' and breadth 'b'.

→ Let us assume a wave in TM mode is propagating in the rectangular waveguide along z-direction.



We know that

$$\nabla^2 E_z = -\omega^2 \mu \varepsilon E_z \rightarrow (1)$$

$$\nabla^2 H_z = -\omega^2 \mu \varepsilon H_z \rightarrow (2)$$

from TM mode we can write,

$$\nabla^2 E_z = -\omega^2 \mu \varepsilon E_z$$

$$\nabla^2 H_z = -\omega^2 \mu \varepsilon (0) = 0 \qquad (\because H_z = 0 \;\&\; E_z \neq 0)$$

$$\therefore \boxed{\nabla^2 E_z = -\omega^2 \mu \varepsilon E_z} \rightarrow (3)$$

$$\Rightarrow \frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2} = -\omega^2 \mu \varepsilon E_z$$

Replace $\frac{\partial}{\partial z} = -\gamma$

$$\Rightarrow \frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \gamma^2 E_z = -\omega^2 \mu \varepsilon E_z$$

$$\Rightarrow \frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \gamma^2 E_z + \omega^2 \mu \varepsilon E_z = 0$$

$$\Rightarrow \frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + (\gamma^2 + \omega^2 \mu \varepsilon) E_z = 0$$

$$\Rightarrow \frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + h^2 E_z = 0 \rightarrow (4)$$

Let, $E_z = XY$ (E is in either 'x' or 'y' direction and hence a consideration is made of this type)

Here, X is a Pure function of 'x'

Y is a Pure function of 'y'

$$\Rightarrow \frac{\partial^2(XY)}{\partial x^2} + \frac{\partial^2(XY)}{\partial y^2} + h^2(XY) = 0$$

$$\Rightarrow Y \cdot \frac{\partial^2 X}{\partial x^2} + X \cdot \frac{\partial^2 Y}{\partial y^2} + h^2(XY) = 0$$

Divide above equation with "XY"

$$\Rightarrow \frac{Y}{XY} \cdot \frac{\partial^2 X}{\partial x^2} + \frac{X}{XY} \cdot \frac{\partial^2 Y}{\partial y^2} + \frac{h^2(XY)}{XY} = 0$$

$$\Rightarrow \frac{1}{X} \cdot \frac{\partial^2 X}{\partial x^2} + \frac{1}{Y} \cdot \frac{\partial^2 Y}{\partial y^2} + h^2 = 0$$ (Variable-separable method is applied)

Let $\frac{1}{X} \cdot \frac{\partial^2 X}{\partial x^2} = -B^2$

$\frac{1}{Y} \cdot \frac{\partial^2 Y}{\partial y^2} = -A^2$

$$\Rightarrow -B^2 - A^2 + h^2 = 0$$

$$\Rightarrow h^2 = A^2 + B^2 \longrightarrow (5)$$

Here, $\frac{1}{X} \cdot \frac{\partial^2 X}{\partial x^2} = -B^2$ is a second order D.eqn w.r.to x and hence the solution is given by,

$$X = (C_1 \cos Bx + C_2 \sin Bx) \longrightarrow (6)$$

Similarly,

$$Y = (C_3 \cos Ay + C_4 \sin Ay) \longrightarrow (7)$$

We have, $E_z = XY$

$$\Rightarrow E_z = (C_1 \cos Bx + C_2 \sin Bx)(C_3 \cos Ay + C_4 \sin Ay) \rightarrow (8)$$

To find C_1, C_2, C_3 and C_4:-

Boundary Conditions:-

(1) Bottom wall:-

$E_z = 0$ for $y = 0 \; \forall \; x = 0$ to a (E on the walls of a R. waveguide is zero)

(2) Left side wall:-

$E_z = 0$ for $x = 0 \; \forall \; y = 0$ to b

(3) Top wall:-

$E_z = 0$ for $y = b \; \forall \; x = 0$ to a

(4) Right side wall:-

$E_z = 0$ for $x = a \; \forall \; y = 0$ to b

Substituting (1)st boundary condition in eq^n-(8)

$$0 = \underbrace{(C_1 \cos Bx + C_2 \sin Bx)}_{\neq 0} C_3$$

Hence, $\boxed{C_3 = 0}$

now eq^n-(8) becomes,

$$E_z = (C_1 \cos Bx + C_2 \sin Bx) C_4 \sin Ay \rightarrow (9)$$

Substituting (2)nd boundary condition in eq^n-(9)

$E_z = 0$ for $x = 0$ $\forall$ $y = 0$ to b

$$\Rightarrow 0 = C_1 \underbrace{C_4 \sin Ay}_{\neq 0} \quad (\because y \text{ is variable})$$

Hence, $\boxed{C_1 = 0}$

now eqⁿ-(9) becomes;

$$E_z = C_2 \sin Bx \cdot C_4 \sin Ay \rightarrow (10)$$

Substituting (3)rd boundary condition in eqⁿ-(10)

$$0 = \underbrace{C_2 \sin Bx}_{\neq 0} \cdot C_4 \sin Ab \quad (\because x \text{ is variable})$$

Hence, $C_4 \sin Ab = 0$

$$\sin Ab = 0$$

$$Ab = \pm n\pi$$

$$\boxed{A = \pm \frac{n\pi}{b}}$$

now, eqⁿ-(10) becomes,

$$E_z = C_2 \sin Bx \cdot C_4 \sin\left(\frac{n\pi}{b}\right)y \rightarrow (11)$$

Substituting (4)th boundary condition in eqⁿ-(11)

$$0 = C_2 \sin Ba \cdot \underbrace{C_4 \sin\left(\frac{n\pi}{b}\right)y}_{\neq 0} \quad (\because y \text{ is variable})$$

Hence, $C_2 \sin Ba = 0$

$$\sin Ba = 0$$

$$aB = \pm m\pi$$

$$B = \frac{\pm m\pi}{a}$$

$$\therefore \boxed{B = \pm \frac{m\pi}{a}}$$

now, eqn-(11) becomes

$$E_z = C_2 \sin\left(\frac{m\pi}{a}\right)x \cdot C_4 \sin\left(\frac{n\pi}{b}\right)y$$

$$\Rightarrow E_z = \overset{C}{C_2 C_4} \sin\left(\frac{m\pi}{a}\right)x \cdot \sin\left(\frac{n\pi}{b}\right)y \longrightarrow (12)$$

We know that

$$E_x = -\frac{\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}$$

$$\Rightarrow E_x = -\frac{\gamma}{h^2}\frac{\partial}{\partial x}\left(C \sin\left(\frac{m\pi}{a}\right)x \cdot \sin\left(\frac{n\pi}{b}\right)y\right) - 0$$

($\because H_z = 0$ for a TM wave)

$$\Rightarrow \boxed{E_x = -\frac{\gamma}{h^2} C\left(\frac{m\pi}{a}\right)\cos\left(\frac{m\pi}{a}\right)x \cdot \sin\left(\frac{n\pi}{b}\right)y}$$

similarly,

$$E_y = -\frac{\gamma}{h^2}\frac{\partial E_z}{\partial y} + \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x}$$

$$\Rightarrow E_y = -\frac{\gamma}{h^2}\frac{\partial}{\partial y}\left(C \sin\left(\frac{m\pi}{a}\right)x \cdot \sin\left(\frac{n\pi}{b}\right)y\right) - 0$$

$$= -\frac{\gamma}{h^2} C \sin\left(\frac{m\pi}{a}\right)x \cdot \left(\frac{n\pi}{b}\right)\cos\left(\frac{n\pi}{b}\right)y$$

$$\boxed{E_y = -\frac{\gamma}{h^2} C \sin\left(\frac{m\pi}{a}\right)x \cdot \left(\frac{n\pi}{b}\right)\cos\left(\frac{n\pi}{b}\right)y}$$

Similarly,

$$H_x = -\frac{\gamma}{h^2}\frac{\partial H_z}{\partial x} + \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial y}$$

$$\Rightarrow H_x = -\frac{\gamma}{h^2}(0) + \frac{j\omega\varepsilon}{h^2}\frac{\partial}{\partial y}\left(C \sin\left(\frac{m\pi}{a}\right)x \cdot \sin\left(\frac{n\pi}{b}\right)y\right)$$

$$\Rightarrow \boxed{H_x = \frac{j\omega\varepsilon}{h^2} \cdot C \sin\left(\frac{m\pi}{a}\right)x \left(\frac{n\pi}{b}\right) \cos\left(\frac{n\pi}{b}\right)y}$$

Similarly,

$$H_y = -\frac{\gamma}{h^2}\frac{\partial H_z}{\partial y} - \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x}$$

$$\Rightarrow H_y = -\frac{\gamma}{h^2}(0) - \frac{j\omega\varepsilon}{h^2}\left(\frac{\partial}{\partial x}\left(C \sin\left(\frac{m\pi}{a}\right)x \cdot \sin\left(\frac{n\pi}{b}\right)y\right)\right)$$

$$\Rightarrow \boxed{H_y = -\frac{j\omega\varepsilon}{h^2} C \sin\left(\frac{n\pi}{b}\right)y \left(\frac{m\pi}{a}\right) \cos\left(\frac{m\pi}{a}\right)x}$$

In total,

$$E_x = -\frac{\gamma}{h^2} C\left(\frac{m\pi}{a}\right) \cos\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y$$

$$E_y = -\frac{\gamma}{h^2} C\left(\frac{n\pi}{b}\right) \sin\left(\frac{m\pi}{a}\right)x \cdot \cos\left(\frac{n\pi}{b}\right)y$$

$$H_x = \frac{j\omega\varepsilon}{h^2} C\left(\frac{n\pi}{b}\right) \sin\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y$$

$$H_y = -\frac{j\omega\varepsilon}{h^2} C\left(\frac{m\pi}{a}\right) \sin\left(\frac{n\pi}{b}\right)y \cos\left(\frac{m\pi}{a}\right)x$$

Steps to be followed:-

① $H_z = 0$ from definition

② $\nabla^2 E_z = -\omega^2 \mu \varepsilon E_z$

$\nabla^2 H_z = -\omega^2 \mu \varepsilon H_z$

$\Rightarrow 0 = -\omega^2 \mu \varepsilon (0) \Rightarrow 0$

③ $\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + h^2 E_z = 0$

④ Variable & separable method: $E_z = XY$

⑤ Boundary conditions of Rectangular waveguide.

⑥ find E_x, E_y, H_x and H_y.

Analysis of TE mode:-

TE wave : $E_z = 0$ and $H_z \neq 0$

We know that

$$\nabla^2 E_z = -\omega^2 \mu \varepsilon E_z \rightarrow ①$$

$$\nabla^2 H_z = -\omega^2 \mu \varepsilon H_z \rightarrow ②$$

The equation-① is cancelled, consider eqn-②

$$\boxed{\nabla^2 H_z = -\omega^2 \mu \varepsilon H_z} \rightarrow ③$$

$$\Rightarrow \frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \frac{\partial^2 H_z}{\partial z^2} = -\omega^2 \mu \varepsilon H_z$$

$$\Rightarrow \frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \gamma^2 H_z + \omega^2 \mu \varepsilon H_z = 0$$

$$\Rightarrow \frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + (\gamma^2 + \omega^2 \mu \varepsilon) H_z = 0$$

$$\Rightarrow \frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + h^2 H_z = 0 \rightarrow (4)$$

Consider, $H_z = XY$

$$\Rightarrow \frac{\partial^2(XY)}{\partial x^2} + \frac{\partial^2(XY)}{\partial y^2} + h^2(XY) = 0$$

$$\Rightarrow Y \cdot \frac{\partial^2 X}{\partial x^2} + X \cdot \frac{\partial^2 Y}{\partial y^2} + h^2(XY) = 0$$

Dividing the above equation with "XY" we get

$$\Rightarrow \frac{1}{X} \cdot \frac{\partial^2 X}{\partial x^2} + \frac{1}{Y} \frac{\partial^2 Y}{\partial y^2} + h^2 = 0$$

Let, $\frac{1}{X} \cdot \frac{\partial^2 X}{\partial x^2} = -B^2$

$\frac{1}{Y} \frac{\partial^2 Y}{\partial y^2} = -A^2$

$$\Rightarrow -B^2 - A^2 + h^2 = 0$$

$$\Rightarrow h^2 = A^2 + B^2 \rightarrow (5)$$

$$X = (C_1 \cos Bx + C_2 \sin Bx) \rightarrow (6)$$

$$Y = (C_3 \cos Ay + C_4 \sin Ay) \rightarrow (7)$$

We have, $H_z = XY$

$$\therefore H_z = (C_1 \cos Bx + C_2 \sin Bx)(C_3 \cos Ay + C_4 \sin Ay) \rightarrow (8)$$

Boundary conditions:-

Bottom wall:-

$E_x = 0$ for $y = 0$ $\forall$ $x = 0$ to a

Leftside wall:-

$E_y = 0$ for $x = 0$ $\forall$ $y = 0$ to b

TOP wall:-

$E_x = 0$ for $y = b$ $\forall$ $x = 0$ to a

Rightside wall:-

$E_y = 0$ for $x = a$ $\forall$ $y = 0$ to b

($\bar{E}$ on the walls of a R.waveguide is zero but $\bar{H}$ is not equal to zero)

(If students sitting inside a class represent $\bar{E}$ then noise coming out of the class is $\bar{H}$, though $\bar{E}$ is not on the walls, $\bar{H}$ exists)

Substituting ① st boundary condition in eqn-⑧

before this, we have to find E_x component

We know that

$$E_x = -\frac{\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}$$

$$\Rightarrow E_x = \frac{-j\omega\mu}{h^2}\frac{\partial H_z}{\partial y} \quad (\because E_z = 0 \text{ for TE wave})$$

Substituting 'E_x' value in eqn-⑧ we get

$$E_x = \frac{-j\omega\mu}{h^2}\Big[(C_1\cos Bx + C_2\sin Bx)(AC_3\sin Ay - AC_4\cos Ay)\Big]$$

$$0 = \frac{-j\omega\mu}{h^2}\underbrace{(C_1\cos Bx + C_2\sin Bx)}_{\neq 0}(-AC_4)$$

$(\because x$ is variable$)$

$$\therefore \boxed{C_4 = 0}$$

now eqn-⑧ becomes,

$$H_z = (C_1 \cos Bx + C_2 \sin Bx)(C_3 \cos Ay) \longrightarrow ⑨$$

Substituting 2nd boundary condition in eqn-⑨

We know that

$$E_y = -\frac{\gamma}{h^2}\frac{\partial E_z}{\partial y} + \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x}$$

$$\Rightarrow E_y = \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x}$$

$$\Rightarrow E_y = \frac{j\omega\mu}{h^2}\left[(-BC_1 \sin B + BC_2 \cos Bx)(C_3 \cos Ay)\right]$$

$$\Rightarrow 0 = \frac{j\omega\mu}{h^2}(+C_2 B)\underbrace{(C_3 \cos Ay)}_{\neq 0}$$

$$\Rightarrow +C_2 B = 0$$

$$\Rightarrow \boxed{C_2 = 0}$$

now eqn-⑨ becomes,

$$H_z = (C_1 \cos Bx)(C_3 \cos Ay) \longrightarrow ⑩$$

Substituting ③rd boundary condition in eqn-⑩

We know that

$$E_x = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}$$

$$\Rightarrow E_x = \frac{-j\omega\mu}{h^2}\left[(C_1 \cos Bx)(A C_3 \sin Ay)\right]$$

$$\Rightarrow 0 = \frac{-j\omega\mu}{h^2}\left[\underbrace{(C_1 \cos Bx)}_{\neq 0}(A C_3 \sin Ab)\right]$$

$\Rightarrow \quad A C_3 \sin Ab = 0$

$\Rightarrow \quad \sin Ab = 0$

$\Rightarrow \quad Ab = \pm m\pi$

$\Rightarrow \quad \boxed{A = \frac{m\pi}{b}}$

Now, eqn-(10) becomes,

$$H_z = (C_1 \cos Bx)\left(C_3 \cos\left(\frac{m\pi}{b}\right)y\right) \longrightarrow (11)$$

Substituting (4)th boundary condition in eqn-(11)

We know that

$$E_y = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial y} + \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x}$$

$$\Rightarrow 0 = \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x}$$

$$\Rightarrow 0 = \frac{j\omega\mu}{h^2}(-B C_1 \sin Ba)\underbrace{\left(C_3 \cos\left(\frac{m\pi}{b}\right)y\right)}_{\neq 0}$$

$\Rightarrow \quad \sin Ba = 0$

$\Rightarrow \quad Ba = n\pi$

$\Rightarrow \quad \boxed{B = \frac{n\pi}{a}}$

now eqn-(11) becomes

$$H_z = C \cos\left(\frac{n\pi}{a}\right)x \cdot \cos\left(\frac{m\pi}{b}\right)y \quad (\because C = C_1 C_3) \longrightarrow (12)$$

$$E_x = -\frac{\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}$$

$$\Rightarrow E_x = -\frac{\gamma}{h^2}(0) - \frac{j\omega\mu}{h^2}\frac{\partial}{\partial y}\left[C\cos\left(\frac{n\pi}{a}\right)x \cdot \cos\left(\frac{m\pi}{b}\right)y\right]$$

$$\Rightarrow E_x = -\frac{j\omega\mu}{h^2}\left[C\cos\left(\frac{n\pi}{a}\right)x \cdot -\left(\frac{m\pi}{b}\right)\sin\left(\frac{m\pi}{b}\right)y\right]$$

$$\Rightarrow \boxed{E_x = \frac{j\omega\mu}{h^2}\,C\left(\frac{m\pi}{b}\right)\cos\left(\frac{n\pi}{a}\right)x\,\sin\left(\frac{m\pi}{b}\right)y}$$

$$E_y = -\frac{\gamma}{h^2}\frac{\partial E_z}{\partial y} + \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x}$$

$$\Rightarrow E_y = \frac{j\omega\mu}{h^2}\frac{\partial}{\partial x}\left[C\cos\left(\frac{n\pi}{a}\right)x \cdot \cos\left(\frac{m\pi}{b}\right)y\right]$$

$$\Rightarrow E_y = \frac{j\omega\mu}{h^2}\left[C\cos\left(\frac{m\pi}{b}\right)y \cdot -\left(\frac{n\pi}{a}\right)\sin\left(\frac{n\pi}{a}\right)x\right]$$

$$\Rightarrow \boxed{E_y = -\frac{j\omega\mu}{h^2}\,C\left(\frac{n\pi}{a}\right)\cos\left(\frac{m\pi}{b}\right)y\,\sin\left(\frac{n\pi}{a}\right)x}$$

$$H_x = -\frac{\gamma}{h^2}\frac{\partial H_z}{\partial x} + \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial y}$$

$$\Rightarrow H_x = -\frac{\gamma}{h^2}\left[\frac{\partial}{\partial x}\left(C\cos\left(\frac{n\pi}{a}\right)x \cdot \cos\left(\frac{m\pi}{b}\right)y\right)\right]$$

$$\Rightarrow H_x = -\frac{\gamma}{h^2}\left[C\cos\left(\frac{m\pi}{b}\right)y\left(\frac{n\pi}{a}\right)\left(-\sin\left(\frac{n\pi}{a}\right)x\right)\right]$$

$$\Rightarrow \boxed{H_x = \frac{\gamma}{h^2}\left[c\left(\frac{n\pi}{a}\right)\cos\left(\frac{m\pi}{b}\right)y\ \sin\left(\frac{n\pi}{a}\right)x\right]}$$

$$H_y = -\frac{\gamma}{h^2}\frac{\partial H_z}{\partial y} - \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x}$$

$$\Rightarrow H_y = -\frac{\gamma}{h^2}\frac{\partial}{\partial y}\left[c\cos\left(\frac{n\pi}{a}\right)x \cdot \cos\left(\frac{m\pi}{b}\right)y\right]$$

$$\Rightarrow H_y = -\frac{\gamma}{h^2}\left[c\cos\left(\frac{n\pi}{a}\right)x \cdot -\left(\frac{m\pi}{b}\right)\sin\left(\frac{m\pi}{b}\right)y\right]$$

$$\Rightarrow \boxed{H_y = \frac{\gamma}{h^2}\left[c\left(\frac{m\pi}{b}\right)\cos\left(\frac{n\pi}{a}\right)x\ \sin\left(\frac{m\pi}{b}\right)y\right]}$$

Summary:-

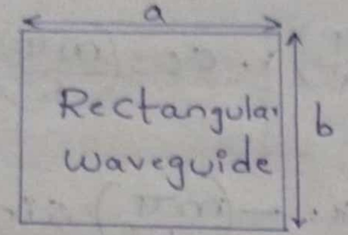
1. Define TE wave
2. Helmholtz wave eqn's
3. $H_z = (C_1\cos Bx + C_2\sin Bx)(C_3\cos Ay + C_4\sin Ay)$

   $H_z = c\cos\left(\frac{n\pi}{a}\right)x \cdot \cos\left(\frac{m\pi}{b}\right)y$
4. Boundary conditions
5. Substituting Boundary conditions in E_x, E_y
6. obtain the field component H_z
7. obtain the field components E_x, E_y, H_x and H_y.

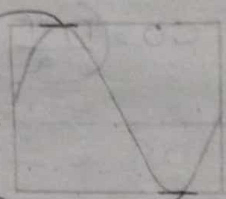
Z

→ When a wave enters into a rectangular waveguide, it follows different patterns and exhibits a wide range of behaviour.

TE$_{mn}$ mode:-

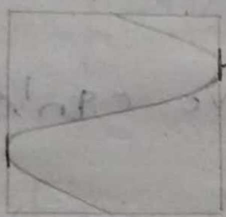


m → no.of halfwave variations along X-direction



only half-wave is touching

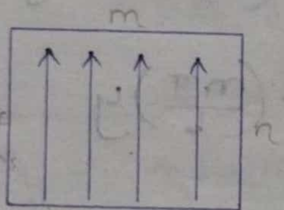
n → no.of halfwave variations along Y-direction



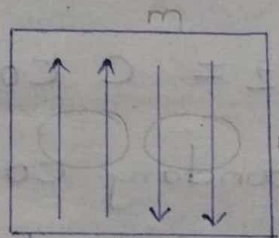
only half-wave is touching

eg:-

single type of variation along m-direction



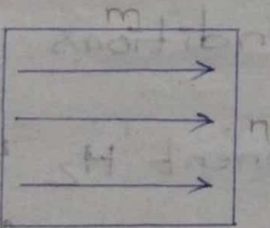
$\equiv TE_{10}$



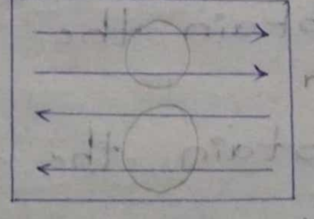
$\equiv TE_{20}$

two types of variations along m-direction

single type of variation along n-direction

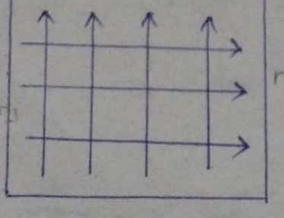


$\equiv TE_{01}$

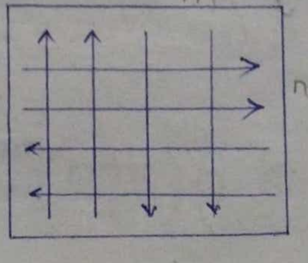


$\equiv TE_{02}$

single type of variation along m&n directions



$\equiv TE_{11}$



$\equiv TE_{22}$

	TE_{10}	TE_{01}	TE_{11}
E_x	×	✓	✓
E_y			
H_x	✓	×	
H_y	×	✓	✓

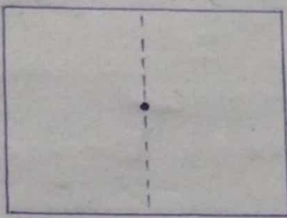
TM_{mn} mode:-

$m \rightarrow$ no.of halfwave variations along X-direction

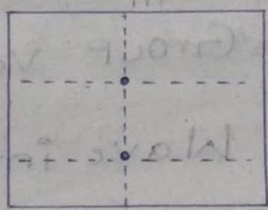
$n \rightarrow$ no.of halfwave variations along Y-direction

eg:-

Single type of variation along m-direction

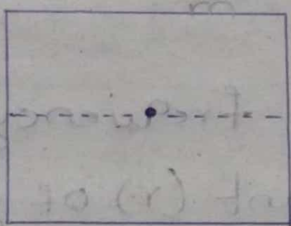


$\equiv TM_{10}$



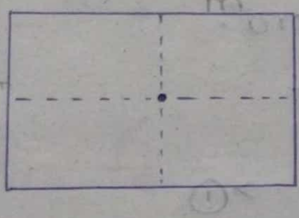
$\equiv TM_{12}$

Single type of variation along n-direction



$\equiv TM_{01}$

Single type of variation along m&n directions



$\equiv TM_{11}$

	TM_{10}	TM_{01}	TM_{11}
E_x	×	×	✓
E_y	×	×	✓
H_x	×	×	✓
H_y	×	×	✓

Note:-

* In TE_{MN} mode, we will refer to electric field.
* In TM_{MN} mode, we will refer to magnetic field.

Characteristics of TE and TM waves in a Rectangular waveguide:-

The following are the characteristics of TE and TM waves in a Rectangular waveguide:

① Cut-off frequency (f_c)

② Cut-off Wavelength (λ_c)

③ Guided Wavelength (λ_g)

④ Phase velocity (V_P)

⑤ Group velocity (V_g)

⑥ Wave impedance (η)

Same for both TE & TM waves

① Cut-off frequency (f_c):-

It is defined as "the frequency at which the propagation constant (γ) of a rectangular waveguide becomes zero".

We know that

$$h^2 = \gamma^2 + \omega^2\mu\varepsilon \longrightarrow ①$$

$$h^2 = A^2 + B^2 \longrightarrow ②$$

from ①; $\gamma^2 = h^2 - \omega^2\mu\varepsilon$

$$= \sqrt{h^2 - \omega^2\mu\varepsilon}$$

$$= \sqrt{A^2 + B^2 - \omega^2\mu\varepsilon}$$

$$= \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\varepsilon} \quad \left(\because A = \pm\frac{m\pi}{a},\ B = \pm\frac{n\pi}{b}\right)$$

At $f = f_c$ (or) $\omega = \omega_c \Rightarrow \boxed{\gamma = 0}$

$$\Rightarrow 0 = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega_c^2 \mu \varepsilon}$$

$$\Rightarrow \omega_c \sqrt{\mu\varepsilon} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$\Rightarrow \omega_c = \frac{1}{\sqrt{\mu\varepsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$\Rightarrow 2\pi f_c = \frac{1}{\sqrt{\mu\varepsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$\Rightarrow f_c = \frac{1}{2\pi\sqrt{\mu\varepsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$\Rightarrow f_c = \frac{c\cancel{\pi}}{2\cancel{\pi}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} \quad \left(\because c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}\right)$$

$$\Rightarrow f_c = \frac{c}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$\therefore \boxed{f_c = \frac{c}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$$

TE_{10}:- $f_c = \frac{c}{2}\sqrt{\left(\frac{1}{a}\right)^2 + 0} = \frac{c}{2a}$

TE_{01}:- $f_c = \frac{c}{2}\sqrt{0 + \left(\frac{1}{b}\right)^2} = \frac{c}{2b}$

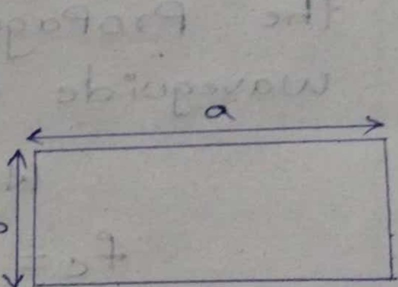
TE_{11}:- $f_c = \frac{c}{2}\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2}$

TM_{10}:- doesnot exist

TM_{01}:- doesnot exist

TM_{11}:- $f_c = \frac{c}{2}\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2}$

Dominant mode:-



Consider a rectangular waveguide with width 'a' and height 'b'.

For all Rectangular waveguides width is always greater than height i.e., $(a > b)$. The mode in which the cut-off frequency (f_c) is less (or minimum is referred to as "Dominant mode". From the derived expressions, it is clear that 'f_c' incase of TE_{10} mode and TM_{11} mode are/is minimum. Hence, TE_{10} mode and TM_{11} modes are referred to as Dominant modes. When You're given dominant mode indirectly You are Provided with the values of m & n.

Degenerative modes:-

Modes whose cut-off frequencies are same are referred to as Degenerative modes.

eg:- TE_{12} & TE_{21}

Note:- For a Wave to enter into a rectangular Waveguide, 'f' should be greater than 'f_c'
i.e., $\boxed{f > f_c}$

Here, $f \rightarrow$ frequency of wave

$f_c \rightarrow$ cut-off frequency

② cut-off Wavelength (λ_c):-

It is defined as "the wavelength at which the Propagation constant (γ) of a rectangular waveguide becomes zero".

We have,

$$f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$\Rightarrow \frac{c}{\lambda_c} = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} \qquad \left(\because \begin{array}{l} c = f\lambda \\ f = c/\lambda \end{array}\right)$$

$$\Rightarrow \lambda_c = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}}$$

$$\therefore \boxed{\lambda_c = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}}}$$

note:-

$f > f_c \rightarrow$ Wave Propagation exists

$\lambda < \lambda_c \rightarrow$ " " "

③ Guided wavelength(λ_g):-

It is defined as "the distance travelled by a wave, inorder to produce a phaseshift of 360° (or) 2π radians".

∴ We know that

$$\beta = \frac{2\pi}{\lambda}$$

$$\Rightarrow \lambda = \frac{2\pi}{\beta}$$

$$\Rightarrow \lambda_g = \frac{2\pi}{\beta} \longrightarrow ①$$

We also know that,

$$h^2 = \gamma^2 + \omega^2\mu\varepsilon \longrightarrow ②$$

$$h^2 = A^2 + B^2 \longrightarrow ③$$

$$\gamma = \sqrt{\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\varepsilon}$$

$$\Rightarrow \alpha + j\beta = \sqrt{\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\varepsilon}$$

if $\alpha = 0$ then,

$$\Rightarrow j\beta = \sqrt{\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\varepsilon}$$

At $\omega = \omega_c \Rightarrow \gamma = 0$

from ② we can write

$$h^2 = \omega_c^2 \mu \varepsilon \longrightarrow ④$$

from eqn's - ② and ③

$$j\beta = \sqrt{\omega_c^2 \mu\varepsilon - \omega^2 \mu\varepsilon}$$

$$\Rightarrow j\beta = \sqrt{-\omega^2 \mu\varepsilon \left(1 - \frac{\omega_c^2}{\omega^2}\right)}$$

$$\Rightarrow j\beta = j\omega\sqrt{\mu\varepsilon}\sqrt{1 - \frac{\omega_c^2}{\omega^2}}$$

$$\Rightarrow \beta = \omega\sqrt{\mu\varepsilon}\sqrt{1 - \left(\frac{2\pi f_c}{2\pi f}\right)^2}$$

$$\Rightarrow \beta = \omega\sqrt{\mu\varepsilon}\sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

$$\Rightarrow \beta = \frac{2\pi f}{c}\sqrt{1 - \left(\frac{f_c}{f}\right)^2} \qquad \left(\because \begin{array}{l} \omega = 2\pi f \\ c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \end{array}\right)$$

$$\therefore \boxed{\beta = \frac{2\pi f}{c}\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

from ①; $\lambda_g = \frac{2\pi}{\beta}$

$$\Rightarrow \lambda_g = \frac{\cancel{2\pi}}{\frac{\cancel{2\pi} f}{c}\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

$$\Rightarrow \lambda_g = \frac{c}{f\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

$$\Rightarrow \lambda_g = \frac{\lambda_0}{\sqrt{1 - \left(\frac{\lambda_0}{\lambda_c}\right)^2}}$$

$$\boxed{\begin{array}{l} c = f\lambda \\ \Rightarrow f = \frac{c}{\lambda} \\ \text{(or)} \\ \lambda = \frac{c}{f} \end{array}}$$

$$\therefore \boxed{\lambda_g = \frac{\lambda_0}{\sqrt{1 - \left(\frac{\lambda_0}{\lambda_c}\right)^2}}}$$

Relation between λ_g, λ_0 and λ_c :-

We have,

$$\lambda_g = \frac{\lambda_0}{\sqrt{1-(\lambda_0/\lambda_c)^2}}$$

$$\Rightarrow \sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2} = \frac{\lambda_0}{\lambda_g}$$

$$\Rightarrow 1-\left(\frac{\lambda_0}{\lambda_c}\right)^2 = \left(\frac{\lambda_0}{\lambda_g}\right)^2$$

$$\Rightarrow 1 = \left(\frac{\lambda_0}{\lambda_g}\right)^2 + \left(\frac{\lambda_0}{\lambda_c}\right)^2 = \lambda_0^2\left[\frac{1}{\lambda_g^2}+\frac{1}{\lambda_c^2}\right]$$

$$\Rightarrow \frac{1}{\lambda_0^2} = \frac{1}{\lambda_g^2} + \frac{1}{\lambda_c^2}$$

$$\therefore \boxed{\frac{1}{\lambda_0^2} = \frac{1}{\lambda_g^2} + \frac{1}{\lambda_c^2}}$$

note:-

$f > f_c$ & $\lambda < \lambda_c \longrightarrow$ WavePropagation exists

We have, $\lambda_g = \dfrac{\lambda_0}{\sqrt{1-(\lambda_0/\lambda_c)^2}}$

$$\boxed{\lambda_g > \lambda_0}$$

④ Phase velocity (v_p):-

It is defined as "the rate at which the wave changes its Phase".

(Or)

It is the velocity with which the Phase of a wave changes.

We know that

$$\lambda_g = v_p t = \frac{v_p}{f}$$

$$\Rightarrow \frac{2\pi}{\beta} = \frac{v_p}{f}$$

$$\Rightarrow V_P = \frac{2\pi f}{\beta}$$

$$\Rightarrow V_P = \frac{\omega}{\beta}$$

$$\Rightarrow V_P = \frac{\omega}{\omega\sqrt{\mu\varepsilon}\sqrt{1-\frac{\omega_c^2}{\omega^2}}}$$

$$\Rightarrow V_P = \frac{1}{\sqrt{\mu\varepsilon}\sqrt{1-\omega_c^2/\omega^2}}$$

$$\Rightarrow V_P = \frac{c}{\sqrt{1-(f_c/f)^2}}$$

$$\Rightarrow V_P = \frac{c}{\sqrt{1-(\lambda_0/\lambda_c)^2}}$$

$$\therefore \boxed{V_P = \frac{c}{\sqrt{1-(\lambda_0/\lambda_c)^2}}}$$

note:-

$f > f_c \rightarrow$ Wave Propagation exist

$V_P > c \rightarrow$ Phase velocity is always greater than velocity of light.

⑤ Group velocity (V_g):-

It is defined as " the rate at which a wave Propagates through a rectangular waveguide".

$$\boxed{V_g = \frac{d\omega}{d\beta}}$$

We know that

$$\beta = \sqrt{\omega_c^2\mu\varepsilon - \omega^2\mu\varepsilon}$$

$$\Rightarrow \frac{d\beta}{d\omega} = \frac{1}{\cancel{2}\sqrt{\omega_c^2\mu\varepsilon - \omega^2\mu\varepsilon}} \cdot \cancel{2}\omega\mu\varepsilon$$

$$\Rightarrow \frac{d\beta}{d\omega} = \frac{\omega\mu\varepsilon}{\sqrt{\omega_c^2\mu\varepsilon - \omega^2\mu\varepsilon}}$$

$$\Rightarrow \frac{d\omega}{d\beta} = \frac{\cancel{\omega}\cancel{\sqrt{\mu\varepsilon}}\sqrt{1 - \frac{\omega_c^2}{\omega^2}}}{\cancel{\omega}\cancel{\mu\varepsilon}\sqrt{\mu\varepsilon}}$$

$$\Rightarrow \frac{d\omega}{d\beta} = \frac{1}{\sqrt{\mu\varepsilon}}\sqrt{1 - \frac{\omega_c^2}{\omega^2}}$$

$$\Rightarrow \frac{d\omega}{d\beta} = c\sqrt{1 - \frac{\omega_c^2}{\omega^2}} = c\sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

$$\therefore \boxed{\frac{d\omega}{d\beta} = c\sqrt{1 - \left(\frac{f_c}{f}\right)^2} \qquad \frac{d\omega}{d\beta} = c\sqrt{1 - \left(\frac{\lambda_0}{\lambda_c}\right)^2}}$$

$$\boxed{V_p \cdot V_g = c^2}$$

note:- $f > f_c \longrightarrow$ wave propagation exists

$V_p >> v_c \longrightarrow$ Phase velocity is always greater than light velocity.

$V_g << v_c \longrightarrow$ group velocity is always less than light velocity.

⑥ Impedance(η):-

→ It is different for TE wave and TM wave.

→ It is defined as "the ratio of strength of electric field to the strength of magnetic field."

$$\boxed{\eta = \frac{E_x}{H_y} = -\frac{E_y}{H_x}}$$

For TE wave

$$\eta_{TE} = \frac{-\frac{\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}}{-\frac{\gamma}{h^2}\frac{\partial H_z}{\partial y} - \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x}}$$

$$\Rightarrow \eta_{TE} = \frac{\cancel{-}\frac{j\omega\mu}{\cancel{h^2}}\cancel{\frac{\partial H_z}{\partial y}}}{\cancel{-}\frac{\gamma}{\cancel{h^2}}\cancel{\frac{\partial H_z}{\partial y}}} \quad \left(\because E_z = 0 \text{ for a TE wave}\right)$$

$$\Rightarrow \eta_{TE} = \frac{j\omega\mu}{\gamma}$$

$\gamma = \alpha + j\beta$

if $\alpha = 0$ then $\gamma = j\beta$

now, $\eta_{TE} = \frac{j\omega\mu}{j\beta} = \frac{\omega\mu}{\beta} = \frac{\omega\mu\sqrt{\mu}}{\omega\sqrt{\mu\varepsilon}\sqrt{1-(\omega_c^2/\omega^2)}}$

$$\Rightarrow \eta_{TE} = \frac{\sqrt{\frac{\mu}{\varepsilon}}}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$\Rightarrow \eta_{TE} = \frac{\eta}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}$$

$$\therefore \boxed{\eta_{TE} = \frac{\eta}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}}$$

Here, $\eta \rightarrow$ freespace impedance $= \sqrt{\frac{\mu_0}{\varepsilon_0}} = \sqrt{\frac{4\pi\times10^{-7}}{8.854\times10^{-12}}}$

$= 120\pi$ (or 377Ω)

note:-

$f > f_c$ then $\boxed{\eta_{TE} > \eta}$

For TM Wave:-

$$\eta_{TM} = \frac{-\frac{\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}}{-\frac{\gamma}{h^2}\frac{\partial H_z}{\partial y} - \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x}}$$

$$\Rightarrow \eta_{TM} = \frac{\cancel{-}\frac{\gamma}{\cancel{h^2}}\cancel{\frac{\partial E_z}{\partial x}}}{\cancel{-}\frac{j\omega\varepsilon}{\cancel{h^2}}\cancel{\frac{\partial E_z}{\partial x}}}$$

$(\because H_z = 0$ for a TM wave$)$

$$\Rightarrow \eta_{TM} = \frac{\gamma}{j\omega\varepsilon}$$

$$\Rightarrow \eta_{TM} = \frac{\cancel{j}\beta}{\cancel{j}\omega\varepsilon} = \frac{\beta}{\omega\varepsilon} = \frac{\cancel{\omega}\sqrt{\mu\varepsilon}\sqrt{1-\frac{\omega_c^2}{\omega^2}}}{\cancel{\omega}\varepsilon}$$

$$\Rightarrow \eta_{TM} = \sqrt{\frac{\mu}{\varepsilon}}\sqrt{1-\frac{\omega_c^2}{\omega^2}}$$

$$\Rightarrow \eta_{TM} = \eta\sqrt{1-\left(\frac{f_c}{f}\right)^2} = \eta\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}$$

$$\therefore \boxed{\eta_{TM} = \eta\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}$$

note:-

$f > f_c$ then $\boxed{\eta_{TM} < \eta}$

For TEM wave:-

$$\eta_{TEM} = \frac{-\frac{\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}}{-\frac{\gamma}{h^2}\frac{\partial H_z}{\partial y} - \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x}}$$

$$\Rightarrow \eta_{TEM} = \frac{0}{0} = 0 \quad (\because E_z = H_z = 0 \text{ for a TEM wave})$$

But You are not supposed to write, $\eta_{TEM} = 0$. The impedance of a TEM wave is equal to freespace impedance.

$$\therefore \boxed{\eta_{TEM} = \eta} = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi \text{ (or) } 377\,\Omega$$

Summary:-

1. $f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$

2. $\lambda_c = \frac{2}{\sqrt{(m/a)^2 + (n/b)^2}}$

3. $\lambda_g = \frac{\lambda_0}{\sqrt{1-(\lambda_0/\lambda_c)^2}}$

4. $\frac{1}{\lambda_0^2} = \frac{1}{\lambda_g^2} + \frac{1}{\lambda_c^2}$

5. $V_p = \frac{c}{\sqrt{1-(\lambda_0/\lambda_c)^2}}$

6. $V_g = \frac{d\omega}{d\beta} = c\sqrt{1-\left(\frac{f_c}{f}\right)^2}$ (or) $c\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}$

7. $\eta_{TE} = \frac{\eta}{\sqrt{1-(\lambda_0/\lambda_c)^2}}$

8. $\eta_{TM} = \eta\sqrt{1-(\lambda_0/\lambda_c)^2}$

Power Transmission in Rectangular waveguides:-

→ Assume that a Rectangular waveguide is terminated in such a way that there is no reflection of energy from its walls.

→ The waveguide is infinitely long compared to wavelength.

$$P_{tr} = \oint P \, ds \rightarrow (1)$$

Here, P_{tr} --> transmitted power

From Power Poyinting theorem

$$P = \bar{E} \times H^* \rightarrow (2)$$

from (1) and (2); $P_{tr} = \oint \bar{E} \times H^* . ds \rightarrow (3)$

Always we have to consider average Power.

$$\therefore P_{avg} = \frac{1}{2} P_{tr}$$

$$P_{avg} = \frac{1}{2} \oint \bar{E} \times H^* ds \rightarrow (4)$$

We know that

$$\eta = \frac{E_x}{H_y} \Rightarrow H_y = \frac{E_x}{\eta} \rightarrow (5)$$

from (3) and (5);

$$P_{tr} = \oint \frac{|E|^2}{\eta} ds$$

(or)

from (4) and (5);

$$\boxed{P_{avg} = \frac{1}{2} \oint \frac{|E|^2}{\eta} ds = \frac{1}{2} \oint \frac{|H|^2}{\eta} ds \quad \text{(or)} \quad P_{avg} = \frac{1}{2} \iint \frac{|E|^2}{\eta} dx \, dy = \frac{1}{2} \iint \frac{|H|^2}{\eta} dx \, dy}$$

For TE wave:-

$$P_{avg.} = \frac{1}{2}\iint \frac{|E|^2}{\eta_{TE}}\, dx\, dy$$

$$= \frac{1}{2}\iint \frac{|E|^2}{\frac{\eta}{\sqrt{1-(\lambda_0/\lambda_c)^2}}}\, dx\, dy$$

$$= \frac{1}{2}\iint \frac{|E|^2}{\eta} \cdot \sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}\, dx\, dy$$

For TM wave:-

$$P_{tr} = \frac{1}{2}\iint \frac{|E|^2}{\eta_{TM}}\, dx\, dy$$

$$P_{tr} = \frac{1}{2}\iint \eta_{TM} |H|^2\, dx\, dy$$

$$= \frac{1}{2}\iint \eta \sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}\, |H|^2\, dx\, dy$$

PowerLosses in Rectangular waveguide:-

Consider a Rectangular waveguide. There are two types of Losses:

(i) Losses in the guided walls

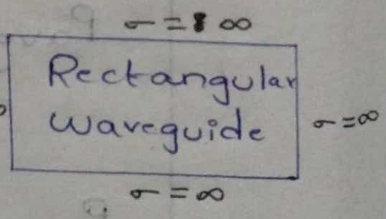
(ii) Losses due to dielectric material

(i) Losses in the guided walls:-

Powerloss may occur at conducting (guided) walls of the waveguide. If the conductivity of the guided walls of the waveguide is infinite, there there would be no Power loss. There would be no reflection of energy.

But ideally, this is not Possible.

if $\sigma = \infty$, then from ohm's law



$$V = IR$$

$$\Rightarrow I = \frac{V}{R}$$

$$\Rightarrow I = \frac{V}{0} = \infty$$

If the conductivity is less than infinite(∞), there would be some Power loss in the waveguide

(ii) Losses due to dielectric material:- (Perfect dielectric)

A Rectangular waveguide is a hollow metallic tube, consisting of two Parallel conducting plates in between which the space is filled with air. Air acts as a dielectric medium. There will be some loss in Power due to this dielectric medium.

It is know that, for a wave Propagate in a rectangular waveguide, $f > f_c$. Suppose that

$f < f_c$:-

Propagation Constant $(\gamma) = \alpha + j\beta$

When $f < f_c$, the imaginary Part ($j\beta$) gets Vanished and the wave is said to be fully attenuated.

i.e., $\boxed{\gamma = \alpha}$

We know that

$$h^2 = \gamma^2 + \omega^2\mu\varepsilon$$

$$\Rightarrow \gamma^2 = h^2 - \omega^2\mu\varepsilon$$

$$\Rightarrow \gamma = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\varepsilon}$$

$$\Rightarrow \gamma = Re\left\{\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\varepsilon}\right\} \equiv \alpha$$

for $f < f_c$, $\boxed{\alpha = \frac{54.6}{\lambda_c}}$

$f > f_c$:-

Wave Propagation exists through the Rectangular waveguide.

For a dielectric material,

$$\text{Loss tangent} = \left|\frac{\sigma}{\omega\varepsilon}\right| << 1$$

$$\Rightarrow \boxed{\sigma << \omega\varepsilon}$$

Attenuation due to dielectric medium is given by,

$$\alpha_d = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$$

$$\Rightarrow \alpha_d = \frac{\sigma}{2}\eta$$

$$\therefore \boxed{\alpha_d = \frac{\sigma}{2}(\eta)}$$

Here, $\eta \rightarrow$ freespace impedance $= 377\Omega$ (or) 120π

<u>PowerLoss due to imperfect dielectric:-</u> (Good dielectric)

$$\alpha_g = \frac{P_L}{2P_{tr}}$$

$$\Rightarrow \alpha_g = \frac{R_s \oint_s |H|^2 ds}{2\eta_g \oint_s |H|^2 ds}$$

$$\therefore \boxed{\alpha_g = \frac{R_s \oint_s |H|^2 ds}{2\eta_g \oint_s |H|^2 ds}}$$

Here, α_g --> Attenuation due to guided walls

P_{tr} --> Transmitted Power

$P_L \rightarrow$ Path Loss - Losses occuring in the Pathe followed by wave in the waveguide

03/05/21

UNIT III :- Microwave tubes

Limitations and Losses of conventional tubes at microwave frequencies:-

conventional tubes:-

The triodes, Pentodes and tetrodes are known as Conventional tubes. These tubes are only useful at low microwave frequencies. The Vaccum tube Was the first active electronic device, capable of actually controlling and amplifying a small signal. Small vaccum tubes were available for microwave and millivolt signals but have been replaced by transistors.

Limitations of conventional tubes at microwave frequencies:-

→ The size of electronic devices required for generation of microwave energy, becomes very smaller at microwave frequencies.

→ Because of small size, these devices increased the noise levels and results in lesser Power handling capacity.

→ So, at the microwave frequencies, the microwave tubes are used because they can provide higher output Power, lesser noise, better reliability with reduced output Power levels.

→ Due to some characteristics, the conventional tubes and transistors are not used at high frequencies, as mentioned below:

a) interelectrode capacitances

b) Lead inductance effect

c) Gain bandwidth limitation

d) Transit time effect

e) Skin effect

f) Dielectric Losses

a) Interelectrode capacitance:-

→ The figure below shows the interelectrode capacitance between the grid and the cathode (C_{gk}) in parallel with the signal source.

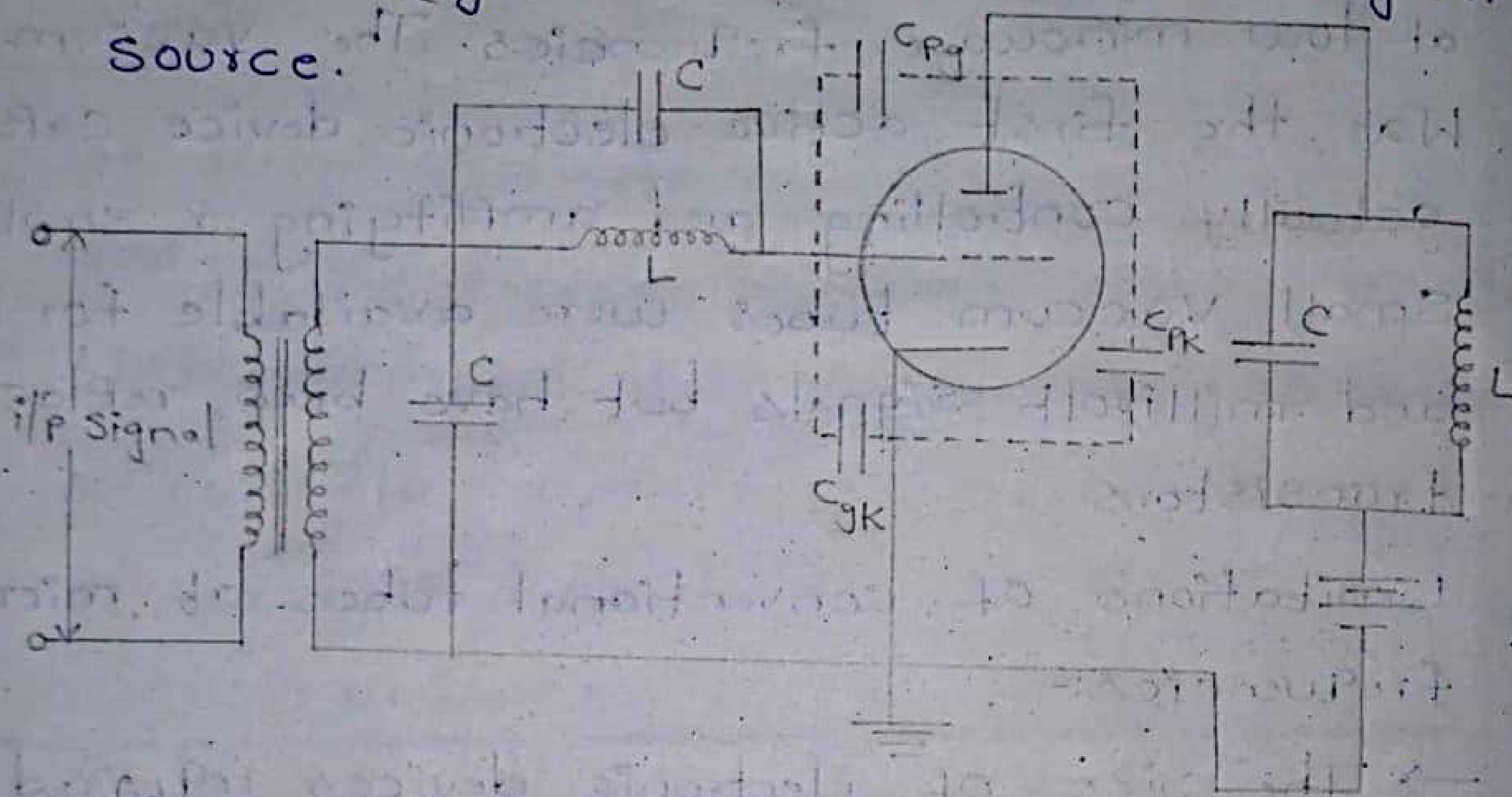


fig:- Interelectrode capacitance

→ The reactance of the capacitor is given by the relation:

$$\boxed{X_c = \frac{1}{2\pi f C}}$$

→ As the interelectrode capacitance decreases, the reactance of the interelectrodes increases.

→ As the frequency of the input signal increases, the effective grid to cathode impedance of the tube decreases because of a decrease in the reactance of the interelectrode capacitance.

→ When the signal frequency is greater than 100MHz, then the reactance of the grid to cathode capacitance is so small that much of the signal is short-circuited with the tube.

→ Since the electrode capacitances are effectively in Parallel With the tuned circuits, as shown in the above circuit, they will also affect the frequency at which the tuned circuit resonates.

→ This effect is minimized by using the smaller electrodes and by increasing the distance between electrodes.

b) Lead inductance effect:-

→ The lead inductances within a tube are effectively in Parallel with the interelectrode capacitances.

→ The reactance of the inductor is given by the relation:

$$\boxed{X_L = 2\pi f L}$$

→ As the lead inductance increases, the reactance of the circuit also increases.

→ This effect raise the frequency limit of the tube.

→ The inductance of cathode lead is common to both the grid and Plate circuits.

→ This Provides a Path for degenerative feedback which reduces the overall circuit efficiency.

→ This effect is minimized by using the larger sized short leads without base pins.

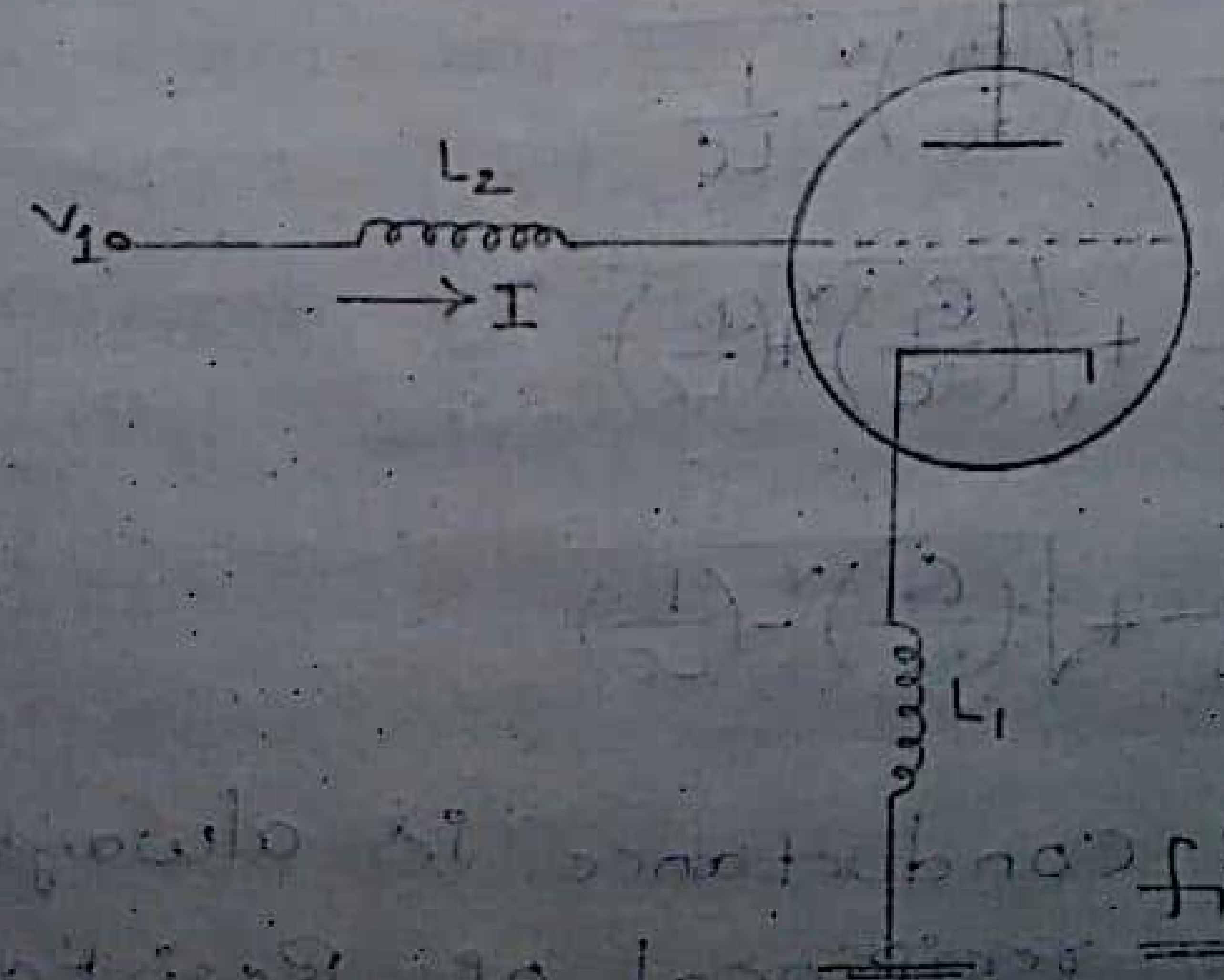


fig:- Lead inductance

(iii) Gain bandwidth limitation:-

→ To achieve the maximum gain, the vaccum tubes generally use the circuit as shown below:

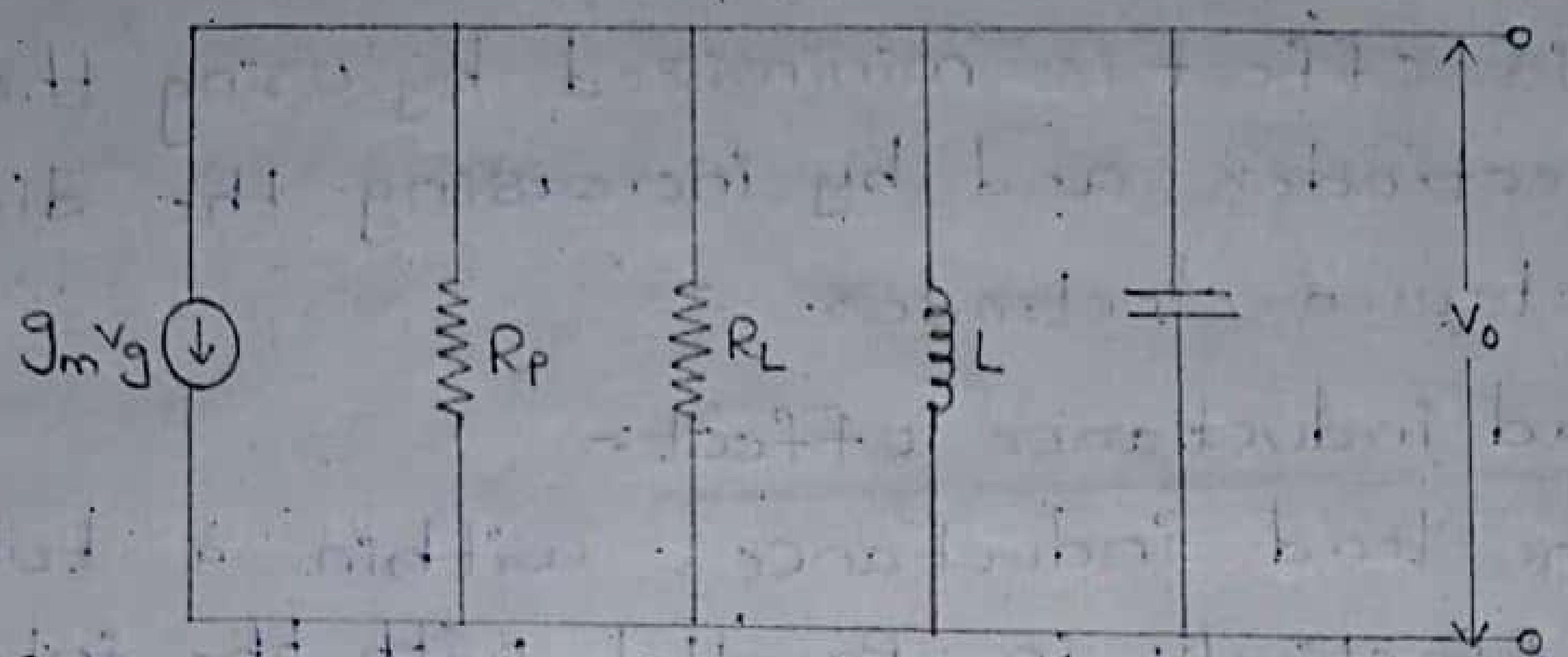


fig:- Equivalent circuit

→ Replacing R_P and R_L by R.

$$R = \frac{1}{R_P} + \frac{1}{R_L}$$

$$G = \frac{V_0(S)}{V_i(S)} = Z_0(S)$$

$$\frac{1}{Z_0(S)} = Y_0(S) = CS + \frac{1}{LS} + \frac{1}{R} = \frac{S^2LCR + LS + R}{RLS}$$

$$\Rightarrow Z_0(S) = \frac{S/C}{S^2 + \frac{3S}{CR} + \frac{1}{LC}}$$

→ From the characteristic equation of the denominator, the roots give the values of lowest and highest and frequencies.

$$\omega_1 = -\frac{G}{2C} - \sqrt{\left(\frac{G}{2C}\right)^2 - \frac{1}{LC}}$$

$$\omega_h = -\frac{G}{2C} + \sqrt{\left(\frac{G}{2C}\right)^2 + \left(\frac{-1}{LC}\right)}$$

$$= -\frac{G}{2C} + \sqrt{\left(\frac{G}{2C}\right)^2 - \left(\frac{1}{LC}\right)}$$

$G = \frac{1}{R}$ ($\because$ conductance is always the reciprocal of Resistance)

$$\text{Bandwidth} = \omega_n - \omega_1 = \frac{G}{C} \text{ where } \left(\frac{G}{2C}\right)^2 \gg \frac{1}{LC}.$$

The maximum gain at resonance is given by,

$$A_{max} = \frac{g_m}{G}$$

$\therefore$ Gain bandwidth Product = $A_{max} \cdot$ Bandwidth

$$= \frac{g_m}{G} \times \frac{G}{C}$$

$$= \frac{g_m}{C}$$

→ As shown in the above relation, the Gain-Bandwidth Product is independent of frequency.

→ Higher gain for a given tube is achieved only by using the narrow bandwidth.

→ This restriction is applicable only to its resonant circuit.

→ To obtain an overall high gain over a broad bandwidth, in microwave devices, slow wave structures are used.

(d) Transit time effect :-

→ Transit time is the time required for electrons to travel from the cathode to the anode plate.

→ If we consider the circuit of a simple vaccum tube as shown in the figure, where 'd' is the distance between two plates, 'i_p' is the plate current, 'V' is applied input voltage, 'V_0' is the output voltage.

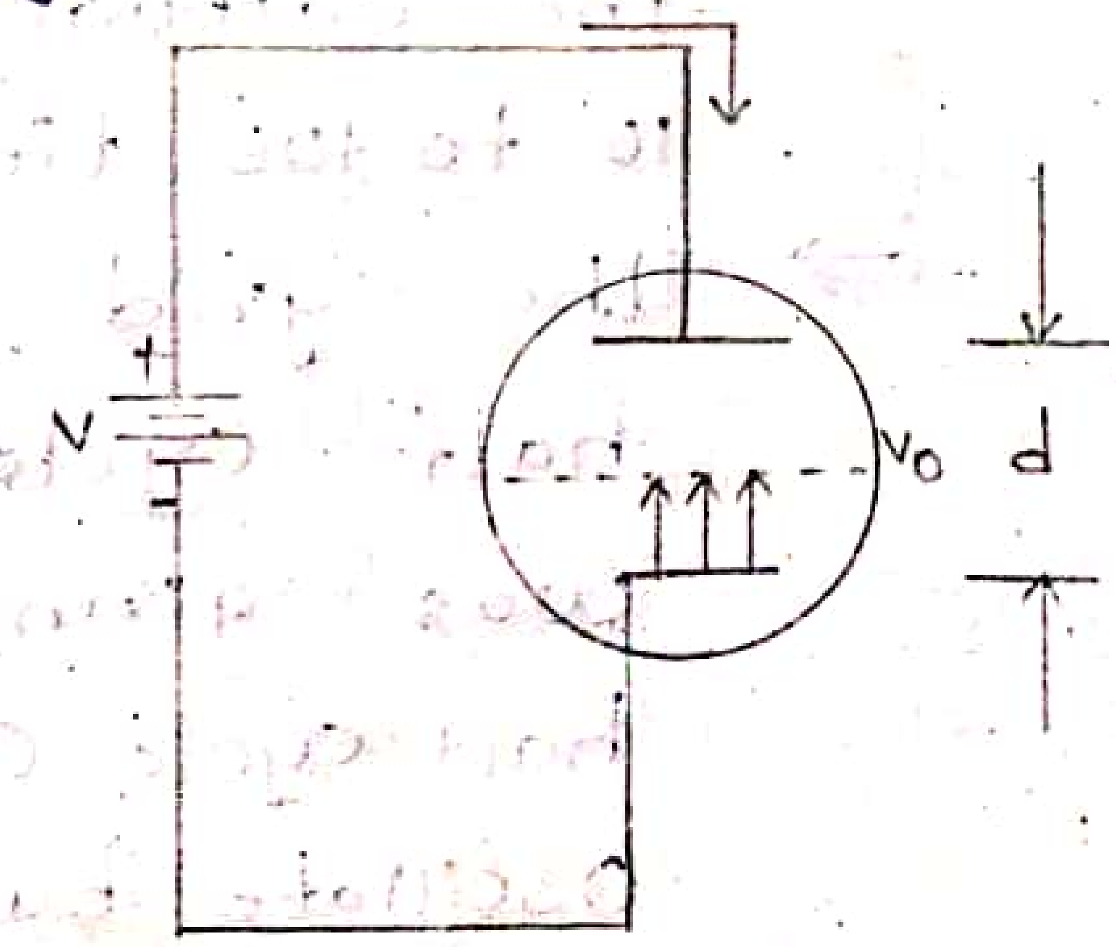


fig:- Transit time effect

Calculation for Transit Time

By definition, transit time is given by:

$$\boxed{\tau = \frac{d}{v_0}}$$ Where v_0 is the velocity of e^-.

Static energy of electrons = eV

Kinetic energy of electrons = eV

Kinetic energy of electrons = $\frac{1}{2}mv_0^2$

We know that, under equilibrium state, the static energy of electrons is equal to the kinetic energy of electrons.

$$eV = \frac{1}{2}mv_0^2$$

$$\Rightarrow v_0 = \sqrt{\frac{2eV}{m}}$$

$$\therefore \boxed{\tau = \frac{d}{\sqrt{\frac{2eV}{m}}}}$$

→ At low frequencies, the transit time is negligible because distance between anode and cathode is very small.

→ But at higher frequencies, the transit time is large as compared to the period of microwave signal. The potential between the cathode and grid may alternate from 10 to 100 times during the electron transmit.

→ The grid potential during the negative half cycle thus removes energy that was given to the electron during positive half cycle. Consequently, the electrons may oscillate back and forth in the cathode grid space (or) return to the cathode.

→ The overall effect/result of transit time effect is to reduce the overall efficiency of the Vaccume tube.

→ To minimise this effect the separation between electrodes can be decreased and the plate to cathode Potential 'V' can be increased.

(c) Skin effect:-

→ This effect introduces at high frequencies, when the current flows from small cross-sectional area to outer surface of the conductor.

→ As given in the figure below, "S" is the skindepth (wall thickness of the conductor) and A_{eff} is the effective area over which the current flows.

fig:-

$$\text{Skindepth} = \delta = \sqrt{2/\omega\mu\sigma}$$

$$\delta \propto \frac{1}{\sqrt{\omega}} \quad \text{and}$$

$$\delta \propto A_{eff}$$

$$A_{eff} \propto \frac{1}{\sqrt{f}}$$

Resistance is given by the relation,

$$R = \frac{\rho l}{A_{eff}}$$

$$R = \rho l \cdot \sqrt{f}$$

As the frequency increases the resistance of the conductor increases, due to the higher frequency losses are produced.

(f) Dielectric losses :-

→ These are different insulating materials which are used as a glass envelope, silicon Plastic encapsulations in different microwave devices. The loss in any of these material is in general related to Power loss and is given by,

$$\boxed{P = \pi f \cdot V_0^2 \varepsilon_r \tan(\sigma)}$$

Where ε_r → Relative Permitivity of dielectric

δ → skin depth

P → Power loss

$\tan(\sigma)$ → Loss angle of dielectric

→ At higher frequencies, the Power loss increases. To eliminate these losses the surface area of glass should be decreased and the tube base should be eliminated.

Re-entrant cavity Resonators :-

→ At frequencies above 3MHz, transistor-based oscillators and amplifiers become obsolete due to the "skin effect" and "stray reactances".

→ To efficiently generate oscillations and amplification at higher frequencies, cavity resonators are used instead.

→ Increased bandwidth is the main advantage of re-entrant cavity resonators.

Now, Let us see about cavity resonators.

What are Cavity Resonators:-

→ Cavity resonators are hollow, closed compartments made of conducting material.

→ RF signals are given as input and output within the compartment through input and output ports.

→ The compartment is analogous to an inductor and its mouth acts as the capacitor for radio frequencies.

→ There are several types of cavity resonators, characterized based on their structure and function:

- Regulated cavity resonators
- Unregulated cavity resonators
- Co-axial cavity resonators
- Capacitive cavity resonators
- Waveguide cavity resonators
- Re-entrant cavity resonators

→ Re-entrant cavity resonators are used for oscillation filtering and amplification in the 3MHz - 300MHz frequency range.

The Structure of a Re-entrant Cavity Resonator

→ A re-entrant cavity resonator is made from two cavity resonators connected perpendicularly by another rectangular waveguide at both ends. Increased bandwidth is the main advantage of the re-entrant cavity resonator, which makes this type of resonator applicable as a wide-band amplifier and oscillator in the frequency range of 3MHz to 300MHz.

→ Efficient energy transfer occurs from the election beam to the high-quality factor cavity resonator when electrons cross the cavity field region in minimum time.

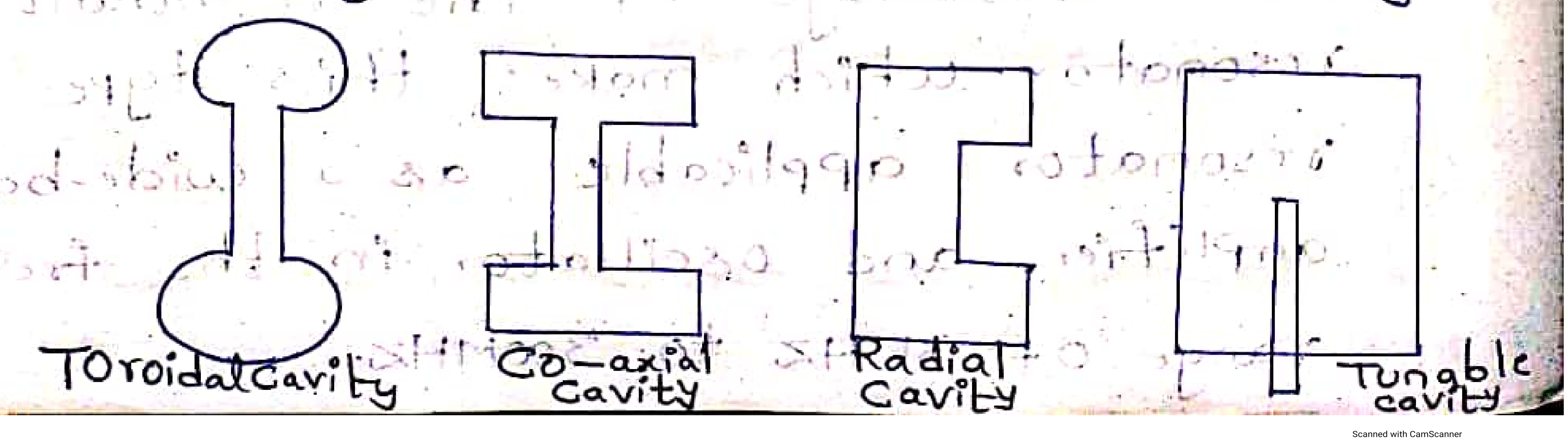
→ The electric field is concentrated across gap 'g' on the capacitance region, allowing the electrons to flow through it.

→ Electric energy stored in the cavity can be increased by increasing the capacitance, C. This type of re-entrant cavity resonator is tuned by varying the short plunger. The resonant length can be varied by using the short-plunger as well.

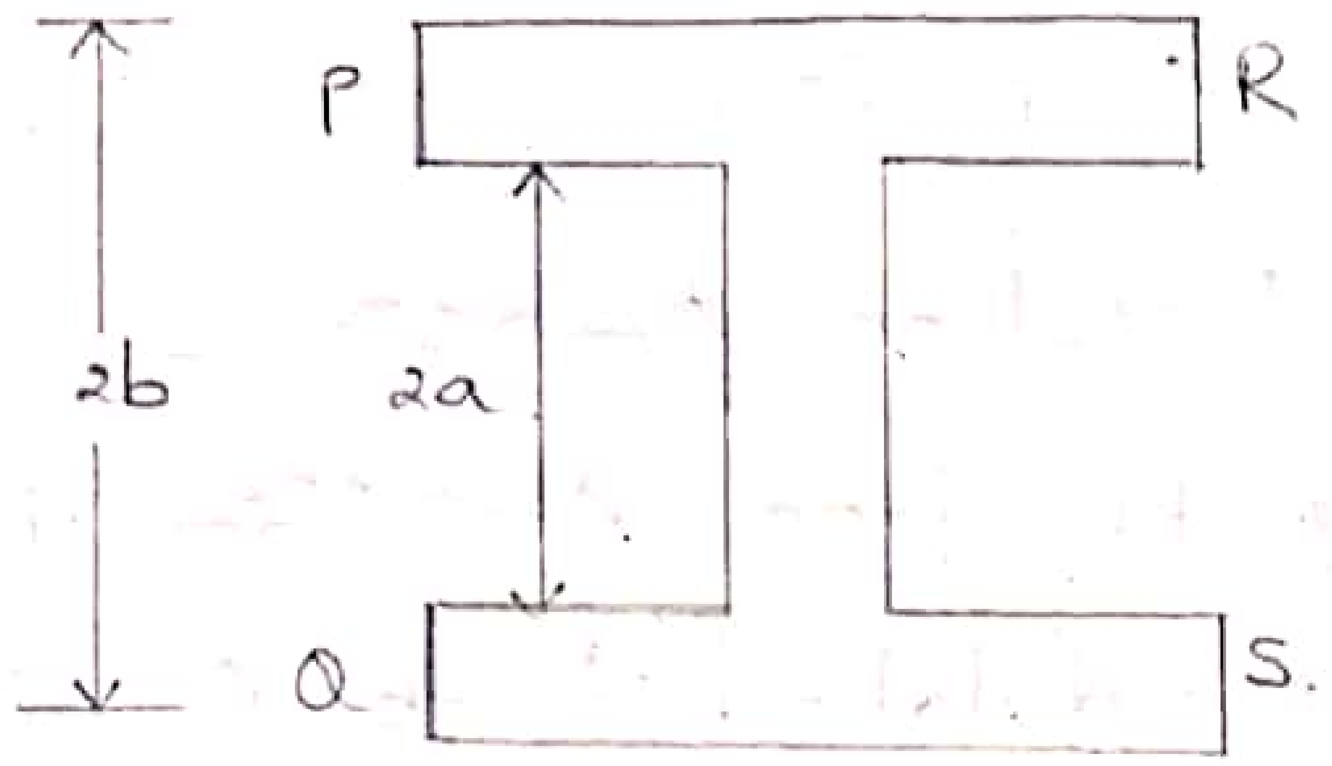
→ If the re-entrant cavity's length is greater than the gap thickness, then such a sturucture would be considered a co-axial line with the radii of the inner and outer conductor.

→ At resonance frequency, the gap capacitance (C) and the co-axial line below the gap provide reactances, which are equal and opposite.

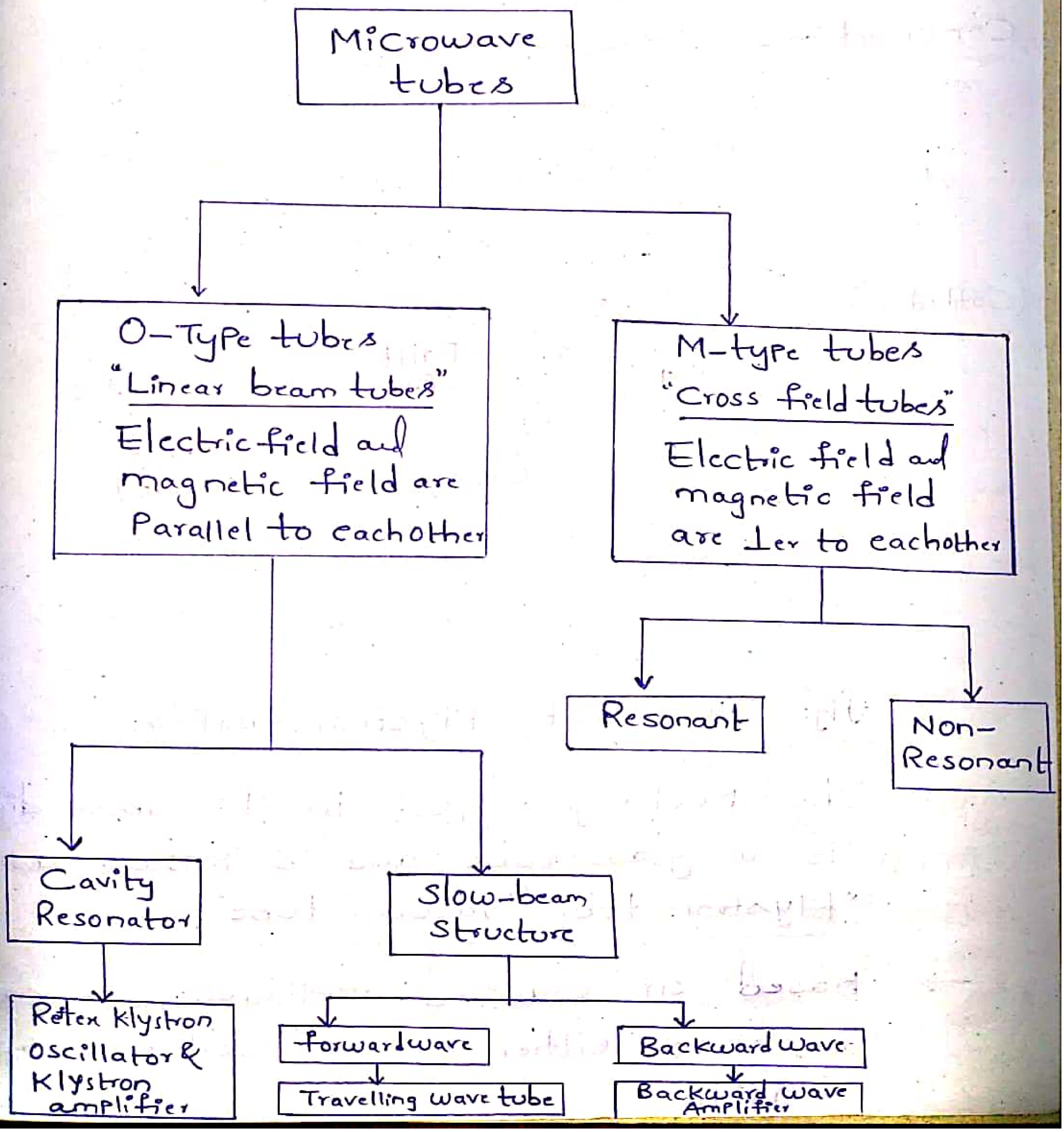
→ Cavity resonators are metallic boundaries extending to interior of the cavity.



Re-entrant Cavity resonators are similar to co-axial line shorted at 2 ends and joined at center by capacitor.



Classification of Microwave tubes:-



** Two cavity klystron Amplifier :-

Klystron :- A klystron is a vaccum tube that can be used as oscillator (or) Amplifier.

Two Cavity klystron Amplifier:-

Two cavity klystron Amplifier is basically a velocity modulated tube. A simplified diagram of Two cavity Klystron Amplifier is shown below:

Construction:-

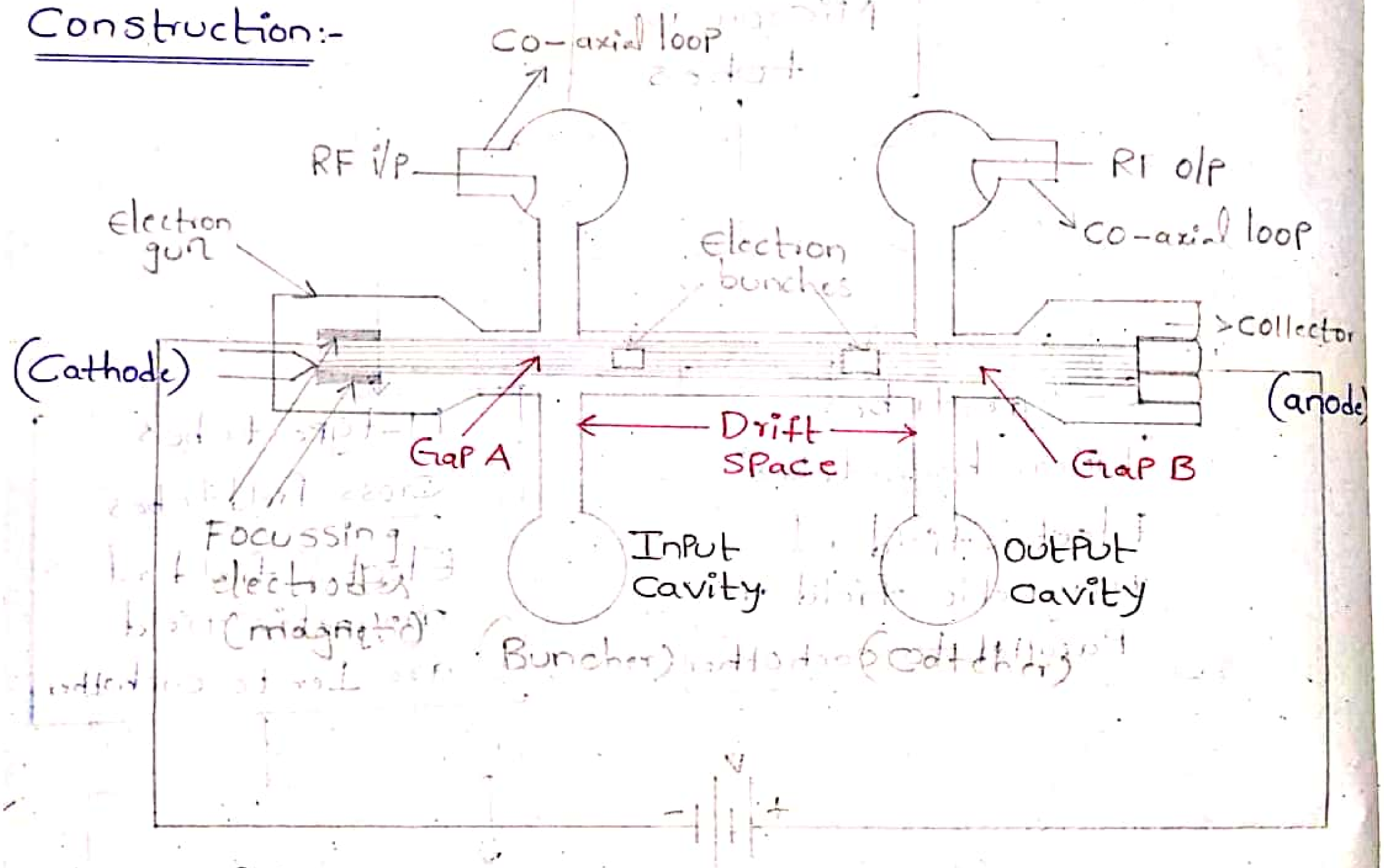


fig:- Two cavity klystron Amplifier

→ The Rectangular part in the above diagram is a glass tube and is known as "klystron tube"/"vaccum tube".

→ Based on required application, it can be used either as an oscillator (or) Amplifier.

→ Here, the klystron tube is used as an Amplifier, with two cavities and hence it is known as Two Cavity klystron Amplifier.

→ one end of the glass tube is connected to -ve supply While the other end of the tube is connected to +ve supply.

→ The -ve terminal is connected to Electrongun Which is referred to as cathode and the +ve terminal is connected to collector Which is referred to as anode.

→ The Two Cavity klystron Amplifier consists of two cavities namely:-

1. Buncher cavity (input cavity)
2. catcher cavity (output cavity)

→ The gap between the two cavities is referred to as "Drift space".

→ The gap between electron gun and the Buncher cavity is referred to as "GapA".

→ The gap between the catcher cavity and collector is referred to as "Gap B".

Operation:-

→ RF signals are applied as input at Buncher cavity and their Amplified version is collected at catcher cavity. Now, Let us see how this happens in the Amplifier.

→ When a voltage 'v' is applied across the terminals, the electron gun starts emitting electrons.

→ These electrons travel from Cathode to Anode. Meanwhile, if at all an RF Signal is applied as input to the Buncher Cavity and if the applied RF signal comes in contact with the moving e^-, the velocity of RF Signal increases.

→ on the other hand, the velocity of the applied RF frequency, increases resulting in the amplification of the signal.

→ The Amplified signal is collected at Electron bunches, which then travels through Catcher Cavity and is finally collected at the Collector.

→ When the electrons are travelling from Cathode to Anode, they have to Pass through 3 stages:- (i) Gap A

(ii) Drift space

(iii) Gap B

→ The electrons collected at Collector are referred to as "Early electrons" (e_e).

→ The electrons between Buncher Cavity and catcher Cavity are referred to as "Reference electrons" (e_R).

→ The electrons between the Electron gun and buncher Cavity are referred to as "Late electrons" (e_l).

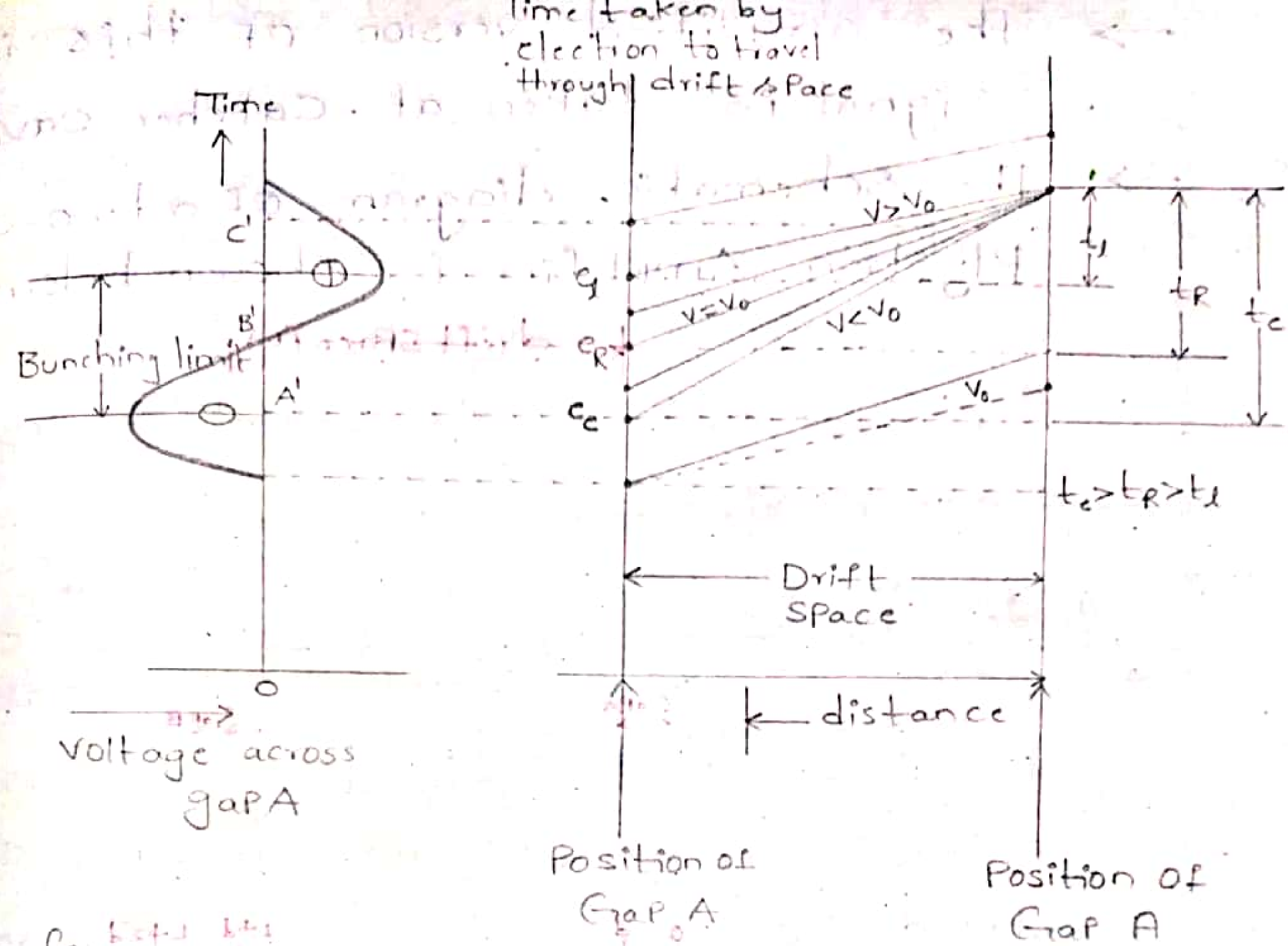


fig:- Applegate diagram of a klystron Amplifier

Applications of Two Cavity klystron Amplifier:-

* TV transmitters.
* Radar communications
* Satellite communications

TWO CAVITY KLYSTRON AMPLIFIER - Velocity Modulation

→ It is known that the Two Cavity klystron Amplifier consists of two cavities namely :- (i) Buncher Cavity
(ii) Catcher Cavity

→ The RF signal which is to be amplified is given to Buncher cavity. Let the input signal be,

$$\boxed{V_s = V_1 \sin\omega t}$$

→ The amplified version of this input signal is taken at catcher cavity.

→ The schematic diagram of a two-cavity Klystron Amplifier is shown below:-

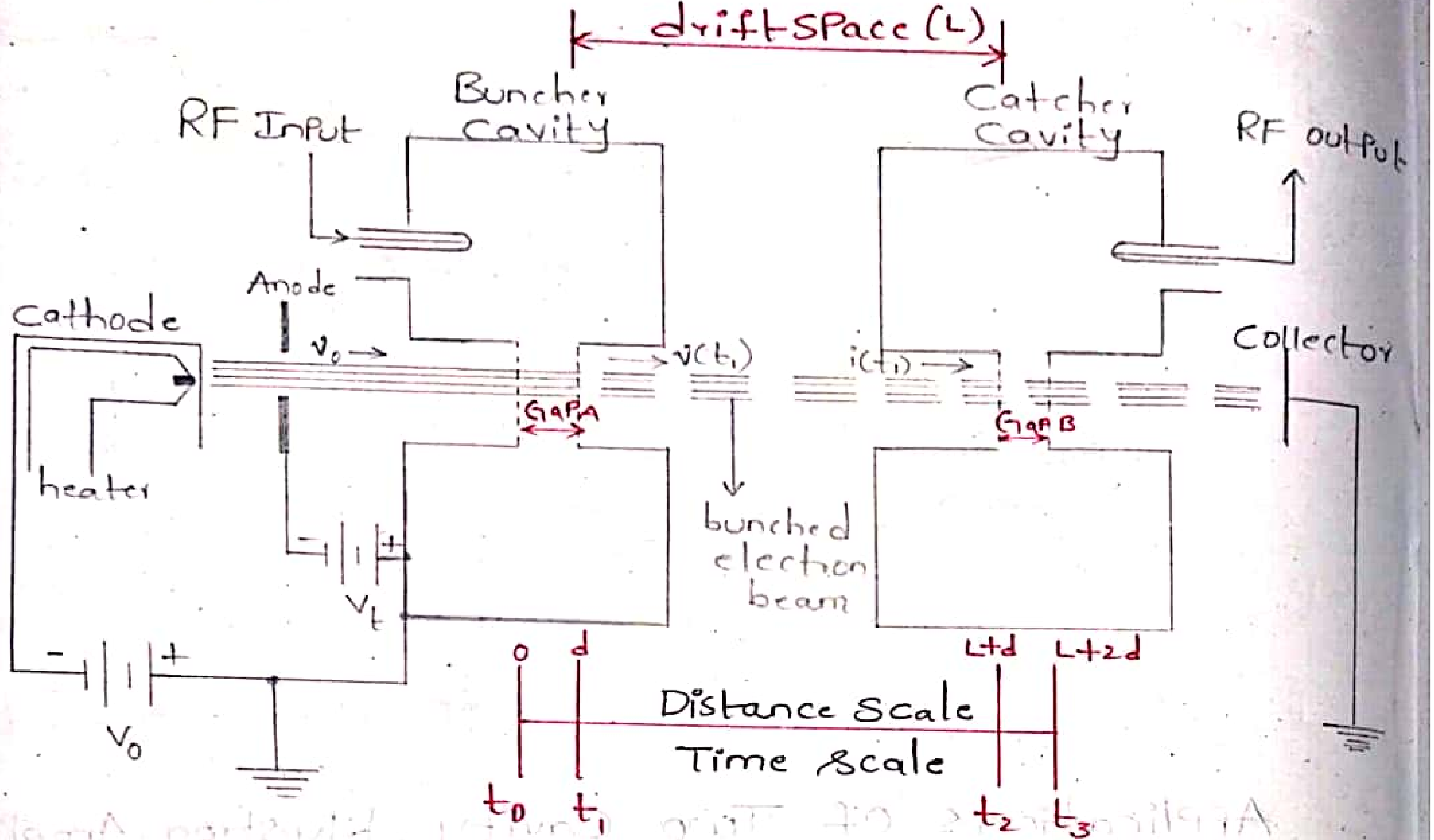


fig:- Schematic diagram of a Two cavity Klystron Amplifier

→ There is a heater in the diagram. When a voltage V_0 is applied to cathode the heater heats the cathode and hence the cathode emitts electrons.

→ These electrons are accelerated by anode and travel towards Buncher cavity.

→ Let the initial velocity of emitted electrons be v_0. once they leave the buncher cavity/ the cavity gap also known as Gap A, the electrons are formed into Bunches. It is known as velocity of velocity $v(t_1)$.

Modulation, Which is considered to be one of the basic working Principles of Two cavity klystron. Amplifier. This velocity Modulation leads to Current modulation in further.

Velocity Modulation:-

The Variation in the Velocity of electrons while moving inside the Rectangular shaped glass tube (klystron tube) is known as Velocity modulation. This velocity modulation Permits bunching of electrons While Propagation. So, the combined energy of bunched e^- is transferred at the output thereby Providing an amplified signal.

Distance Scale:-

$d \rightarrow$ Gap A

$(L+d) \rightarrow$ Gap A + drift space

$(L+2d) \rightarrow$ Gap A + drift space + Gap B

Time Scale:-

t_0 ---> electron entering time of gap A

t_1 ---> electron leaving time of gap A

t_2 ---> electron entering time of gap B

t_3 ---> electron leaving time of gap B.

→ It is called "O-type (original type) tube" (Or) "Linear beam tube".

Linear beam tube indicates that the main purpose of magnetic field here is to focus the electron beam to travel from cathode to collector.

Potential energy of e^- is given by

$$\boxed{\text{Potential energy} = eV_0}$$

Here, $V_0 \rightarrow$ cathode voltage

When the emitted e^- are accelerated by the anode, this potential energy is converted into kinetic energy. The kinetic energy associated with the accelerated electrons is given by,

$$\boxed{\text{Kinetic energy} = \frac{1}{2} m v_0^2}$$

from the above description we can write

$$\text{Potential energy} = \text{Kinetic energy}$$

$$eV_0 = \frac{1}{2} m v_0^2$$

$$v_0 = \sqrt{\frac{2eV_0}{m}}$$

$$\frac{e}{m} = \frac{\text{charge of } e^-}{\text{mass of } e^-} = 1.759 \times 10^{11} \, c/kg$$

$$\Rightarrow v_0 = 0.593 \times 10^{6} \sqrt{V_0} \, (m/sec)$$

$$\therefore \boxed{\text{Velocity of emitted electrons } (v_0) = 0.593 \times 10^{6} \sqrt{V_0} \, (m/sec)}$$

Let the RF input be,

$\boxed{V_s = V_1 \sin\omega t}$ Which is given to Buncher Cavity

Here, $V_1 << V_0$ indicating that the amplitude of the signal which is to be amplified is very very less than Cathode voltage. (Let's say, V_0 (kV) and V_1 (volts)). Let us consider, three cases as below:

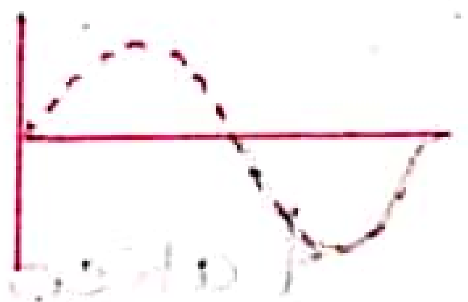
Case (i):- $V_s = 0$

When $V_s = 0$ i.e., no RF input is applied, the electrons travel with a velocity of v_0.

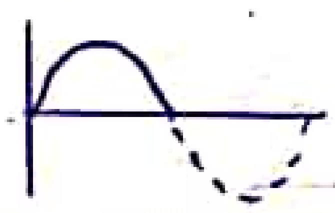
Case (ii):- Positive Half-cycle of RF input

if the gap voltage is positive i.e., during the positive half-cycle of applied RF input, the electrons are accelerated.

Case (iii):- Negative Half-cycle of RF input



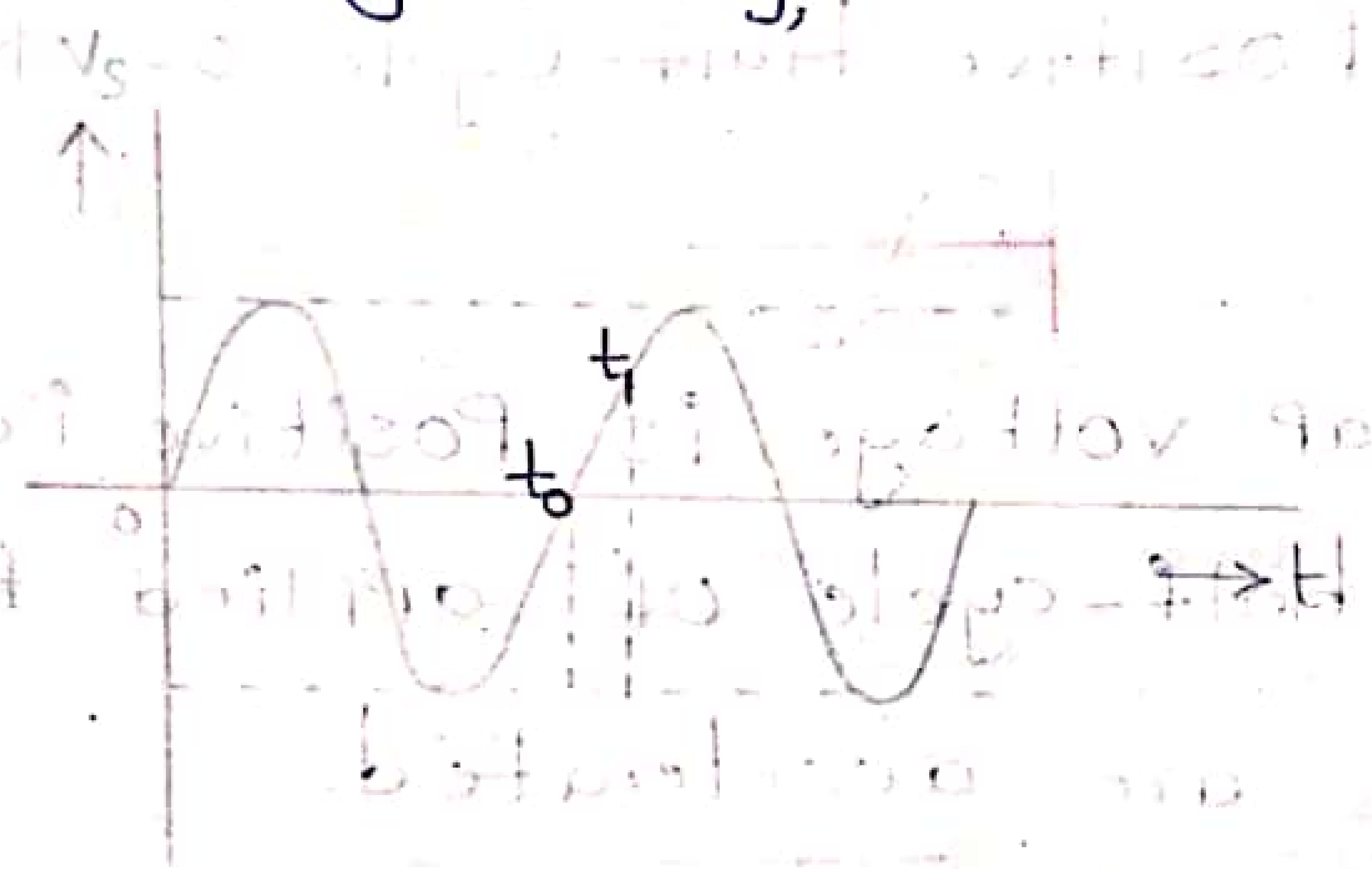
if the gap voltage is negative i.e., during the negative half-cycle of applied RF input, the electrons are decelerated.

$V_s = 0$	Unchanged velocity
	velocity ↑
	velocity ↓

(Principle of Operation)

Due to these changes in velocity, velocity modulation occurs and as a result, electrons start forming into bunches within the drift space (L). These bunched electrons are referred to as "Bunched electron beam". Now, the velocity of electrons is changed from v_0 to $v(t_1)$. Now, we have to findout the changed velocity $v(t_1)$.

The graphical representation of RF input voltage is given by,



The average transit time of buncher cavity,

$$\boxed{\tau = t_1 - t_0}$$

This is the time taken by the electrons to cross the buncher cavity.

The Average gap transit angle is given by,

$$\theta_g = \omega\tau$$

$$= \omega(t_1 - t_0)$$

$$\boxed{\theta_g = \omega(t_1 - t_0)} = \omega\tau$$

Average Buncher Cavity gap voltage during t_0 to t_1 is given by,

$$\langle V_s \rangle = \frac{1}{t_1 - t_0} \int_{t_0}^{t_1} V_1 \sin\omega t \, dt$$

$$= \frac{V_1}{t_1 - t_0} \int_{t_0}^{t_1} \sin\omega t \, dt$$

$$= \frac{-V_1}{\omega(t_1 - t_0)} \left(\cos\omega t\right)_{t_0}^{t_1}$$

$$= \frac{-V_1}{\omega(t_1 - t_0)} \left(\cos\omega t_1 - \cos\omega t_0\right)$$

$$= \frac{V_1}{\omega\tau} \left(\cos\omega t_0 - \cos\omega t_1\right)$$

$$= \frac{V_1}{\omega\tau} \left[\cos\omega t_0 - \cos\omega(\tau + t_0)\right]$$

$$= \frac{V_1}{\omega\tau} \left[\cos\omega t_0 - \cos(\omega t_0 + \omega\tau)\right]$$

$$= \frac{2V_1}{\omega\tau} \sin\left(\omega t_0 + \frac{\omega\tau}{2}\right) \sin\left(\frac{\omega\tau}{2}\right)$$

$$\boxed{\cos A - \cos B = 2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)}$$

$$= \frac{V_1}{\left(\frac{\omega\tau}{2}\right)} \sin\left(\frac{\omega\tau}{2}\right) \sin\left(\omega t_0 + \frac{\tau\omega}{2}\right)$$

$$= V_1 \cdot \frac{\sin\left(\frac{\omega\tau}{2}\right)}{\left(\frac{\omega\tau}{2}\right)} \sin\left(\omega t_0 + \frac{\omega\tau}{2}\right)$$

$$= V_1 \frac{\sin(\theta_g/2)}{(\theta_g/2)} \sin\left(\omega t_0 + \theta_g/2\right)$$

$$= V_1 \beta_i \sin(\omega t_0 + \theta_g/2)$$

Here, β_i = Beam Coupling Coefficient of Buncher Cavity / input cavity

$$\boxed{\beta_i = \frac{\sin(\theta_g/2)}{(\theta_g/2)}}$$

$$\therefore \boxed{\langle V_s \rangle = V_1 \cdot \beta_i \sin(\omega t_0 + \theta_g/2)}$$

The velocity of electrons at time 't_1' is given by,

$$v(t_1) = \sqrt{\frac{2e}{m}\left(V_0 + V_1 \cdot \beta_i \sin(\omega t_0 + \theta_g/2)\right)}$$

$$= \sqrt{\frac{2eV_0}{m}} \left[1 + \frac{V_1 \beta_i}{V_0} \sin(\omega t_0 + \theta_g/2)\right]^{1/2}$$

$$= v_0 \left[1 + \frac{V_1 \beta_i}{2V_0} \sin(\omega t_0 + \theta_g/2)\right]$$

$$\boxed{(1+x)^n = 1 + nx + \frac{n(n-1)}{2!} x^2 + \cdots}$$

Since $V_1 << V_0$, neglect all the higher order terms

$$\therefore \boxed{v(t_1) = v_0 \left[1 + \frac{V_1 \cdot \beta_i}{2V_0} \sin(\omega t_0 + \theta_g/2)\right]}$$

(Velocity modulated eqn)

interms of t_1 is given by,

$$v(t_1) = v_0 \left[1 + \frac{V_1 \cdot \beta_i}{2V_0} \sin(\omega t_1 - \theta_g + \theta_g/2)\right]$$

$$(\because \theta_g = \omega t_1 - \omega t_0)$$

$$\Rightarrow v(t_1) = v_0\left[1 + \frac{V_1 \cdot \beta_i}{2V_0}\sin(\omega t_1 - \theta_g/2)\right]$$

$$\therefore \boxed{v(t_1) = v_0\left[1 + \frac{V_1 \cdot \beta_i}{2V_0}\sin(\omega t_1 - \theta_g/2)\right]}$$ → (Velocity modulated equation interms of t_1.)

TWO CAVITY KLYSTRON AMPLIFIER - Bunching Process

The schematic diagram of a two cavity Klystron Amplifier is shown below:

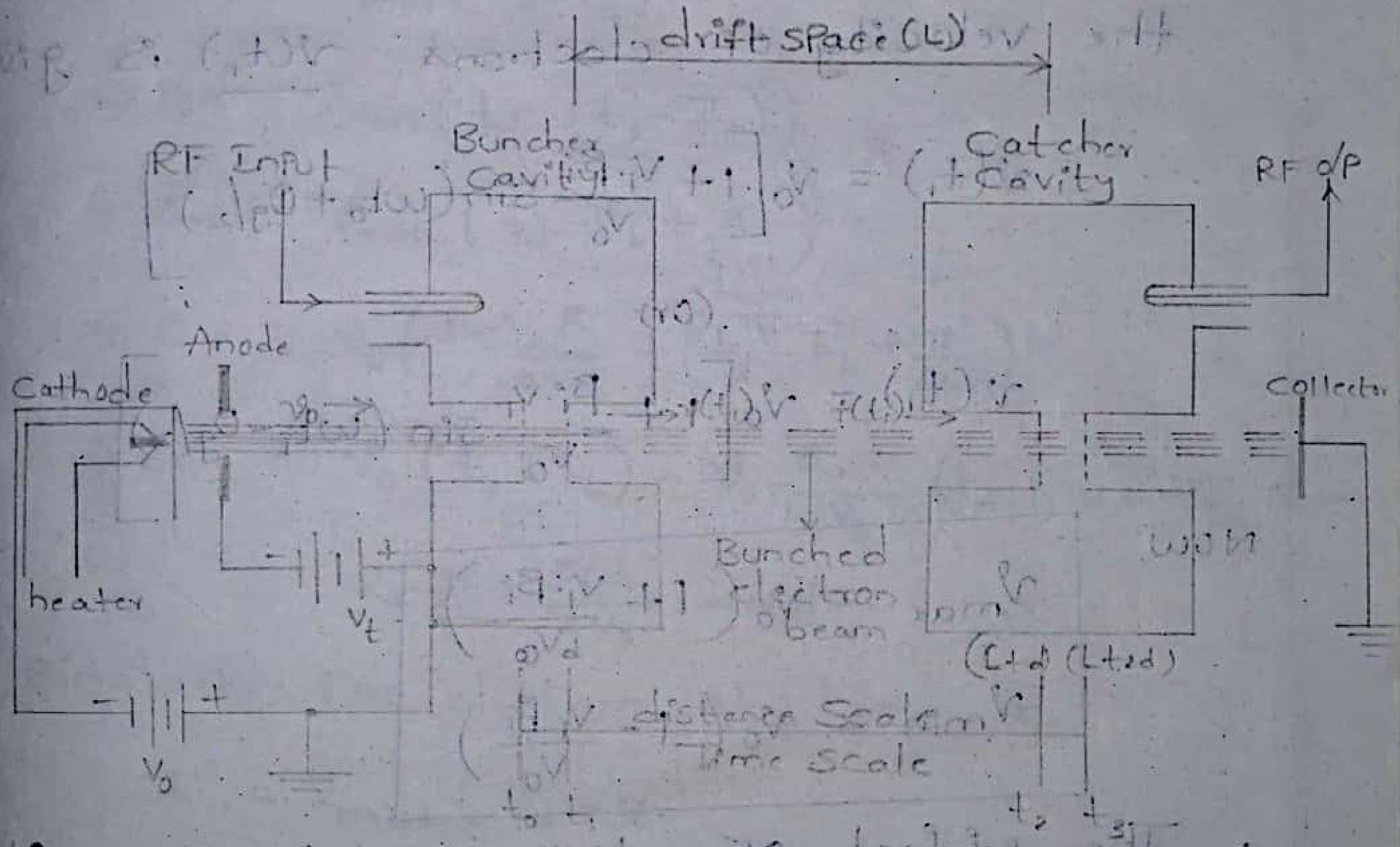


fig:1 schematic diagram of a Two Cavity klystron Amplifier

→ As we discussed earlier, the electrons that pass through the buncher cavity at $V_s = 0$ travel with unchanged velocity 'v_0' and become the bunching center.

→ The electrons that pass through the Buncher Cavity during the +ve half-cycles of RF i/p voltage 'V_s' travel faster than the electrons than the electrons that Passed the gap when $V_s = 0$.

→ The electrons that pass the Buncher cavity during the −ve half-cycles of RF i/p Voltage 'V_s', travel slower than the electrons that Passed the gap when $V_s = 0$.

→ At a distance 'ΔL' from the buncher cavity, the beam electrons have drifted into "dense clusters".

→ Once the electrons leave the buncher cavity the velocity of electrons $v(t_1)$ is given by,

$$v(t_1) = v_0\left[1 + \frac{V_1 \cdot \beta_i}{V_0} \sin(\omega t_0 + \theta_g/_2)\right]$$

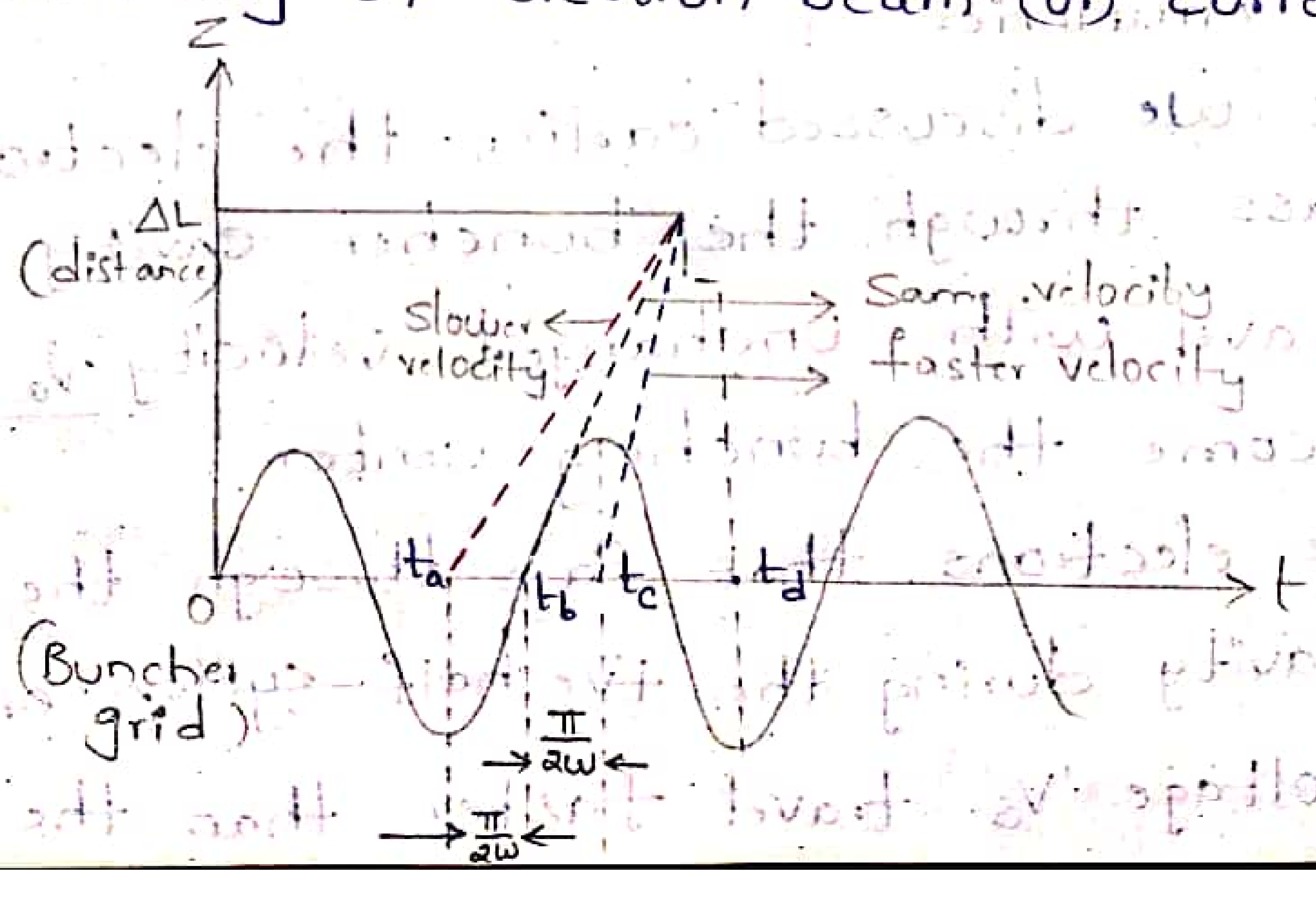
(or)

$$v(t_1) = v_0\left[1 + \frac{\beta_i \cdot V_1}{V_0} \sin(\omega t_1 - \theta_g/_2)\right]$$

Now,

$$\boxed{\begin{aligned} v_{max} &= v_0\left(1 + \frac{V_1 \cdot \beta_i}{V_0}\right) \\ v_{min} &= v_0\left(1 - \frac{V_1 \cdot \beta_i}{V_0}\right) \end{aligned}}$$

→ The effect of velocity modulation Produces bunching of electron beam (or) current modulation.



$$\omega = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi}{\omega}$$

$$\frac{T}{2} = \frac{\pi}{\omega} \Rightarrow \frac{T}{4} = \frac{\pi}{2\omega}$$

$$\text{Velocity} = \frac{\text{distance}}{\text{time}} \Rightarrow \boxed{d = vt}$$

For electron at time 't_b'

$$\Delta L = v_0 (t_d - t_b) \longrightarrow (1)$$

For electron at time 't_a'

$$\Delta L = v_{min} (t_d - t_a)$$

$$\Rightarrow \Delta L = v_{min} \left(t_d - \left(t_b - \frac{\pi}{2\omega}\right)\right)$$

$$\Rightarrow \Delta L = v_{min} \left(t_d - t_b + \frac{\pi}{2\omega}\right)$$

$$\Rightarrow \Delta L = v_0 \left(1 - \frac{V_1 \beta_i}{2V_0}\right) \left(t_d - t_b + \frac{\pi}{2\omega}\right)$$

$$\Rightarrow \Delta L = v_0 (t_d - t_b) - v_0 \frac{\beta_i \cdot V_1}{2V_0} (t_d - t_b) + \frac{\pi}{2\omega} v_0 -$$
$$v_0 \cdot \frac{\pi}{2\omega} \cdot \frac{\beta_i V_1}{2V_0} \longrightarrow (2)$$

Similarly, For electron at time 't_c'

$$\Delta L = v_{max} (t_d - t_c)$$

$$\Rightarrow \Delta L = v_{max} \left(t_d - \left(t_b + \frac{\pi}{2\omega}\right)\right)$$

$$\Rightarrow \Delta L = v_0 \left(1 + \frac{\beta_i V_1}{2V_0}\right) \left(t_d - t_b - \frac{\pi}{2\omega}\right)$$

$$\Rightarrow \Delta L = v_0 (t_d - t_b) + v_0 \cdot \frac{\beta_i V_1}{2V_0} (t_d - t_b) - v_0 \cdot \frac{\pi}{2\omega} -$$
$$v_0 \frac{\pi}{2\omega} \cdot \frac{\beta_i V_1}{2V_0} \longrightarrow (3)$$

The necessary and sufficient condition for electrons at t_a, t_b, t_c to meet at same distance is,

from (2);

$$-v_0 \frac{\beta_i V_1}{2V_0}(t_d - t_b) + \frac{\pi}{2\omega} v_0 - v_0 \frac{\pi}{2\omega} \cdot \frac{\beta_i V_1}{2V_0} = 0$$

(Or)

from (3);

$$v_0 \frac{\beta_i V_1}{2V_0}(t_d - t_b) - v_0 \frac{\pi}{2\omega} - v_0 \frac{\pi}{2\omega} \cdot \frac{\beta_i V_1}{2V_0} = 0$$

$$\Rightarrow \cancel{v_0} \frac{\beta_i V_1}{2V_0}(t_d - t_b) = \cancel{v_0} \frac{\pi}{2\omega}\left(1 + \frac{\beta_i V_1}{2V_0}\right)$$

$$\Rightarrow (t_d - t_b) \frac{\beta_i \cdot V_1}{2V_0} = \frac{\pi}{2\omega}$$

$\left(\because V_1 << V_0 \text{ so neglect } 1 + \frac{\beta_i V_1}{2V_0}\right)$

$$\Rightarrow \boxed{(t_d - t_b) = \frac{\pi V_0}{\omega \beta_i V_1}} \longrightarrow (4)$$

Substituting eqn-(4) in eqn-(1) we get,

$$\Delta L = v_0 (t_d - t_b)$$

$$= v_0 \cdot \frac{\pi V_0}{\omega \beta_i V_1}$$

**

$$\therefore \boxed{\Delta L = v_0 \cdot \frac{\pi V_0}{\omega \beta_i V_1}}$$

Here, $v_0 \longrightarrow$ initial velocity of electrons

$V_0 \longrightarrow$ Cathode Voltage

$\beta_i \longrightarrow$ Beam coupling coefficient

$V_1 \longrightarrow$ Amplitude of the signal to be amplified

TWO CAVITY KLYSTRON AMPLIFIER – Current Modulation

The Schematic diagram of a two cavity klystron Amplifier is shown below:

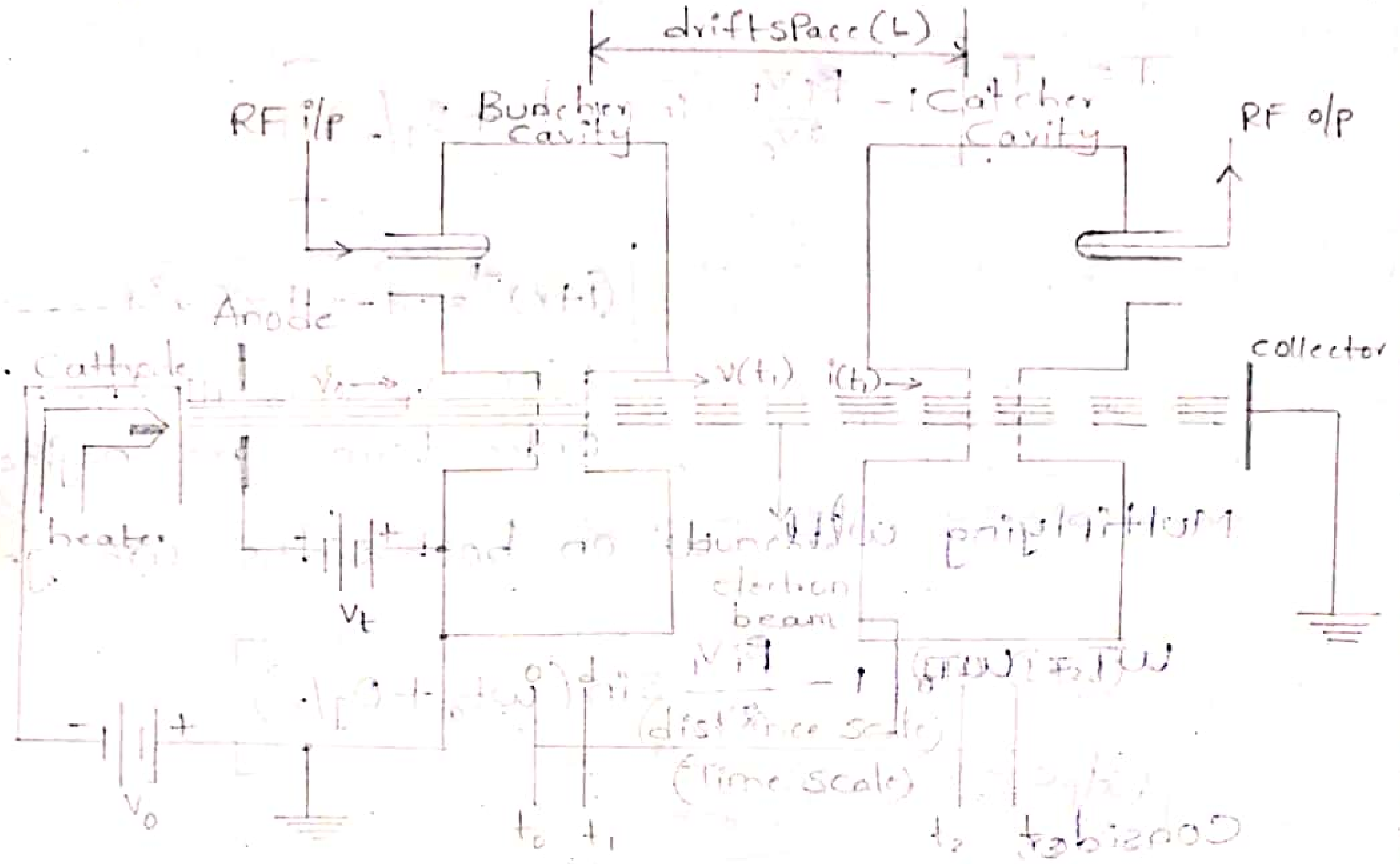


fig:- Schematic diagram of Two cavity Klystron Amplifier

The Velocity of electron's Passing from buncher cavity gap is given by,

$$v(t_1) = v_0\left[1 + \frac{\beta_i V_1}{2V_0}\sin(\omega t_0 + \theta_g/2)\right]$$

(or)

$$v(t_1) = v_0\left[1 + \frac{\beta_i V_1}{2V_0}\sin(\omega t_1 - \theta_g/2)\right]$$

The transit time of electron to travel a distance of L is given by,

$$(t_2 - t_1) = T = \frac{L}{v(t_1)} = \frac{L}{v_0\left[1 + \frac{\beta_i V_1}{2V_0}\sin(\omega t_0 + \theta_g/2)\right]}$$

$$\Rightarrow T = T_0\left[1 + \frac{\beta_i V_1}{2V_0}\sin(\omega t_0 + \theta_g/2)\right]^{-1}$$

Where, $T_0 = \frac{L}{v_0}$ is Dc transit time between the cavities when no velocity modulation occurs.

$$T = T_0\left[1 - \frac{\beta_i V_1}{2V_0}\sin(\omega t_0 + \theta_g/2)\right]$$

$$\boxed{(1+x)^{-1} = 1 - x + x^2 - x^3 + \cdots}$$

Since $V_1 << V_0$ all the higher order terms are neglected

Multiplying with 'ω' on both sides, we get

$$\omega T = \omega T_0\left[1 - \frac{\beta_i V_1}{2V_0}\sin(\omega t_0 + \theta_g/2)\right]$$

Consider,

θ_g = Dc transit angle of buncher cavity

$\boxed{\theta_0 = \omega T_0 = \frac{\omega L}{v_0} = 2\pi N}$ is Dc transit angle between the two cavities and 'N' is the no.of transit cycles between the cavities.

Now,

$$\omega T = \theta_0\left[1 - \frac{\beta_i V_1}{2V_0}\sin(\omega t_0 + \theta_g/2)\right]$$

$$\Rightarrow \omega T = \theta_0 - \frac{\beta_i V_1}{2V_0}\theta_0 \sin(\omega t_0 + \theta_g/2)$$

Let $\boxed{X = \frac{\beta_i V_1}{2V_0}\theta_0}$ is called "Bunching Parameter"

$\Rightarrow \omega T = \theta_0 - X \sin(\omega t_0 + \theta_g/2)$

According to Law of conservation of charge, if a charge 'dQ_0' Passes the buncher Cavity gap in time 'dt_0' then it appears at Catcher cavity gap at later time dt_2.

$$\boxed{I_0|dt_0| = i_2|dt_2|} \qquad \left(i = \frac{dQ}{dt} \Rightarrow dQ = i\,dt\right)$$

Where, I_0 = Dc beam current

i_2 = Current at catcher cavity

$$t_2 = t_0 + \tau + T$$

$$t_2 = t_0 + \tau + T_0\left[1 - \frac{\beta_i V_1}{2V_0}\sin(\omega t_0 + \theta_g/2)\right]$$

$$dt_2 = dt_0 + T_0\left(-\frac{\beta_i V_1}{2V_0}\cos(\omega t_0 + \theta_g/2)\,\omega\, dt_0\right)$$

$$= dt_0 - X\cos(\omega t_0 + \theta_g/2)\,dt_0 \qquad \left(\begin{array}{l}\omega T_0 = \theta_0 \\ X = \frac{\beta_i V_1}{2V_0}\cdot\theta_0\end{array}\right)$$

$$= dt_0\left(1 - X\cos(\omega t_0 + \theta_g/2)\right)$$

We have, $I_0|dt_0| = i_2|dt_2|$

$$\Rightarrow i_2 = \frac{I_0}{|dt_2/dt_0|}$$

$$\Rightarrow i_2 = \frac{I_0}{1 - X\cos(\omega t_0 + \theta_g/2)}$$

$$\Rightarrow i_2(t_0) = \frac{I_0}{1 - X\cos(\omega t_0 + \theta_g/2)}$$

'i_2' interms of t_2 is given by,

$$t_2 = t_0 + \tau + T_0$$

$$\omega t_2 = \omega t_0 + \omega \tau + \omega T_0$$

$$= \omega t_0 + \theta_g + \theta_0$$

$$\omega t_0 = \omega t_2 - \theta_g - \theta_0$$

$$\therefore \boxed{i_2(t_2) = \frac{I_0}{1 - X\cos(\omega t_2 - \theta_0 - \theta_g/2)}}$$

The Current "i_2" at Catcher Cavity is a Periodic waveform with Period, $T = \frac{2\pi}{\omega}$ which Can be exPressed using trignometric fourier Series.

$$\boxed{i_2(t_2) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega t_2) + b_n \sin(n\omega t_2)}$$

$$(-\pi < \omega t_2 < \pi)$$

$a_0, a_n, b_n \rightarrow$ trignometric fourier Series Constants

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} i_2 \, d(\omega t_2)$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} i_2 \cos(n\omega t_2) \, d(\omega t_2)$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} i_2 \sin(n\omega t_2) \, d(\omega t_2)$$

Substituting the Values of i_2 & the exPression $I_0 |dt_0| = i_2 |dt_2|$; in a_0, a_n, b_n we get,

$$\boxed{\begin{aligned} a_0 &= I_0 \\ a_n &= 2 I_0 J_n(nX) \cos(n\theta_g + n\theta_0) \\ b_n &= 2 I_0 J_n(nX) \sin(n\theta_g + n\theta_0) \end{aligned}}$$

Where $J_n(nx)$ is n^{th} order Bessel's function of 1^{st} order Kind.

Now,

$$i_2(t_2) = I_0 + \sum_{n=1}^{\infty} \left\{ 2I_0 J_n(nx) \cos(n\theta_g + n\theta_0) \cdot \cos(n\omega t_2) + 2I_0 J_n(nx) \sin(n\theta_g + n\theta_0) \cdot \sin(n\omega t_2) \right\}$$

$$= I_0 + \sum_{n=1}^{\infty} 2I_0 J_n(nx) \left\{ \cos(n\omega t_2)\cos(n\theta_g + n\theta_0) + \sin(n\omega t_2)\sin(n\theta_g + n\theta_0) \right\}$$

$$= I_0 + \sum_{n=1}^{\infty} 2I_0 J_n(nx) \cos(n\omega t_2 - \underbrace{n\theta_g}_{\omega\tau} - \underbrace{n\theta_0}_{\omega T_0})$$

$$\boxed{\cos A \cdot \cos B + \sin A \cdot \sin B = \cos(A-B)}$$

$$= I_0 + \sum_{n=1}^{\infty} 2I_0 J_n(nx) \cos\left(n\omega(t_2 - \tau - T_0)\right)$$

$$\therefore \boxed{i_2(t_2) = I_0 + \sum_{n=1}^{\infty} 2I_0 J_n(nx) \cos\left(n\omega(t_2 - \tau - T_0)\right)}$$

The fundamental component of current at catcher cavity gap has amplitude,

$$\boxed{I_2 = 2I_0 J_1(x)}$$

This has maximum amplitude at $\boxed{x = 1.841}$ where $J_1(x) = 0.582$

The optimum distance at which maximum amplitude of fundamental component occurs is given by,

$$\underbrace{x}_{\text{(bunching Parameter)}} = \frac{\beta_i V_1}{2V_0} \theta_0 = \frac{\beta_i V_1}{2V_0}(\omega T_0)$$

$$\Rightarrow X = \frac{\beta_i V_1}{2V_0} \cdot \frac{\omega L}{v_0}$$

if $X = 1.841$, then

$$\boxed{L_{optimum} = \frac{3.682\, v_0 V_0}{\omega \beta_i V_1}}$$

TWO Cavity Klystron Amplifier – output Power & Voltage gain

The schematic diagram of a two-cavity klystron Amplifier is shown below:

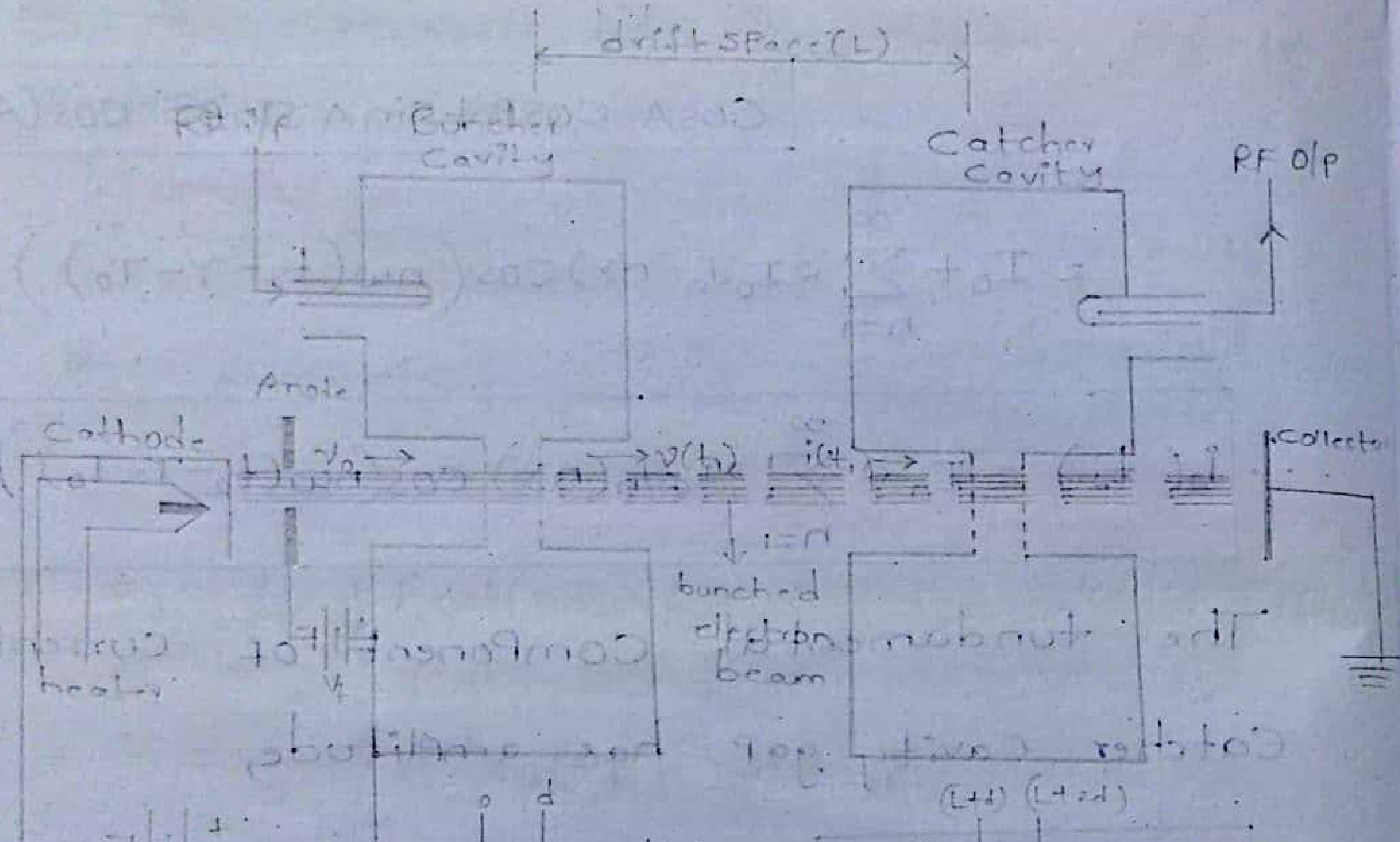


fig.- schematic diagram of a two-cavity klystron Amplifier

→ The maximum bunching should occur approximately midway between the catcher cavity grid.

→ The phase of catcher cavity gap voltage must be maintained in such a way that the bunched electrons, as they pass through the grids, encounter a retarding phase.

→ When the bunched electron beam passes through retarding phase, its kinetic energy is transferred to the field of the catcher cavity.

→ When electrons emerge from catcher grids, they have reduced velocity and finally collected by the collector.

We know that

$$\boxed{i_2 = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega t_2) + b_n \sin(n\omega t_2)};$$

$$-\pi \leq \omega t_2 \leq \pi$$

Where, $a_0 = I_0$

$$a_n = 2I_0 J_n(nx) \cos(n\theta_g + n\theta_0)$$

$$b_n = 2I_0 J_n(nx) \sin(n\theta_g + n\theta_0)$$

Now,

$$i_2 = I_0 + \sum_{n=1}^{\infty} 2I_0 J_n(nx) \cos(n\omega(t_2 - \tau - T_0))$$

fundamental component / 1st harmonics

$$i_2 = I_0 + 2I_0 J_1(x) \cos(\omega(t_2 - \tau - T_0)) + 2I_0 J_2(x) \cos(2\omega(t_2 - \tau - T_0)) + \text{----}$$

DC value of current

2nd harmonics

The fundamental component of induced current at catcher cavity grids is given by

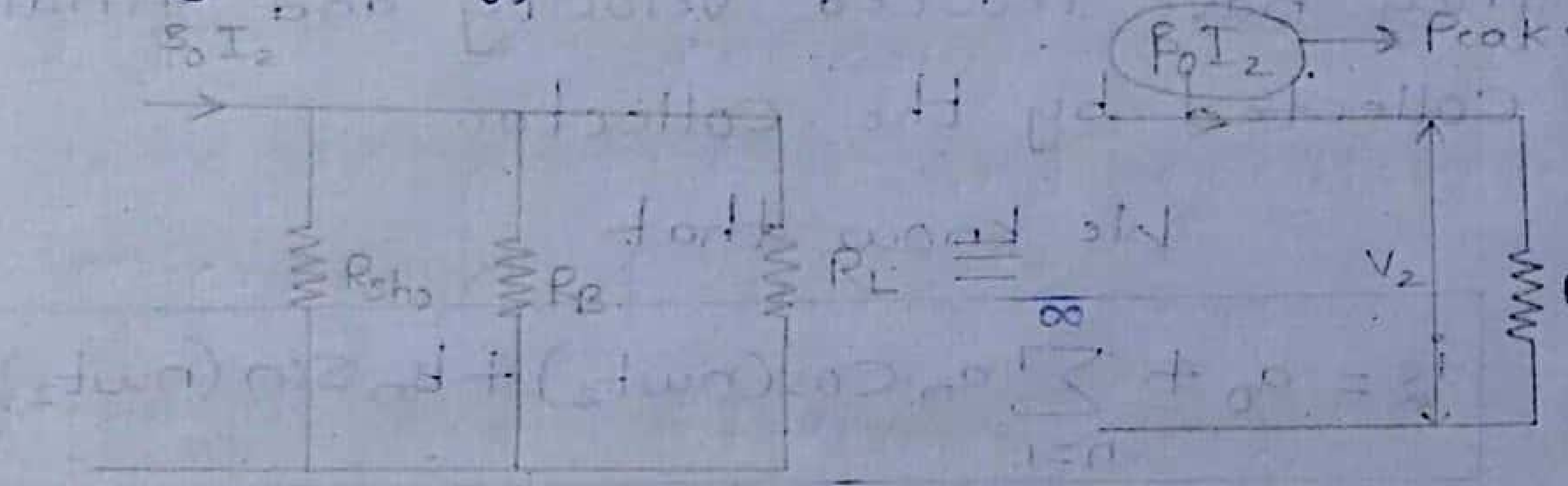
$$\boxed{i_{2(\text{induced})} = \beta_0\, 2I_0 J_1(X) \cos\left(\omega(t_2-\tau-T_0)\right)}$$

Where, $\beta_0 \rightarrow$ Beam coupling coefficient of output cavity

The amplitude of induced current into the catcher cavity gap,

$$\boxed{I_{2(\text{induced})} = \beta_0\, 2I_0 J_1(X) = \beta_0 I_2}$$

The equivalent circuit of catcher cavity is given by,



Here, $R_{sho} \rightarrow$ Resistance of catcher cavity walls

$R_B \rightarrow$ Beam Loading resistance

$R_L \rightarrow$ external Load resistance

$R_{sh} \rightarrow$ effective shunt resistance

output Power, $P_{out} = \dfrac{(\beta_0 I_2)^2}{2} \times R_{sh}$

$$= \frac{(\beta_0 I_2)(\beta_0 I_2 R_{sh})}{2}$$

$$= \frac{\beta_0 I_2 V_2}{2}$$

input Power, $P_{in} = V_0 I_0$

$$\boxed{\text{efficiency}, \eta = \frac{P_{out}}{P_{in}} = \frac{\beta_0 I_2 V_2}{2 V_0 I_0}}$$

For a Practical Two cavity Klystron Amplifier, η is about 15 to 30%

Mutual conductance of two cavity klystron Amplifier (g_m) :-

$$|G_m| = \frac{I_2(\text{induced})}{V_1}$$

$$|G_m| = \frac{\beta_0 I_2}{V_1} = \frac{\beta_0 \cdot 2 I_0 J_1(X)}{V_1} \longrightarrow (1)$$

Bunching Parameter, $X = \frac{\beta_i V_1}{2V_0} \theta_0$

$$\Rightarrow V_1 = \frac{X\, 2V_0}{\beta_i \theta_0} \longrightarrow (2)$$

from (1) and (2);

$$|G_m| = \frac{\beta_0 \cdot 2 I_0 J_1(X)}{2 V_0 \cdot X} \beta_i \theta_0$$

$$\boxed{\frac{|G_m|}{G_0} = \beta_0^2 \frac{J_1(X)}{X} \theta_0} \longrightarrow \text{Normalised mutual conductance}$$

Where, $G_0 = \frac{I_0}{V_0}$ is DC beam conductance

Voltage gain of Two-Cavity Klystron Amplifier:-

$$A_V = \frac{V_2}{V_1} = \frac{\beta_0 I_2 R_{sh}}{V_1} = \frac{\beta_0 \cdot 2 I_0 J_1(X)}{V_1} \cdot R_{sh}$$

$$\Rightarrow \boxed{A_V = \frac{\beta_0^2 \theta_0}{R_0} \cdot \frac{J_1(X)}{X} R_{sh}}$$

Where $R_0 = \frac{V_0}{I_0}$ is DC beam resistance.

Applications of two-cavity klystron Amplifier

The two-cavity klystron finds application in satellite communication

- UHF TV transmitters
- Radar systems
- Wideband high Power communication
- Troposphere scatter transmitters, etc...

** Reflex Klystron :-

A Reflex klystron is a specialized Low Power vaccum tube used to Produce oscillations at microwave frequency. klystrons are basically specialized tubes used as Amplifiers and oscillators at microwave frequency range.

Need of Reflex Klystron:-

→ We have already discussed Two-cavity Klystron in Previous concepts. We know that a two-cavity klystron acts as an amplifier to Provide Amplification of RF signals. So, can that same structure be used for generating oscillations???

→ Basically a two-cavity klystron can be converted into an oscillator, but some disadvantages are associated with it.

→ As we know to design an oscillator, positive feedback must be provided to the input in a way to have a magnitude of loop gain as unity.

→ So, if we design a klystron oscillator using two-cavity klystron, then to have a change in oscillating frequency, the resonant frequency of the two-cavities is also required to be changed. There by leading to cause difficulty in generating oscillations.

→ Thus to overcome the disadvantage, a reflex klystron having a single cavity was invented to have sustained oscillations at microwave frequency.

Construction/structure of Reflex klystron:-

The basic schematic of a reflex klystron is shown below:

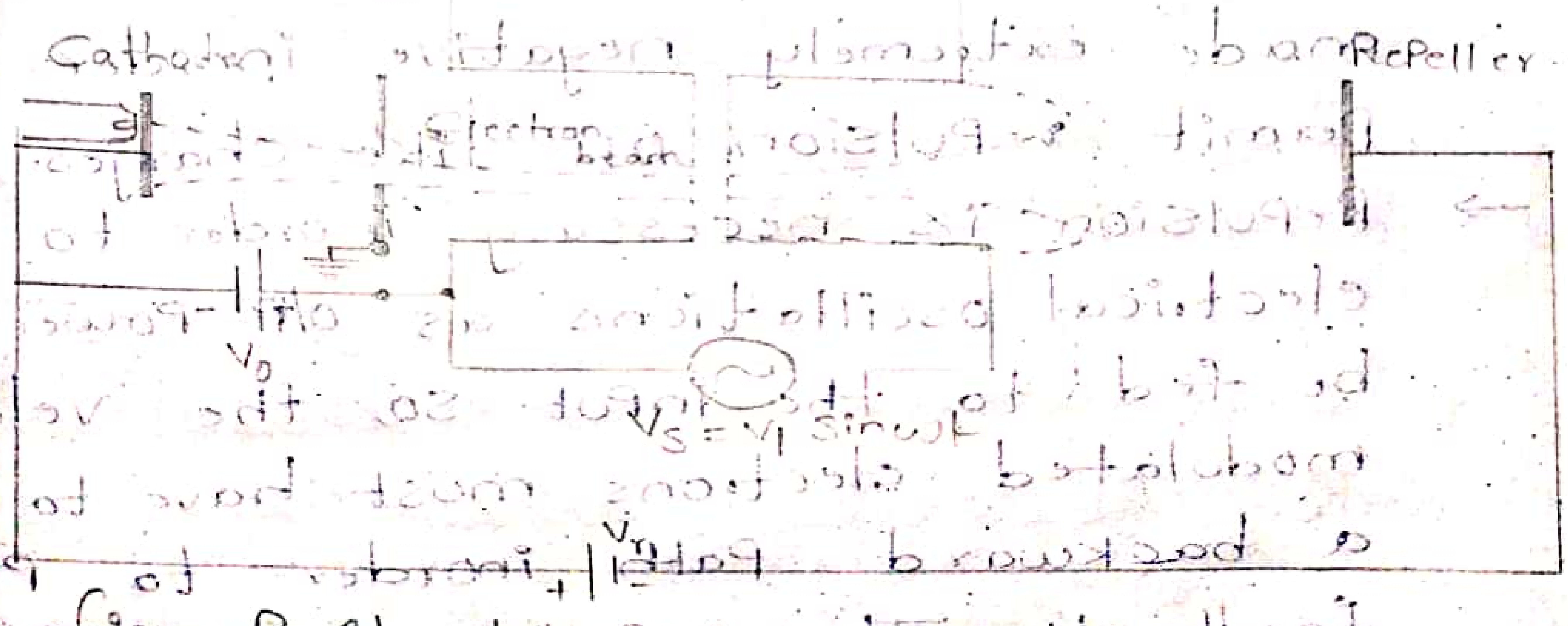


fig:- Reflex klystron

→ The structure consists of a cathode and focusing anode that combinely acts as an election gun for the tube. The cathode emits the electron beam which is focussed inside the tube by the focusing anode.

→ Also, a positive potential is provided as input which sets up an electric field inside the cavity.

→ As it is a single cavity structure, thus single cavity act as buncher and catcher cavity separately. At the time of forward movement of the electron beam, it acts as a buncher cavity. While at the time of backward movement, it is a catcher cavity.

→ A repeller plate that causes backward movement of the electron beam is present at the end of the electron gun. The potential at the repeller is made extremely negative inorder to permit repulsion of like charges.

→ Repulsion is necessary inorder to build electrical oscillations, as o/p power must be fed to the input. So, the velocity modulated electrons must have to travel a backward path inorder to provide feedback. Thus, repeller is used in the structure of klystron.

Operating Principle:-

→ Like two-cavity klystron, a reflex klystron utilizes the phenomenon of "velocity and current modulation" to produce oscillations

→ However, there exists variation in constructional structure and the respective applications of both.

→ A reflex klystron consists of a single cavity that performs the action of both buncher & catcher cavity. As to have oscillations, feedback is needed to be applied at the input which is provided by the oscillator.

→ While moving, electrons undergo velocity modulation and the repeller applies repulsive forces on them. This leads to the formation of a bunch of electrons. Further, this bunching will lead to cause, current modulation.

Working of Reflex klystron:-

As we have already discussed the fundamental principle of operation of a reflex klystron is velocity and current modulation. So, conside the above figure:

* Initially when the election beam is emitted by the electron gun, then the "early electrons" (e_e) experience a very high potential. Due to this, a strong

electric field gets generated inside the cavity gap, leading to cause movement of electrons towards the repeller with a very high velocity.

* Due to high velocity, the electrons penetrate deeper into the region of the repeller and thus require greater time to repel back towards the catcher cavity.

* But when the externally applied potential is almost 0, then electron moves with a uniform velocity with which it was emitted by the gun. These electrons are generally known as "reference electrons" (e_r).

* So, in this case, e_r will not penetrate into the repeller surface and gets repelled by the repeller in lesser time than the early electron.

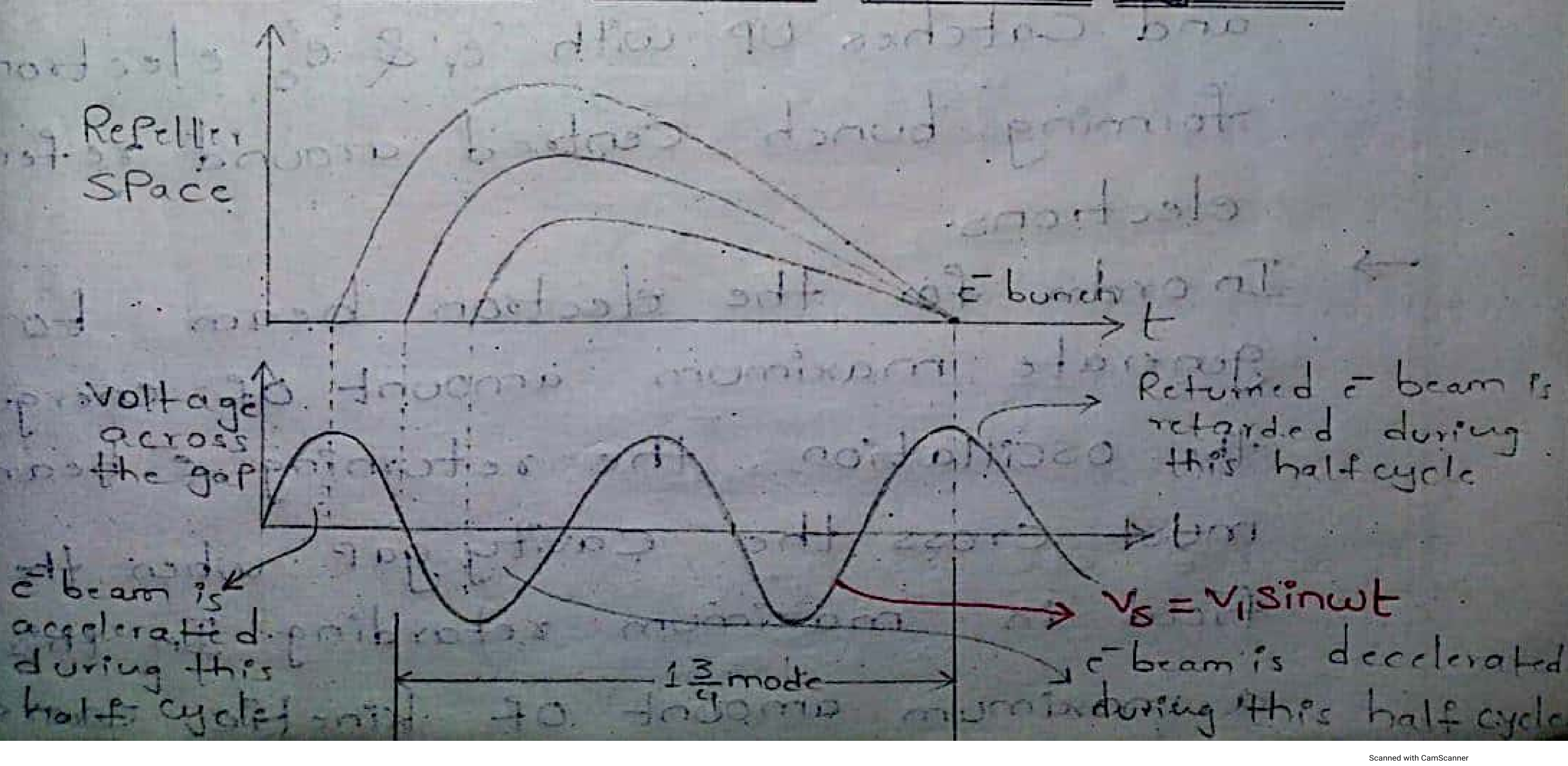
* Further, the electron that is emitted by the gun after reference electron experiences highly negative potential at the cavity.

* This electron is generally known as "late electron" (e_l) and moves with a very low velocity inside the tube. The penetration level of the late electron

into the repeller space is least thus takes a minimal amount of time to get repelled back.

* It is to be noted that, due to deep penetration in the repeller region, e_e will take more time than e_r while returning towards the catcher.
* This change in velocity of moving e^- is known as "velocity modulation", all the electrons get bunched while returning towards the catcher cavity.
* So, in this way bunch of electrons reaches the catcher cavity. This bunching of electrons leads to cause "current modulation" inside the tube. Therefore, at the time of returning, the bunched electrons transfer the maximal of their energy to the catcher cavity.
* Thereby, leading to cause "oscillations" inside the tube.

Applegate diagram of Reflex klystron:-



→ The early electron 'e_e' that passes through the gap before the reference electron 'e_r' experiences a maximum positive voltage across the gap and this electron is accelerated. It moves with greater velocity and penetrate deep into repeller space. The return time for electron e_e is greater as the depth of penetration into the repeller space is more.

→ The reference electron 'e_r' that passes through the gap when the gap voltage is zero and gets unaffected by the gap voltage. This moves towards the repeller and gets reflected by the -ve voltage on the repeller.

→ The late electron 'e_l' that passes through the gap later than reference electron 'e_r' experiences a maximum -ve voltage and moves with a retarding velocity. The return time is shorter as the penetration into repeller space is less and catches up with 'e_r' & 'e_e' electrons forming bunch centred around reference electrons.

→ In order for the electron beam to generate maximum amount of energy to the oscillation, the returning $\bar{e}$ beam must cross the cavity gap when the gap field is maximum retarding. In this way a maximum amount of kinetic energy

Can be transferred from returned electrons to the cavity walls.

→ Bunch occurs once, per cycle, centred around reference electron 'e_r'.

→ The optimum transit times should be

$\boxed{T = n + 3/4}$ Where $n = 0, 1, 2, 3.$

→ $1(3/4)$ is the dominant mode because it has high efficiency.

→ It is a Low power generator of 10-500mw output at a frequency range of 1 to 25GHz

The efficient is about 20 to 30%.

** Reflex klystron - Velocity modulation & bunching Parameter derivation

The schematic diagram of a Reflex - klystron is shown below:

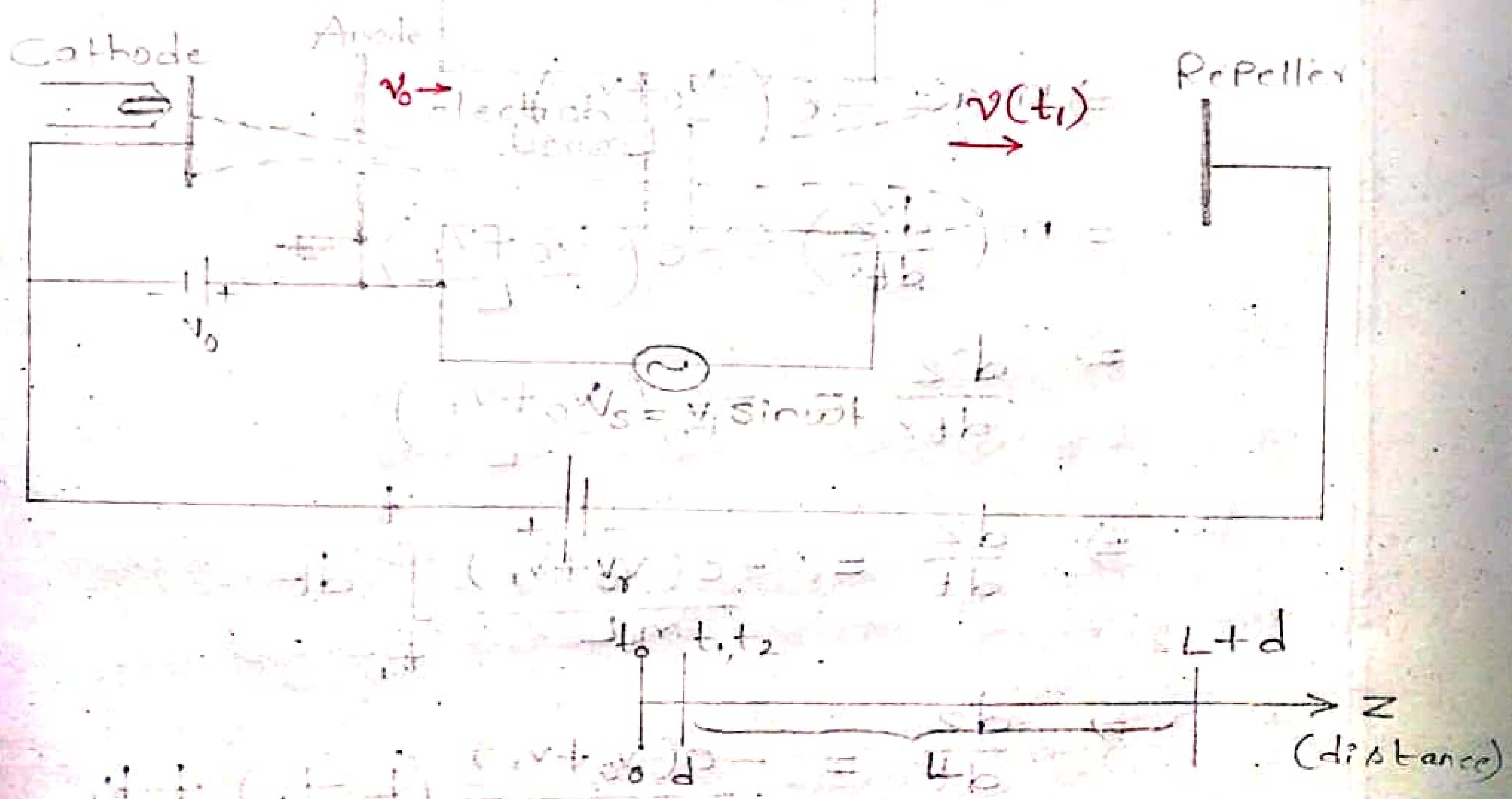


fig - Reflex klystron

The operation of klystron Amplifier is similar to two-cavity klystron Amplifier.

We know that,

The DC beam electron velocity is given by,

$$v_0 = \sqrt{\frac{2eV_0}{m}} = 0.593 \times 10^6 \sqrt{V_0}.$$

The expression for velocity modulation is given by,

$$v(t_1) = v_0 \left[1 + \frac{\beta_i V_1}{2V_0} \sin(\omega t_1 - \theta_g/2)\right]$$

The Retarding electric field is given by,

$$E = \frac{V_0 + V_r + V_1 \sin\omega t}{L}$$

$\because (V_0 + V_r) >> V_1 \sin\omega t$, $\boxed{E = \frac{V_0 + V_r}{L}}$

The force experienced by electron due to Retarding electric field is given by,

$$F = -eE$$

$$\Rightarrow ma = -e\left(\frac{V_0 + V_r}{L}\right)$$

$$\Rightarrow m\left(\frac{d^2z}{dt^2}\right) = -e\left(\frac{V_0 + V_r}{L}\right)$$

$$\Rightarrow \frac{d^2z}{dt^2} = \frac{-e}{m}\left(\frac{V_0 + V_r}{L}\right)$$

$$\Rightarrow \frac{dz}{dt} = \frac{-e(V_0 + V_r)}{mL} \int_{t_1}^{t} dt$$

$$\Rightarrow \frac{dz}{dt} = \frac{-e(V_0 + V_r)}{mL}(t - t_1) + K_1$$

$$\Rightarrow \frac{dz}{dt} = \frac{-e(v_0+v_r)}{mL}(t-t_1)+K_1$$

At $t=t_1$;

$$\frac{dz}{dt} = v(t_1)\ ; \quad \boxed{v(t_1) = K_1}$$

$$\therefore \frac{dz}{dt} = \frac{-e(v_0+v_r)}{mL}(t-t_1)+v(t_1)$$

Again integrating on both sides w.r.to 't' we get,

$$Z = \frac{-e(v_0+v_r)}{mL}\int_{t_1}^{t}(t-t_1)\,dt + v(t_1)\int_{t_1}^{t} dt$$

$$\Rightarrow Z = \frac{-e(v_0+v_r)}{2mL}(t-t_1)^2 + v(t_1)(t-t_1)+K_2$$

At $t=t_1$; $\boxed{K_2 = d}$

$$\therefore Z = \frac{-e(v_0+v_r)}{2mL}(t-t_1)^2 + v(t_1)(t-t_1)+d$$

At $t=t_2$;

$$\cancel{d} = \frac{-e(v_0+v_r)}{2mL}(t_2-t_1)^2 + v(t_1)(t_2-t_1)+\cancel{d}$$

$$\Rightarrow \frac{e(v_0+v_r)}{2mL}(t_2-t_1)^{\cancel{2}} = \cancel{(t_2-t_1)}\,v(t_1)$$

$$\Rightarrow \boxed{t_2-t_1 = \frac{2mL}{e(v_0+v_r)}v(t_1)}$$

Here, $(t_2-t_1) \rightarrow$ Round Trip transit time

$$T = (t_2 - t_1) = \frac{2mL}{e(V_0+V_r)} v_0 \left[1+\frac{\beta_i V_1}{2V_0}\sin(\omega t_1 - \theta_g/2)\right]$$

$$\boxed{T_0' = \frac{2mLv_0}{e(V_0+V_r)}} \rightarrow \text{(DC round trip time)}$$

$$T = T_0'\left[1+\frac{\beta_i V_1}{2V_0}\sin(\omega t_1 + \theta_g/2)\right]$$

$$\omega T = \omega T_0'\left[1+\frac{\beta_i V_1}{2V_0}\sin(\omega t_1 - \theta_g/2)\right]$$

Let, $\boxed{\theta_0' = \omega T_0'} \rightarrow$ (DC round trip transit angle)

Bunching Parameter,

$$\boxed{X' = \frac{\beta_i V_1}{2V_0}\theta_0'}$$

$$\therefore \boxed{\begin{aligned}\omega T &= \omega(t_2 - t_1)\\ &= \theta_0' + X'\sin(\omega t_1 - \theta_g/2)\end{aligned}}$$

** Reflex Klystron - Output Power & efficiency**

The diagram is same as earlier.

The condition for maximum energy transfer to the cavity walls by electron bunch, the round trip transit angle of centre of bunch should be,

$$\boxed{\theta_0' = \omega T_0' = 2\pi N = 2\pi(n - 1/4)}$$

Here, n = any integer representing cycle number

$N = (n - 1/4)$ is the mode number

$n = 1$; $N = \frac{3}{4}$ mode; $\boxed{n = 2 \Rightarrow N = 1\frac{3}{4} \text{ mode}}$

(dominant mode for which maximum efficiency occurs)

The beam current at cavity gap is a periodic waveform and is given by,

$$i_2 = - \sum_{n=1}^{\infty} 2 I_0 J_n(n x') \cos\left[n(\omega t_2 - \theta_0' - \theta_g)\right]$$

(indicating the direction of current is in -ve z-direction)

By expanding the above expression we get,

$$i_2 = -I_0 - 2 I_0 J_1(x') \cos[\omega t_2 - \theta_0' - \theta_g] - 2 I_0 J_2(2x') \cos[2(\omega t_2 - \theta_0' - \theta_g)] - \cdots$$

(Fundamental component (or) first harmonic)

The fundamental component of current induced into the cavity is given by,

$$i_2(\text{ind}) = -\beta_i \, 2 I_0 J_1(x') \cos(\omega t_2 - \theta_0' - \theta_g).$$

The magnitude of current induced into the cavity is given by,

$$\boxed{I_2(\text{indu}) = \beta_i \, 2 I_0 J_1(x')}$$

Where, $\beta_i \rightarrow$ Beam coupling coefficient

Dc. Power is given by,

$$P_{dc} = V_0 I_0$$

Ac Power obtained from the cavity is given by,

$$P_{ac} = \frac{V_1}{\sqrt{2}} \times \frac{I_2(indu)}{\sqrt{2}}$$

$$= \frac{V_1 I_2(indu)}{2}$$

$$= \frac{V_1 \beta_i \cancel{2} I_0 J_1(x')}{\cancel{2}}$$

$$= \beta_i V_1 I_0 J_1(x')$$

$$\boxed{\text{efficiency, } \eta = \frac{P_{ac}}{P_{dc}} = \frac{\beta_i V_1 \cancel{I_0} J_1(x')}{V_0 \cancel{I_0}}}$$

From bunching parameter

$$x' = \frac{\beta_i V_1}{2V_0} \theta_0'$$

$$\Rightarrow V_1 = \frac{x' 2V_0}{\beta_i \theta_0'}$$

Now,

$$\eta = \frac{\cancel{\beta_i} \cdot x' 2 \cancel{V_0} J_1(x')}{\cancel{V_0} \cancel{\beta_i} \theta_0'}$$

$$= \frac{2x' J_1(x')}{\theta_0'}$$

$$\Rightarrow \eta = \frac{2x' J_1(x')}{(2\pi n - \pi/2)} \qquad \left(\because \theta_0' = 2\pi(n - \pi/4) \right.$$
$$\left. = 2\pi n - \pi/2 \right)$$

$$\therefore \boxed{\eta = \frac{2x' J_1(x')}{(2\pi n - \pi/2)}}$$

for dominant mode i.e., for $n = 2$ (or) $N = 1\frac{3}{4}$.

$$x' J_1'(x') = 2.45; \quad x' = 2.408$$
$$J_1(x') = 0.52$$

$$\boxed{\begin{aligned} \eta_{max} &= \frac{2(2.408)(0.52)}{2\pi(2) - \pi/2} \\ &= 22.7\% \end{aligned}}$$

Dc round trip transit time is given by.

$$T_0' = \frac{2mL}{e(V_0 + V_r)} \, v_0$$

$$\Rightarrow \omega T_0' = \frac{2m\omega L}{e(V_0 + V_r)} \cdot v_0$$

$$\Rightarrow \omega T_0' = \frac{2m\omega L}{e(V_0 + V_r)} \cdot \sqrt{\frac{2eV_0}{m}} \qquad \left(\because v_0 = \sqrt{\frac{2eV_0}{m}} \right)$$

$$\Rightarrow (2\pi n - \pi/2) = \frac{2m\omega L}{e(V_0 + V_r)} \sqrt{\frac{2eV_0}{m}}$$

$$\Rightarrow (2\pi n - \pi/2)^2 = \frac{4m^2\omega^2 L^2}{e^2(V_0 + V_r)^2} \cdot \frac{2eV_0}{m}$$

$$\Rightarrow \boxed{\frac{V_0}{(V_0 + V_r)^2} = \frac{(2\pi n - \pi/2)^2}{8\omega^2 L^2} \cdot \frac{e}{m}}$$

Where, $V_0 \rightarrow$ cathode voltage; $V_r \rightarrow$ repeller voltage

$e/m = 1.759 \times 10^{11}$ C/Kg

Reflex Klystron - Electronic Admittance

The schematic diagram of Reflex klystron is same as shown in earlier.

fig: Schematic diagram

The induced current at the cavity in Phasor form is given by,

$$\boxed{i_{2(indu)} = \beta_i\, 2 I_0 J_1(x')\, e^{-j\theta_0'}}$$

The voltage gap across gap at time 't_2' in Phasor form is given by

$$\boxed{V_2 = V_1 e^{-j\pi/2}}$$

The electronic admittance of Reflex klystron is given by,

$$Y_e = \frac{i_{2(indu)}}{V_2}$$

$$= \frac{2\beta_i I_0 J_1(x') e^{-j\theta_0'}}{V_1 e^{-j\pi/2}}$$

$$= \frac{2\beta_i I_0 J_1(x') e^{j(\pi/2 - \theta_0')}}{V_1} \longrightarrow ①$$

Bunching Parameter of Reflex klystron is given by,

$$x' = \frac{\beta_i V_1}{2V_0}\theta_0'$$

$$\Rightarrow V_1 = \frac{2V_0 x'}{\beta_i \theta_0'} \longrightarrow ②$$

Substituting eq^n-② in eq^n-① we get

$$Y_e' = \frac{2\beta_i I_0 J_1(X')}{2V_0 X'} \beta_i \theta_0' \, e^{j(\pi/2 - \theta_0')}$$

$$\boxed{Y_e = \frac{I_0}{V_0} \cdot \frac{\beta_i^{\,2} \theta_0'}{2} \cdot \frac{2J_1(X')}{X'} \, e^{j(\pi/2 - \theta_0')}} \rightarrow \text{DC beam conductance}$$

(exponential form)

Remember, Admittance = Conductance + susceptance
$(G_e + jB_e)$

Here, θ_0' = DC round trip transit angle
$= (n - 1/4) 2\pi$
$= 2\pi N$

Where, 'n' is any integer
'N' represents mode

We know that, the dominant mode of Reflex klystron is $1\frac{3}{4}$ and this has high efficiency.

The equivalent circuit of Reflex klystron is given by,

L&C are energy storage elements

$G_e \rightarrow$ copper/conductance losses inside the cavity.

$G_B \rightarrow$ Beam Loading conductance.

$G_L \rightarrow$ Load conductance

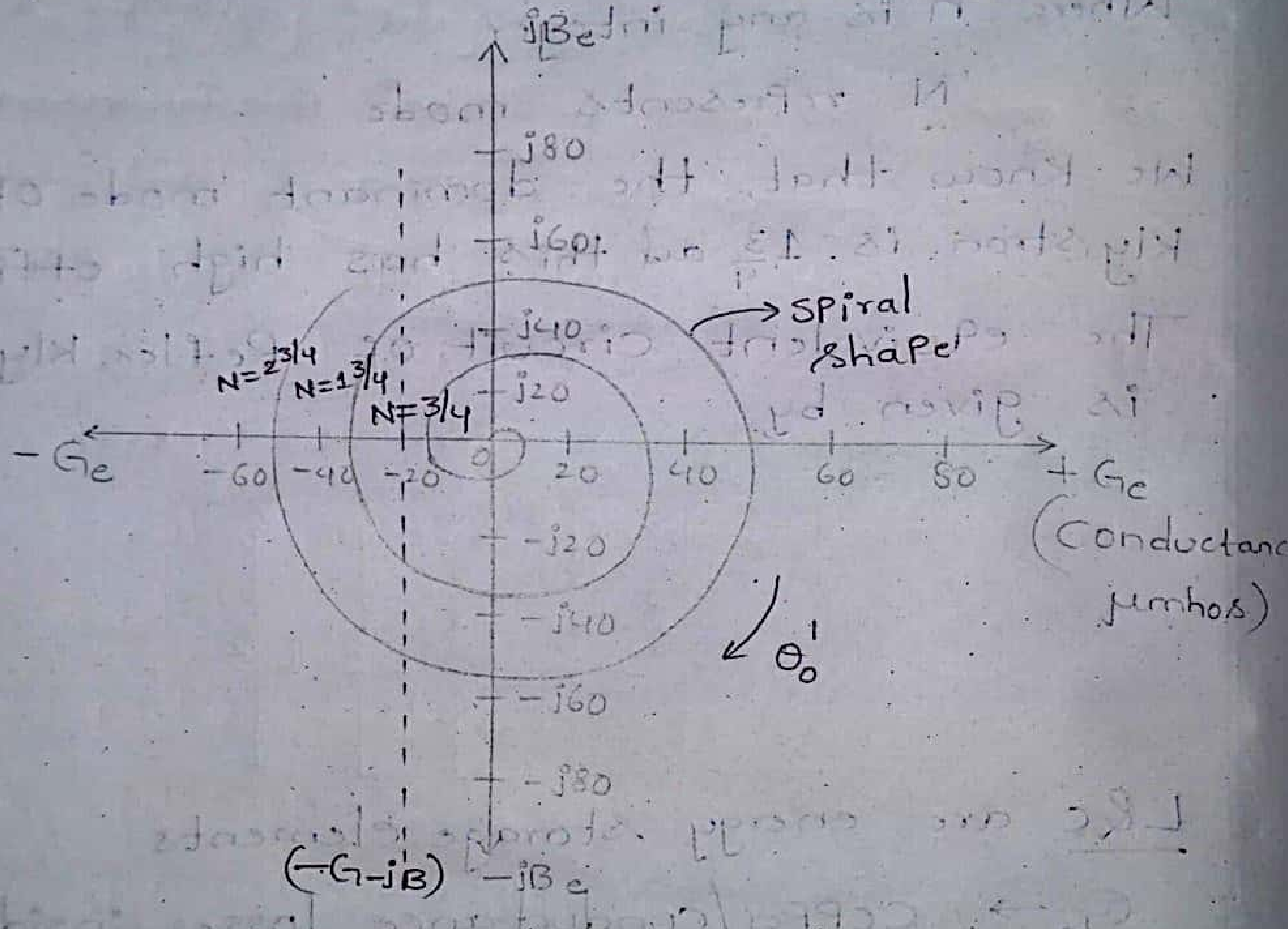
The total conductance, $G = G_e + G_B + G_L$
$= \frac{1}{R_{sh}}$

Where, $R_{sh} \rightarrow$ effective shunt resistance

Note:- The magnitude of -ve real part of Admittance must not be less than the total conductance to maintain oscillations in the cavity.

i.e., $\boxed{|-G_e| \geq G}$

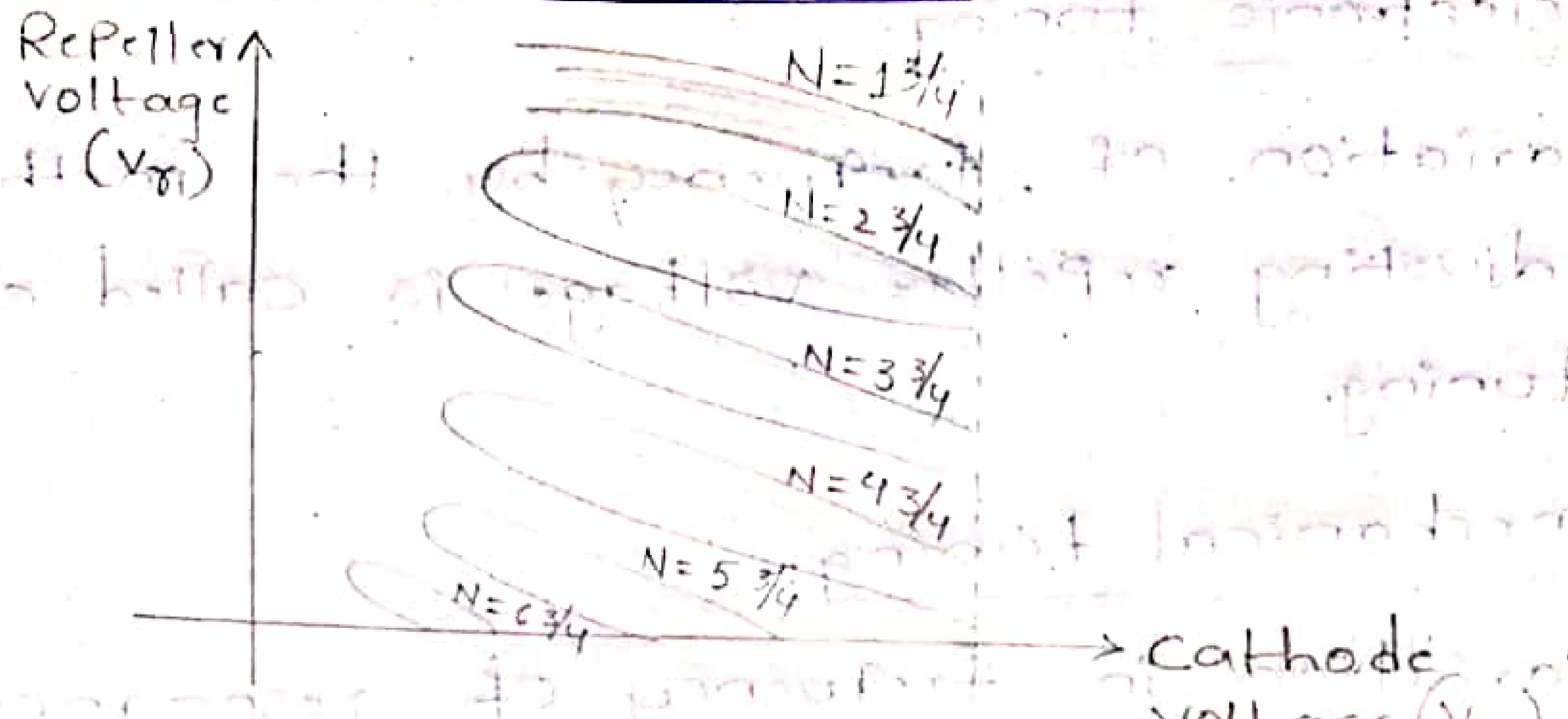
If you plot the exponential form of Admittance in a rectangular form plot a "spiral structure" is formed.



To the values of θ_0', lying to the left of the dotted line drawn, the oscillations will occur. This point should be focussed while considering the electronic Admittance of a Reflex klystron.

Performance characteristics of Reflex klystron

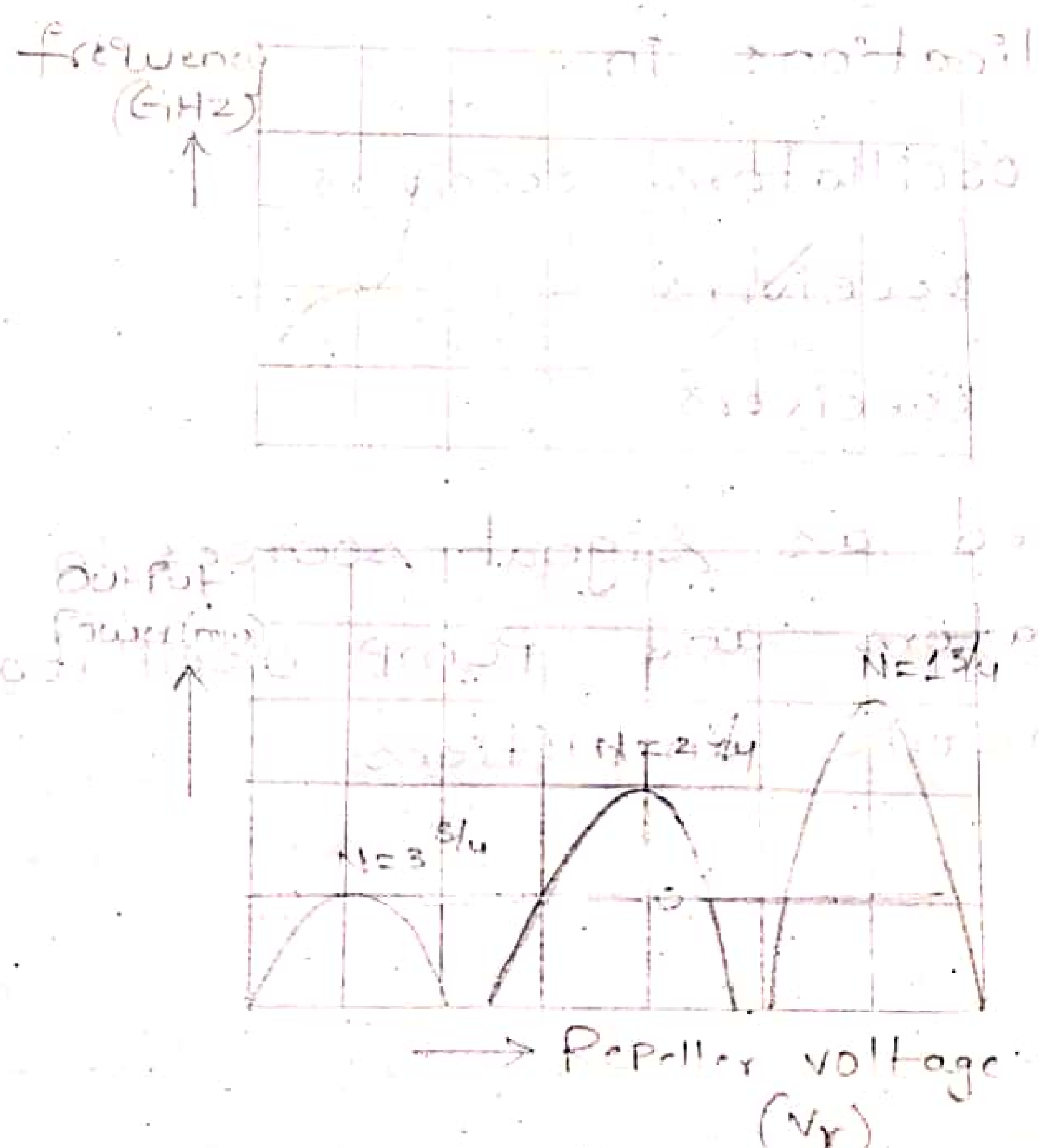
① Voltage characteristics:-



The oscillations are obtained based on specific combinations of (V_0, V_r, V_1).

$$\boxed{T_{optimum} = (n + 3/4)T}$$

② Output Power & frequency characteristics:-



Maximum Power is obtained at N=1 3/4.

Depending on the values of V_r, output Power and frequency varies.

Electronic & mechanical tuning in Reflex Klystron:-

electronic tuning:-

Variation of frequency by the method of adjusting repeller voltage is called electronic tuning.

mechanical tuning:-

Variation in frequency of resonance of cavity by varying its dimension by a mechanical method like adjusting screws is called mechanical tuning.

Applications of reflex klystron:-

As reflex klystrons are oscillators, thus find applications in,

* Local oscillators receivers
* radar receivers
* radio receivers
* Utilized as signal sources in microwave generators and pump oscillators of parametric amplifiers.

———o———

04/05/21

UNIT 5 :- Waveguide components and Applications

Waveguide Microwave functions:-

Consider a Waveguide having 4 Ports. If the Power is applied to one Port, it goes through all the 3 Ports in some proportions where some it might reflect back from the same Port. This concept is clearly depicted in the following figure.

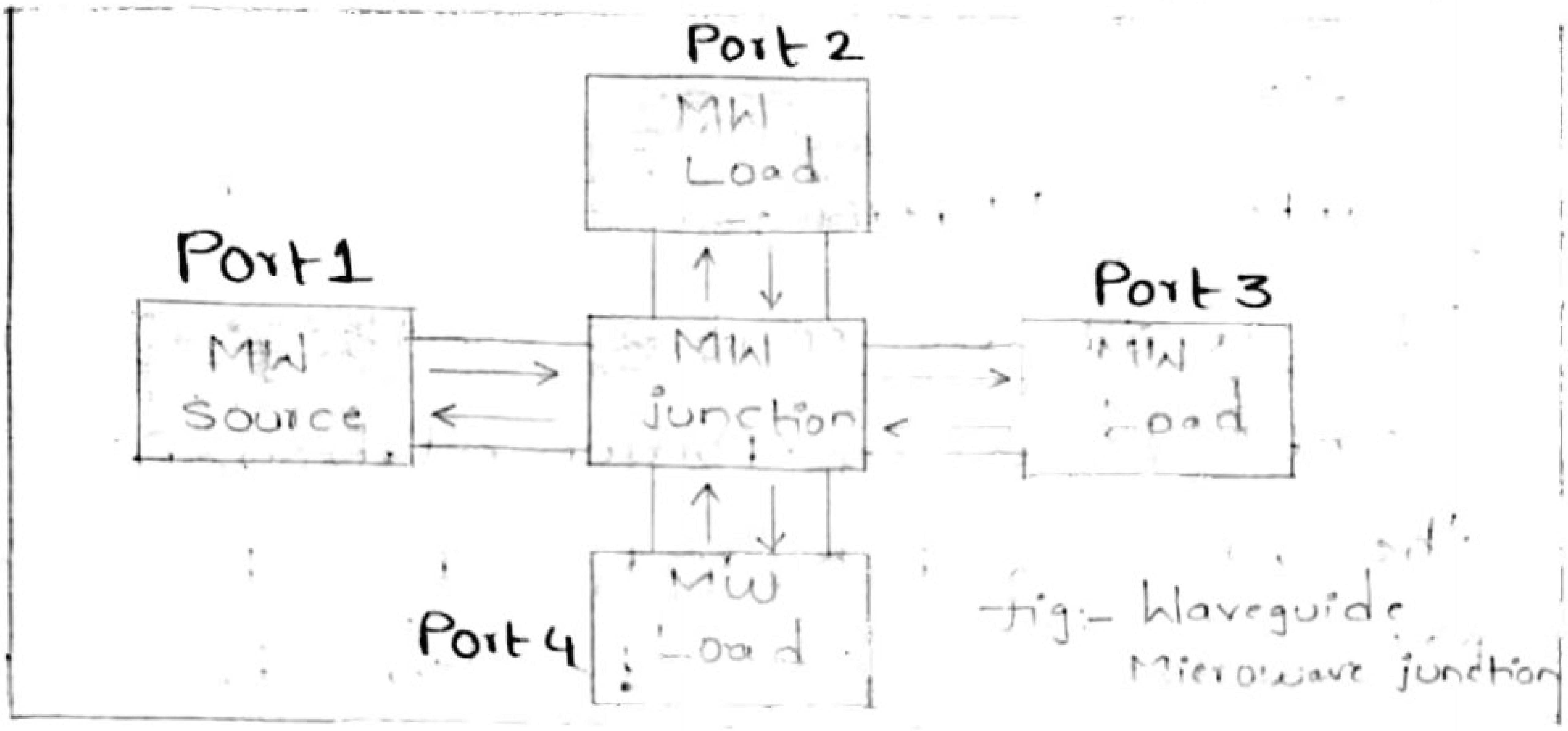


fig:- Waveguide Microwave junction

Scattering Parameters:-

For a two-port network, as shown in the following figure, if the power is applied at one port, as we just discussed, most of the power escapes from the other port, while some of it reflects back to the same port. In the following figure if V_1 or V_2 is applied, then I_1 or I_2 current flows respectively.

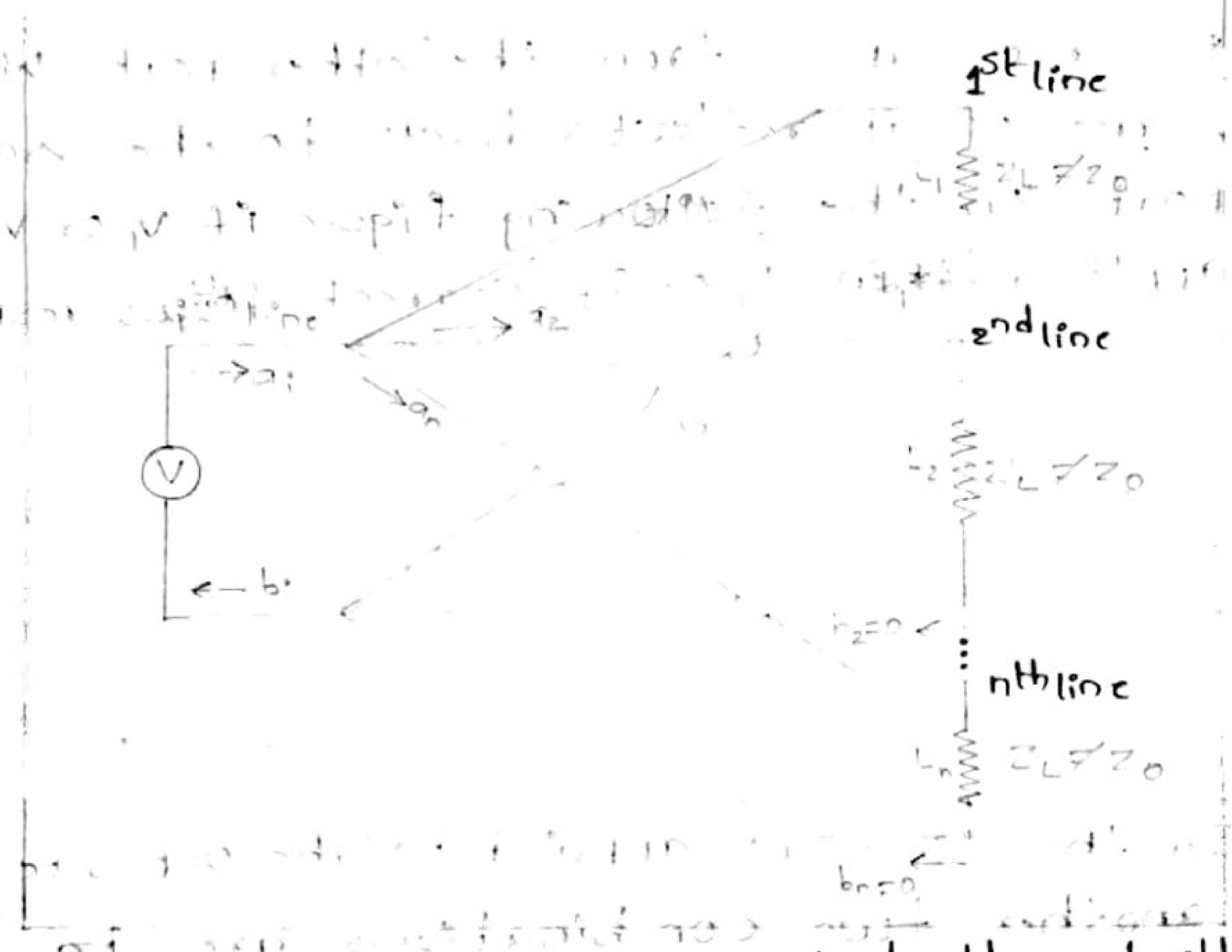
fig: Structure of a 2-Port network

If the source is applied to the opposite port, another two combinations are to be considered. So, for a two-port network, $2 \times 2 = 4$ combinations are likely to occur. The travelling waves with associated powers when scatter out through the ports, the Microwave junction can be defined by S-Parameters (or Scattering Parameters, which are represented in a matrix form, called as "Scattering Matrix".

Scattering Matrix :-

It is a square matrix which gives all the combinations of power relationships between the various input and output ports of a microwave junction. The elements of this matrix are called "Scattering coefficients" (or) "Scattering S Parameters".

consider the following figure.



Here, the source is connected through ith line while a_1 is the indicent wave and b_1 is the reflected wave.

If a relation between b_1 and a_1 is given, it would be as follows:

$b_1 =$ (reflection coefficient) $a_1 = S_{1i}\, a_1$

Where,

$S_{1i} \rightarrow$ Reflection coefficient of 1^{st} line

$1 \rightarrow$ Reflection from 1^{st} line

$i \rightarrow$ source connected at i^{th} line

If the impedance matches, then the Power gets transferred to the load. Unlikely, if the load impedance doesn't match with the characteristic impedance, then the reflection coefficient occurs. That means, Reflection coefficient occurs if

$$\boxed{Z_L \neq Z_0}$$

However, if this mismatch is there for more than one Port example 'n' Ports, then $i = 1$ to n.

Therefore, we have

$$b_1 = S_{11}a_1 + S_{12}a_2 + S_{13}a_3 + \cdots\cdots + S_{1n}a_n$$

$$b_2 = S_{21}a_1 + S_{22}a_2 + S_{23}a_3 + \cdots\cdots + S_{2n}a_n$$

$$\vdots$$

$$b_n = S_{n1}a_1 + S_{n2}a_2 + S_{n3}a_3 + \cdots\cdots + S_{nn}a_n$$

When this whole thing is kept in a matrix form,

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_n \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & \cdots & S_{1n} \\ S_{21} & S_{22} & S_{23} & \cdots & S_{2n} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ S_{n1} & S_{n2} & S_{n3} & \cdots & S_{nn} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_n \end{bmatrix}$$

Here, the column matrix [b], corresponds to the reflected wave or the output while the matrix [a] corresponds to the incident waves (or) the input. The scattering column matrix [s] which is of the order of $n \times n$ contains the reflection coefficients and transmission coefficients. Therefore,

$$\boxed{[b] = [S][a]}$$

Properties of [S] matrix:-

The scattering matrix is indicated as [S] matrix. There are few standard properties for [S] matrix. They are-

1. [S] is always a square matrix of order $n \times n$.

   i.e., $[S]_{n \times n}$

2. [S] is a symmetric matrix

   i.e., $\boxed{S_{ij} = S_{ji}}$

3. [S] is a unitary matrix.

   i.e., $[S][S]^* = [I]$

4. The sum of the products of each term of any row (or) column multiplied by the complex conjugate of the corresponding terms of any other row (or) column is zero. i.e.,

   $$\sum_{i=j}^{n} S_{ik} \cdot S_{ik}^* = 0 \cdot \text{ for } k \neq j$$

$K = (1, 2, 3, \ldots n)$ and $j = (1, 2, 3, \ldots, n)$.

5. If the electrical distance between some k^{th} port and the junction is $\beta_K I_K$, then the coefficients of S_{ij} involving k, will be multiplied by the factor $e^{-j\beta_K I_K}$.

Here, $I \rightarrow$ applied energy

$\beta \rightarrow$ Phase constant

Waveguide Junction:-

→ Waveguide junctions are used to enable power in a waveguide to be split, combined (or) for some extracted.

→ There are a number of different types of waveguide junction that can be used, each type having different properties - the different types of waveguide junction affect the energy contained within the waveguide in different ways.

→ The common types of waveguide junction include the "E-type", "H-type", "Magic type" and "Hybrid Ring junctions".

→ The different forms of waveguide junction have different properties and this means that they are applicable for different applications. Having an understanding of their different properties enables the correct type to be chosen.

Waveguide junction types:-

The main types of waveguide junction are listed below:

E-type T-junction:- The E-type waveguide junction gains its name because the top of the "T" extends from the main waveguide in the same

Plane as the electric field in the waveguide.

H-type T-junction:- The H-type waveguide junction gains its name because top of the "T" in the T-junction is parallel to the plane of the magnetic field, H lines in the waveguide.

Magic T-junction:- The magic T-junction is effectively a combination of the E-type and H-type waveguide junctions.

Hybrid Ring Waveguide junction:- This is another form of waveguide junction that is more complicated than either the basic E-type (or) H-type waveguide junction. It is widely used within radar system as a form of duplexer.

***** E-plane T junction:-**

→ It is mostly considered when we are transmitting electric field through a waveguide.

→ This type of waveguide junction is formed by attaching a single waveguide to the broader dimension of a Rectangular waveguide.

→ It is called an E-type T junction because the junction arm i.e., the top of the "T" extends from the main waveguide in the same direction as the E-field.

→ It is characterized by the fact that the outputs from this form of waveguide junction are 180° out of phase with eachother.

Construction:-

→ The basic construction of the Waveguide junction shows the three Port Waveguide device.

→ Although it may be assumed that the input is the single Port and the two outputs are those on the top section of the "T," actually any port can be used as the input, the other two being outputs.

→ Each Port is considered as one arm. Intotal, there are 3 arms.

→ The two Ports (Port 1 & Port 2) are on the same straight line and hence they are considered to be "collinear Ports". Mostly Collinear Ports are used as o/p Ports.

→ The Port left alone, is considered to be i/p Port.

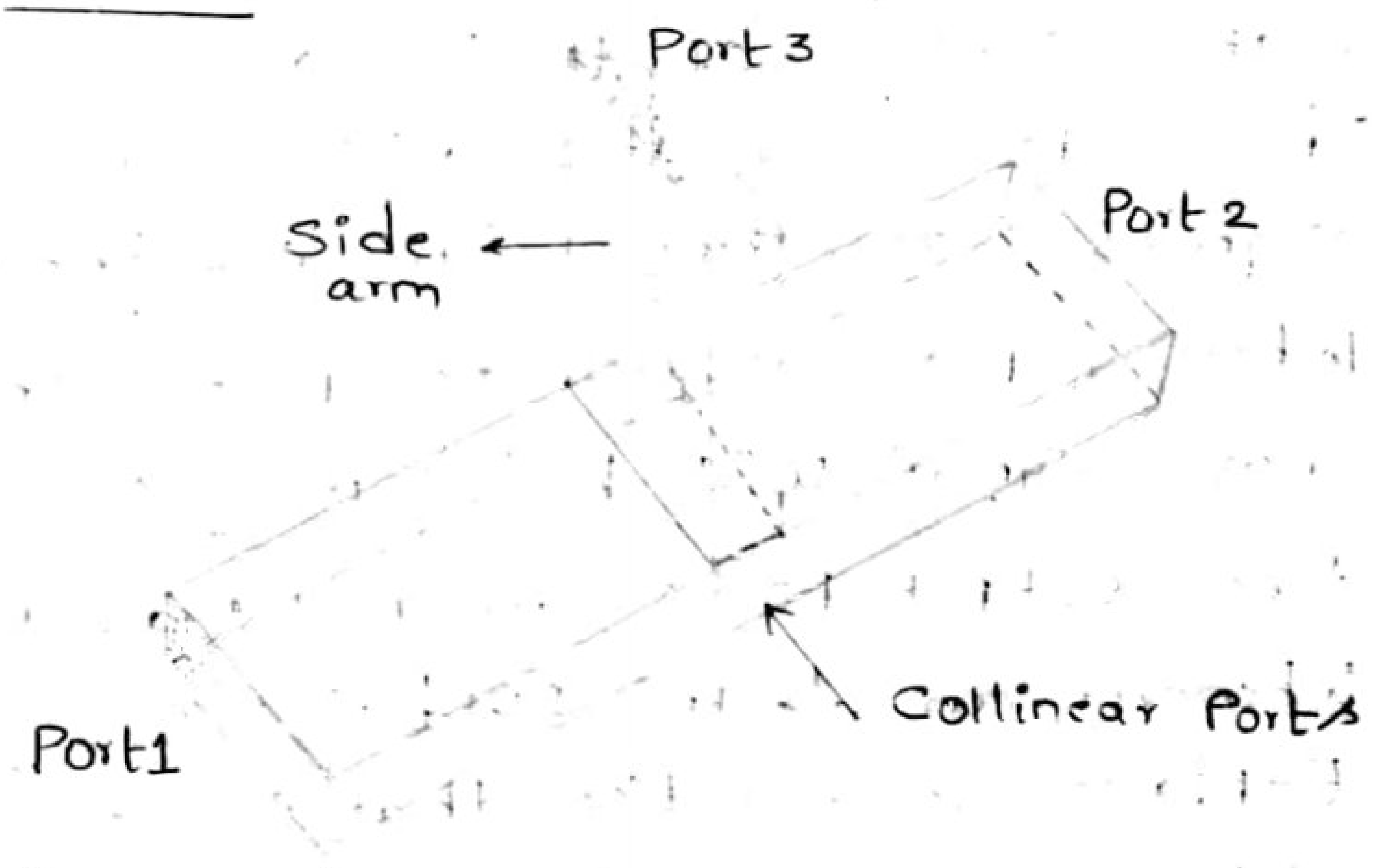


fig:- E-Plane T-junction

Operation:-

→ To see how the Waveguide junction operates, and how the 180° phase shift occurs, it is necessary to look at the drawn electric field. The magnetic field is omitted from the diagram for simplicity.

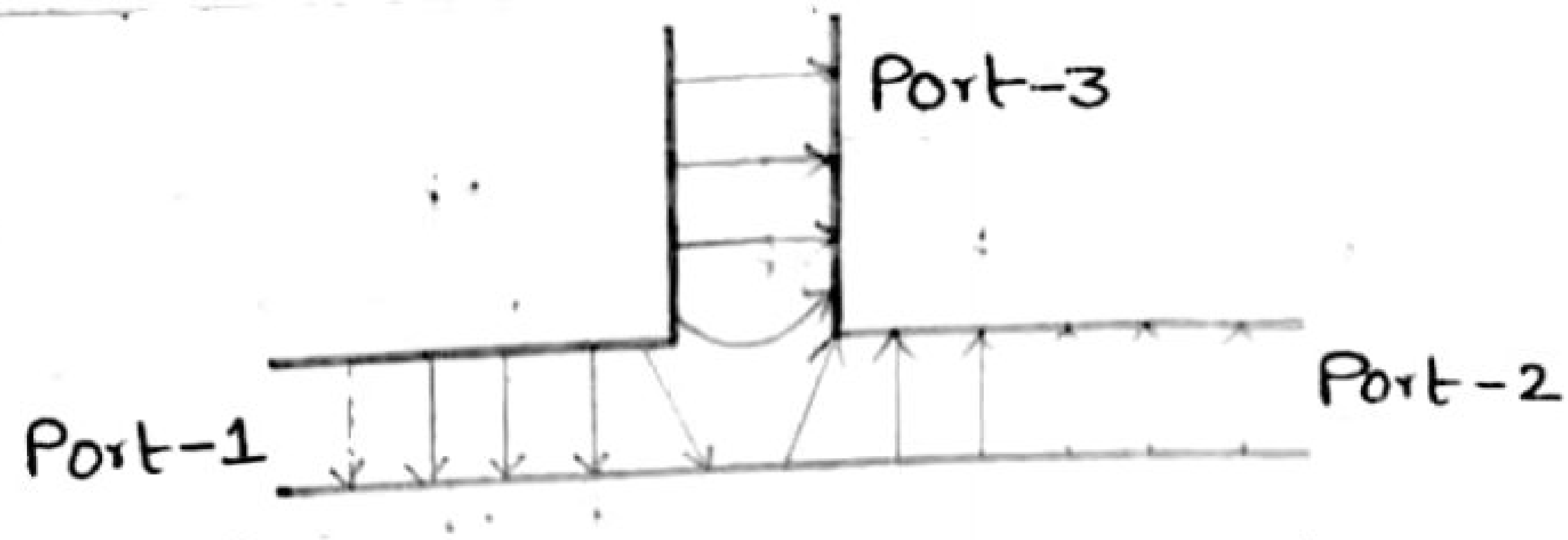


fig:- E-type junction fields

→ It can be seen from the electric field that When it approaches the T-junction itself the electric field lines becomes distorted and bend.

→ They split so that the "Positive" end of the line remains with the top side of the right hand section in the diagram, but the "negative" end of the field lines remain with the top side of the left hand section.

→ In this way, the signals appearing at either section of the "T" are out of Phase. These phase relationships are Preserved, if signals enter from either of the other Ports.

→ When input is given to Port 3, the micro-wave signal will be coming out from the two output Ports i.e., Port 1 and Port 2.

→ Whenever an electric field is coming from E-Plane Tee junction, that is from E-arm (or) Side arm (or) Port 3 it will be coming out from the two o/p Ports (Port 1 & Port 2) but with 180° Phase shift.

→ It can be shown more clearly through a graph as shown below:

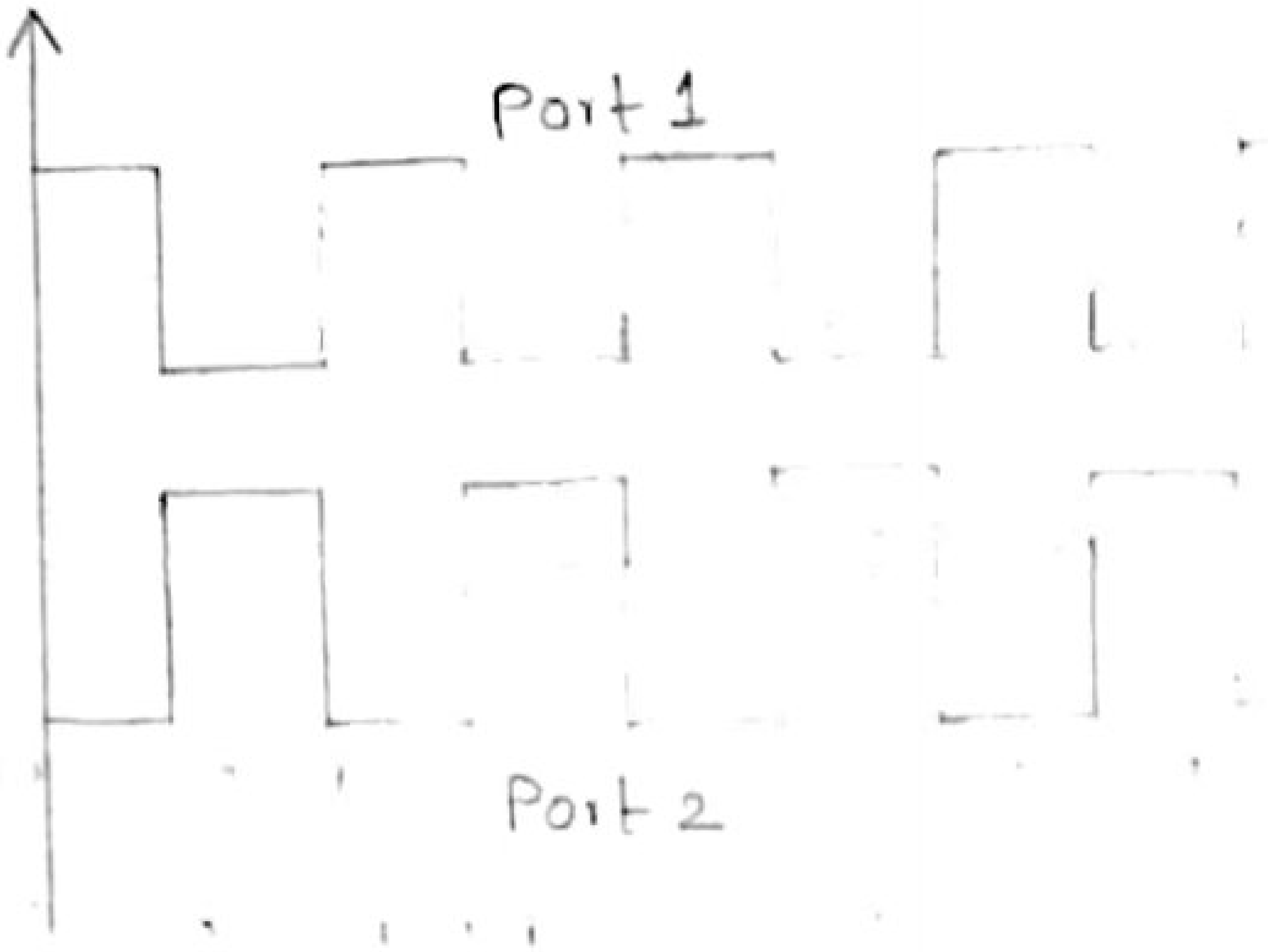
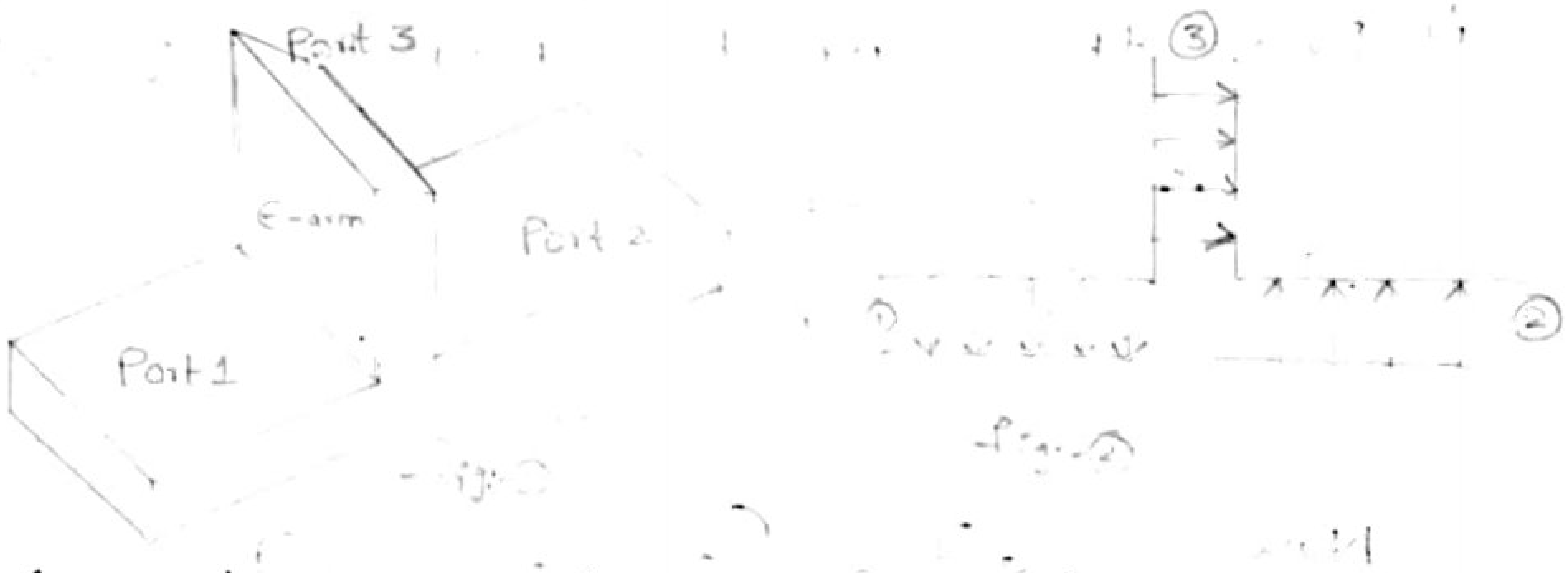


fig:- E-plane 180° phase shift

→ The side arm is parallel to the electric field lines. So, that E-plane Tee junction is also known as "Voltage-Series junction".

S-Matrix Calculations - (E-Plane Tee)



→ A rectangular slot is cut along the broader dimension of a long waveguide and a side arm is attached.

→ Ports 1 & 2 are the collinear ports and port 3 is the E-arm.

→ When wave is made to propagate into Port 3, the two outputs, Port 1 & Port 2 will have a phase shift of 180°.

→ The scattering matrix of E-Plane Tee can be used to describe its ports.

(i) [S] is a 3X3 matrix, since there are 3 ports.

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

(ii) Scattering coefficients $\boxed{S_{23} = -S_{13}}$

(180° phase shift) Since outputs at Port1 and Port2 are out of phase by 180° with an input at port 3.

(iii) If Port3 is perfectly matched to the junction and there are no reflections at Port3, then

$$\boxed{S_{33} = 0}$$

(iv) From the symmetric Property, $\boxed{S_{ij} = S_{ji}}$

$$\therefore \quad S_{12} = S_{21}$$

$$S_{13} = S_{31}$$

$$S_{23} = S_{32} = -S_{13}$$

Now,
$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & -S_{13} \\ S_{13} & -S_{13} & 0 \end{bmatrix}$$

(v) From the unitary Property, we have

$$[S]\,[S]^* = [I]$$

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}^* = [I]$$

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array}\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & -S_{13} \\ S_{13} & -S_{13} & 0 \end{bmatrix} \begin{array}{c} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* \\ S_{12}^* & S_{22}^* & -S_{13}^* \\ S_{13}^* & -S_{13}^* & 0 \end{bmatrix} \\ \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \end{array} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\underline{\underline{R_1 C_1}}$:- $S_{11} \cdot S_{11}^* + S_{12} \cdot S_{12}^* + S_{13} \cdot S_{13}^* = 1$

$$\Rightarrow |S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1$$

$\underline{\underline{R_2 C_2}}$:- $S_{12} \cdot S_{12}^* + S_{22} \cdot S_{22}^* + (-S_{13})(-S_{13})^* = 1$

$$\Rightarrow |S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 = 1$$

$\underline{\underline{R_3 C_3}}$:- $S_{13} \cdot S_{13}^* + (-S_{13})(-S_{13})^* + 0 = 1$

$$\Rightarrow |S_{13}|^2 + |S_{13}|^2 = 1$$

$$\Rightarrow 2|S_{13}|^2 = 1$$

$$\Rightarrow |S_{13}|^2 = \frac{1}{2}$$

$$\Rightarrow S_{13} = \frac{1}{\sqrt{2}}$$

$\therefore \boxed{S_{13} = 1/\sqrt{2}}$

values are S_{11} S_{22} S_{12} ?

Let's evaluate, $R_1 C_1 = R_2 C_2$

$$|S_{11}|^2 + \cancel{|S_{12}|^2} + \cancel{|S_{13}|^2} = \cancel{|S_{12}|^2} + |S_{22}|^2 + \cancel{|S_{13}|^2}$$

$$\Rightarrow |S_{11}|^2 = |S_{22}|^2$$

$$\Rightarrow S_{11} = S_{22}$$

$\therefore \boxed{S_{11} = S_{22}}$

Now Let's calculate, $R_3 C_1$

$\underline{\underline{R_3 C_1}}$:- $S_{13} \cdot S_{11}^* + (-S_{13}) \cdot (S_{12}^*) + 0 \cdot S_{13}^* = 0$

$$\Rightarrow S_{13} \cdot S_{11}^* - S_{13} \cdot S_{12}^* = 0$$

$$\Rightarrow S_{13}(S_{11}^* - S_{12}^*) = 0$$

$$\Rightarrow S_{11}^* - S_{12}^* = 0$$

$$\Rightarrow S_{11}^* = S_{12}^*$$

$$\Rightarrow S_{11}^* = S_{12}^*$$

$$\Rightarrow S_{11} = S_{12}$$

$$\therefore \boxed{S_{11} = S_{12}}$$

We have, $\boxed{S_{11} = S_{12} = S_{22}}$

Now substitute, $S_{11} = S_{12}$ in the eqn of R_1C_1.

$$\Rightarrow R_1C_1 \Rightarrow |S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1$$

$$\Rightarrow |S_{11}|^2 + |S_{11}|^2 + \left|\frac{1}{\sqrt{2}}\right|^2 = 1$$

$$\Rightarrow 2|S_{11}|^2 + \left|\frac{1}{\sqrt{2}}\right|^2 = 1$$

$$\Rightarrow 2|S_{11}|^2 + \frac{1}{2} = 1$$

$$\Rightarrow 2|S_{11}|^2 = \frac{1}{2}$$

$$\Rightarrow |S_{11}|^2 = \frac{1}{4}$$

$$\Rightarrow S_{11} = 1/2$$

$$\therefore \boxed{S_{11} = 1/2}$$

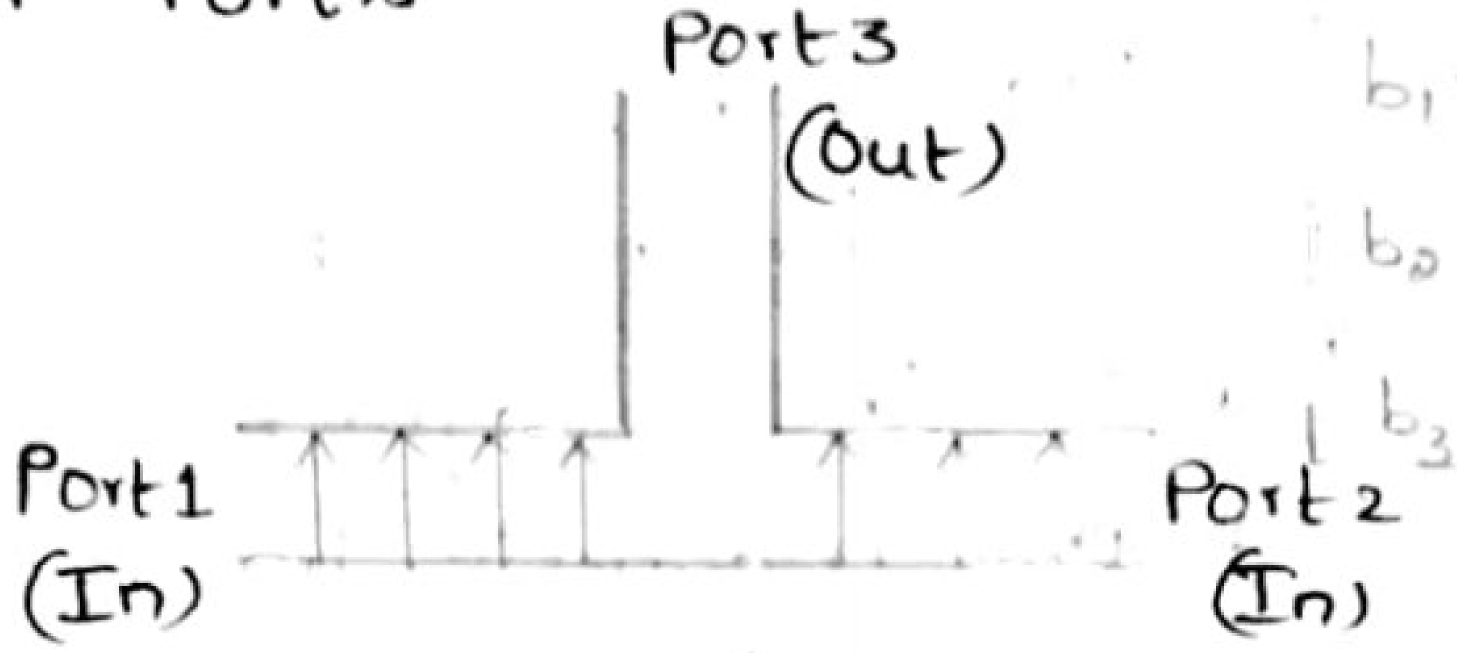
i.e., $\boxed{S_{11} = S_{12} = S_{22} = 1/2}$

$$\therefore [S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & -S_{13} \\ S_{13} & -S_{13} & 0 \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 & 1/\sqrt{2} \\ 1/2 & 1/2 & -1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 \end{bmatrix}$$

This is the S-matrix of E-Plane Tee junction.

Analysis:-

① If input is applied to Port1 and Port2 with same magnitude and same phase, then the output at Port3 is the difference of the i/p Ports.



$b_1 = \frac{a_1}{2} + \frac{a_2}{2} + \frac{a_3}{\sqrt{2}}$

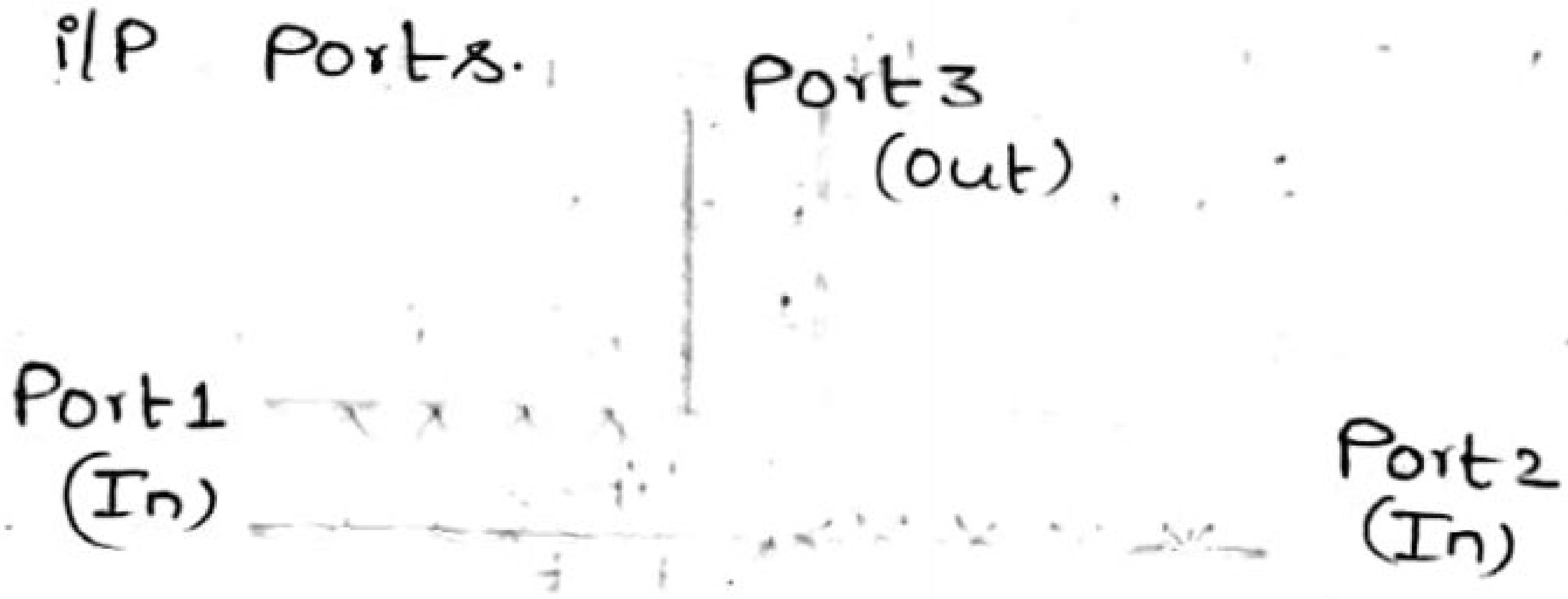
$b_2 = \frac{a_1}{2} + \frac{a_2}{2} - \frac{a_3}{\sqrt{2}}$

$b_3 = \frac{a_1}{\sqrt{2}} - \frac{a_2}{\sqrt{2}}$

Here, i/p is applied to Ports 1 & Port 2 i.e, $a_1 = a_2 = a$ (say) and hence $a_3 = 0$

$\therefore b_3 = \frac{a_1}{\sqrt{2}} - \frac{a_2}{\sqrt{2}} = \frac{a}{\sqrt{2}} - \frac{a}{\sqrt{2}} = 0$ (No o/p at Port 3)

② If input is applied to Port 1 and Port 2 with same magnitude but out of Phase, then the output at Port 3 is the sum of the i/p Ports.

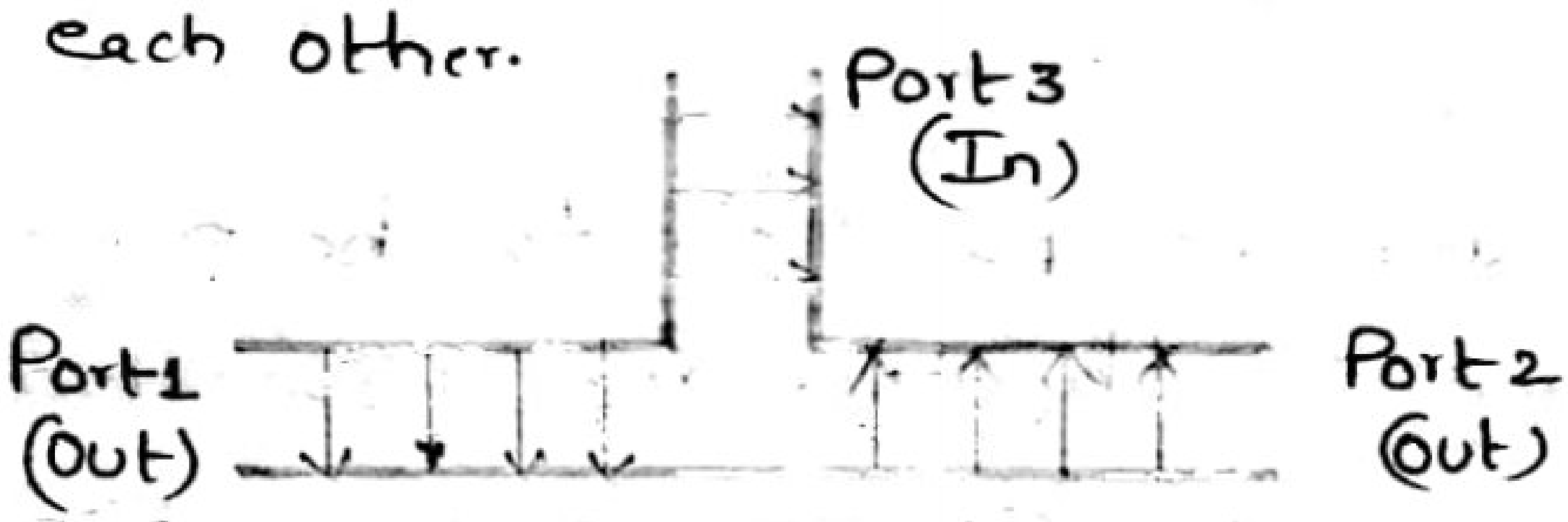


$a_1 = -a_2$

$a_1 = a, \; \& \; a_2 = -a$

$\therefore b_3 = \frac{a}{\sqrt{2}} + \frac{a}{\sqrt{2}} = \frac{2a}{\sqrt{2}} = \sqrt{2}\,a$ (Sum of the i/p Ports)

③ If input is applied at Port 3, then the output is considered at Ports 1 & Port 2 and it will be 180° out of phase with each other.



$a_1 = 0$

$a_2 = 0$

$b_1 = 0 + 0 + \frac{a_3}{\sqrt{2}} = \frac{a}{\sqrt{2}}$

$b_2 = 0 + 0 - \frac{a_3}{\sqrt{2}} = -\frac{a}{\sqrt{2}}$

$b_3 = 0 - 0 = 0$

H-Plane Tee junction:-

- This type of Waveguide junction is formed by attaching a simple waveguide to the along the broader end of a Rectangular waveguide.
- This type of Waveguide junction is called an H-type T junction because the long axis of the main top of the "T" arm is Parallel to the plane of the magnetic lines of force in the waveguide.
- It is characterized by the fact that the two outputs from the top of the "T" section in the waveguide are in phase with eachother.

Construction:-

- It consists of totally 3 Ports: Port 1, Port 2 and Port 3 respectively.
- Ports 1 and 2 are "collinear Ports" and are considered to be output Ports.
- Port 3 is considered to be input Port.

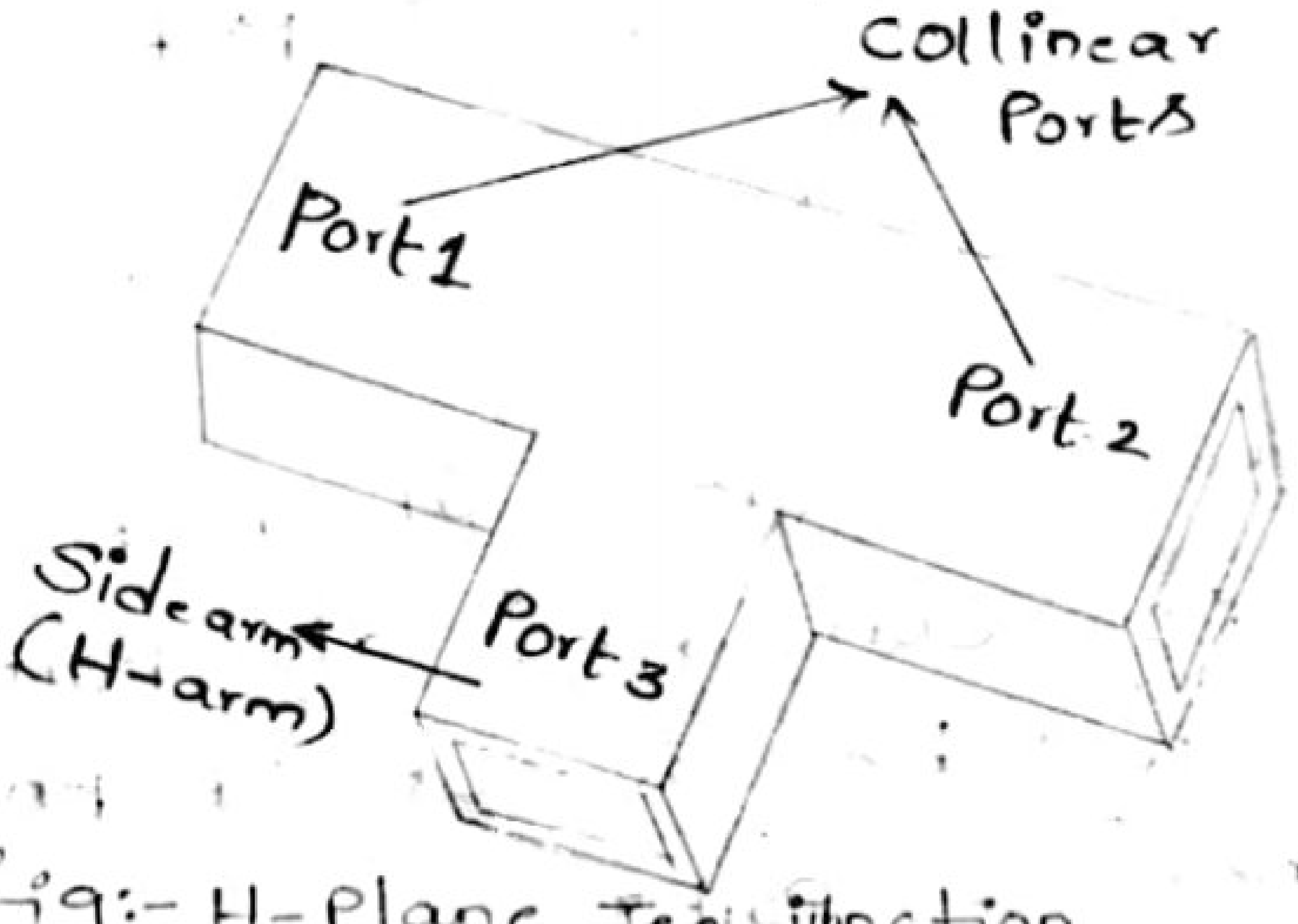
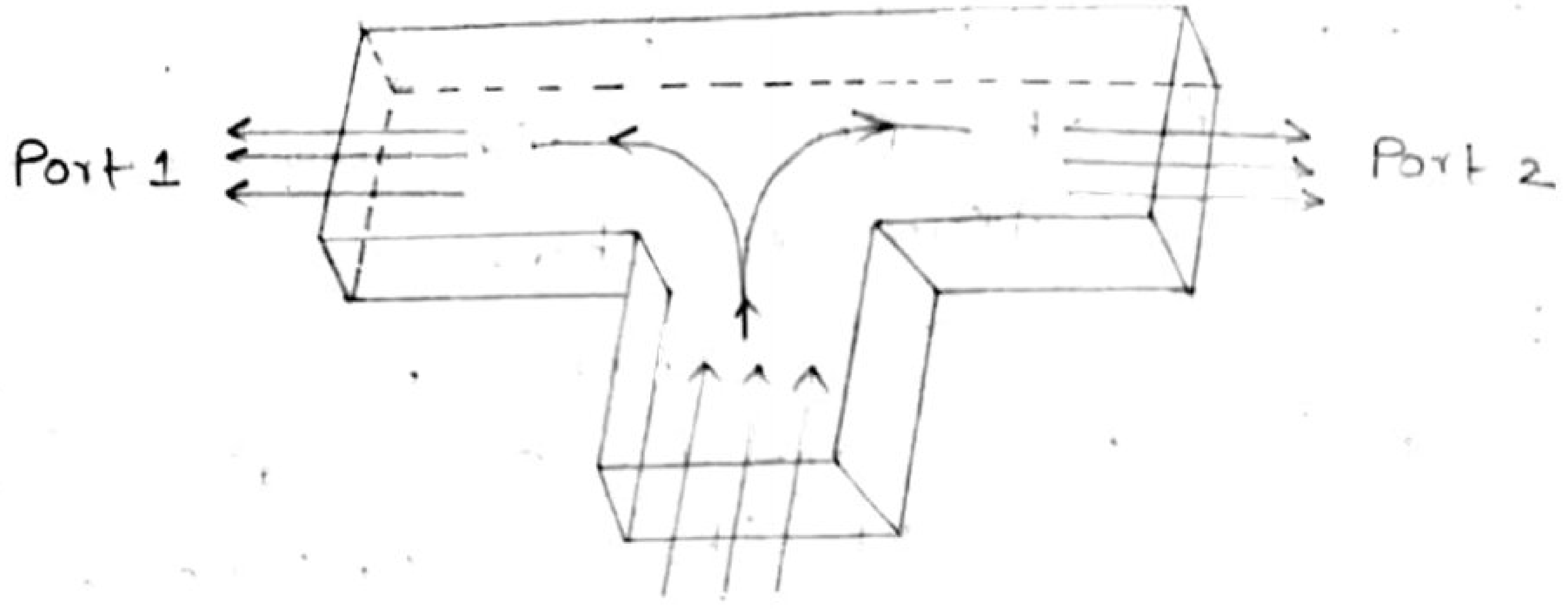


fig:- H-Plane Tee junction

- The Side arm is Parallel to the magnetic field lines, so, the H-Plane T-junction is also known as "Current shunt junction".

S-Matrix Calculations-(H-Plane Tee)



When a microwave signal is propagating through Port 3, it is equally distributed in Port 1 as well as in Port 2, which are considered as output ports and whose output will be in phase with each other.

$$\therefore \boxed{S_{13} = S_{23}}$$

The scattering matrix of the H-Plane Tee, can be used to describe its ports.

(i) [S] is a 3x3 matrix, since there are 3 Ports.

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

(ii) Scattering coefficients, $\boxed{S_{13} = S_{23}}$ since the outputs at Port 1 and Port 2 are inphase with each other with an input at Port 3.

(iii) If Port 3 is perfectly matched to the junction and there are no reflections at Port 3, then

$$\boxed{S_{33} = 0}$$

(iv) From the Property of symmetry, $S_{ij} = S_{ji}$

$$\therefore \; S_{12} = S_{21}$$

$$S_{13} = S_{31}$$

$$S_{23} = S_{32}$$

Now, $$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{13} \\ S_{13} & S_{13} & \cancel{S_{33}}\,0 \end{bmatrix}$$

(V) From the unitary property, we have

$$[S][S]^* = [I]$$

$$\Rightarrow \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{13} \\ S_{13} & S_{13} & \cancel{S_{33}}\,0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* \\ S_{12}^* & S_{22}^* & S_{13}^* \\ S_{13}^* & S_{13}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\underline{R_1C_1}$:- $\quad S_{11} \cdot S_{11}^* + S_{12} \cdot S_{12}^* + S_{13} \cdot S_{13}^* = 1$

$$\Rightarrow |S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1$$

$\underline{R_2C_2}$:- $\quad S_{12} \cdot S_{12}^* + S_{22} S_{22}^* + S_{13} \cdot S_{13}^* = 1$

$$\Rightarrow |S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 = 1$$

$\underline{R_3C_3}$:- $\quad S_{13} \cdot S_{13}^* + S_{13} S_{13}^* + 0 = 1$

$$\Rightarrow |S_{13}|^2 + |S_{13}|^2 = 1$$

$$\Rightarrow 2|S_{13}|^2 = 1$$

$$\Rightarrow |S_{13}|^2 = 1/2$$

$$\Rightarrow |S_{13}| = 1/\sqrt{2}$$

$$\therefore \boxed{S_{13} = 1/\sqrt{2}}$$

Consider, $R_1 C_1 = R_2 C_2$

$$\Rightarrow |S_{11}|^2 + \cancel{|S_{12}|^2} + \cancel{|S_{13}|^2} = \cancel{|S_{12}|^2} + |S_{22}|^2 + \cancel{|S_{13}|^2}$$

$$\Rightarrow |S_{11}|^2 = |S_{22}|^2$$

$$\Rightarrow S_{11} = S_{22}$$

$$\therefore \boxed{S_{11} = S_{22}}$$

Consider, $R_2 C_3 = 0$

$$S_{11} \cdot S_{13}^* + S_{12} \cdot S_{13}^* + S_{13} \cdot 0 = 0$$

$$\Rightarrow S_{11} \cdot S_{13}^* + S_{12} \cdot S_{13}^* = 0$$

$$\Rightarrow S_{13}^* (S_{11} + S_{12}) = 0$$

$$\Rightarrow S_{11} + S_{12} = 0$$

$$\Rightarrow S_{11} = -S_{12}$$

$$\therefore \boxed{S_{11} = -S_{12} \quad (\text{or}) \quad S_{12} = -S_{11}}$$

Now, substitute $S_{12} = -S_{11}$ in $R_1 C_1$

$$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1$$

$$\Rightarrow |S_{11}|^2 + |S_{11}|^2 + |S_{13}|^2 = 1$$

$$\Rightarrow 2|S_{11}|^2 + \left|\frac{1}{\sqrt{2}}\right|^2 = 1$$

$$\Rightarrow 2|S_{11}|^2 + \frac{1}{2} = 1$$

$$\Rightarrow 2|S_{11}|^2 = 1/2$$

$$\Rightarrow |S_{11}|^2 = \frac{1}{4}$$

$$\Rightarrow S_{11} = \frac{1}{2}$$

$$\therefore \boxed{S_{11} = 1/2 \text{ and } S_{12} = -1/2 \text{ and } S_{22} = 1/2}$$

∴ S-matrix of H-Plane Tee Junction is given by,

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{13} \\ S_{13} & S_{13} & 0 \end{bmatrix} = \begin{bmatrix} 1/2 & -1/2 & 1/\sqrt{2} \\ -1/2 & 1/2 & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix}$$

$$\therefore \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 1/2 & -1/2 & 1/\sqrt{2} \\ -1/2 & 1/2 & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

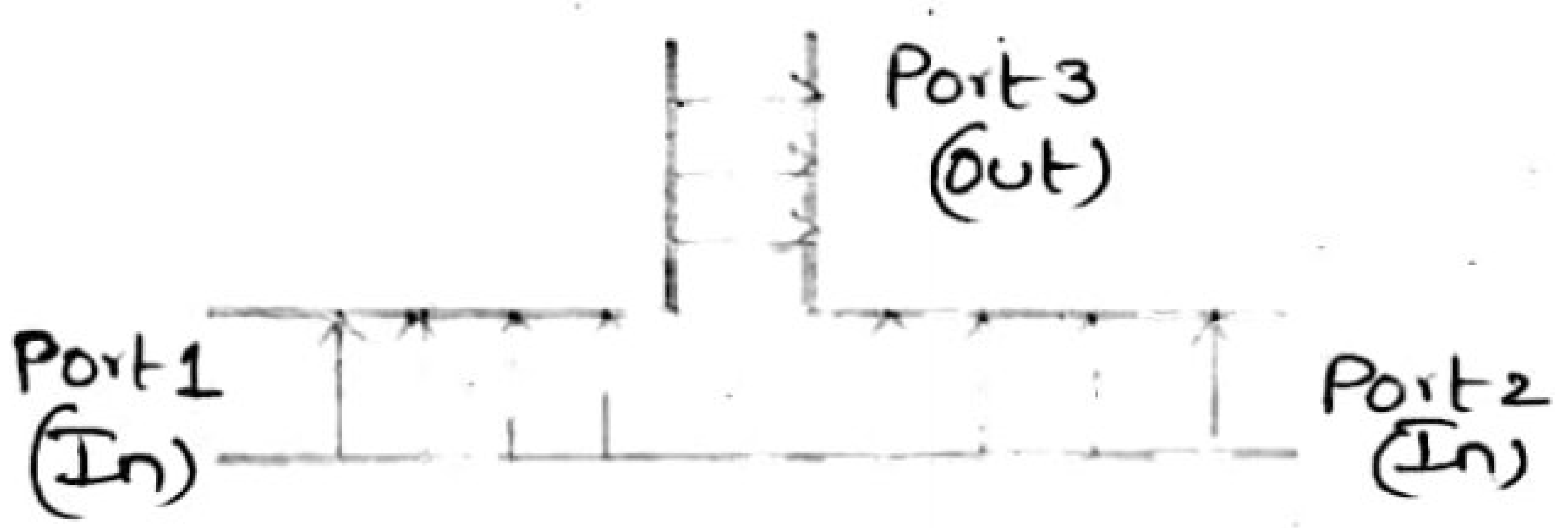
$$b_1 = \frac{a_1}{2} - \frac{a_2}{2} + \frac{a_3}{\sqrt{2}}$$

$$b_2 = \frac{-a_1}{2} + \frac{a_2}{2} + \frac{a_3}{\sqrt{2}}$$

$$b_3 = \frac{a_1}{\sqrt{2}} + \frac{a_2}{\sqrt{2}} + 0$$

Analysis :-

① If input is applied at Port 1 and Port 2 with same magnitude and same Phase, then the output at Port 3 will be the sum of the two input Ports.

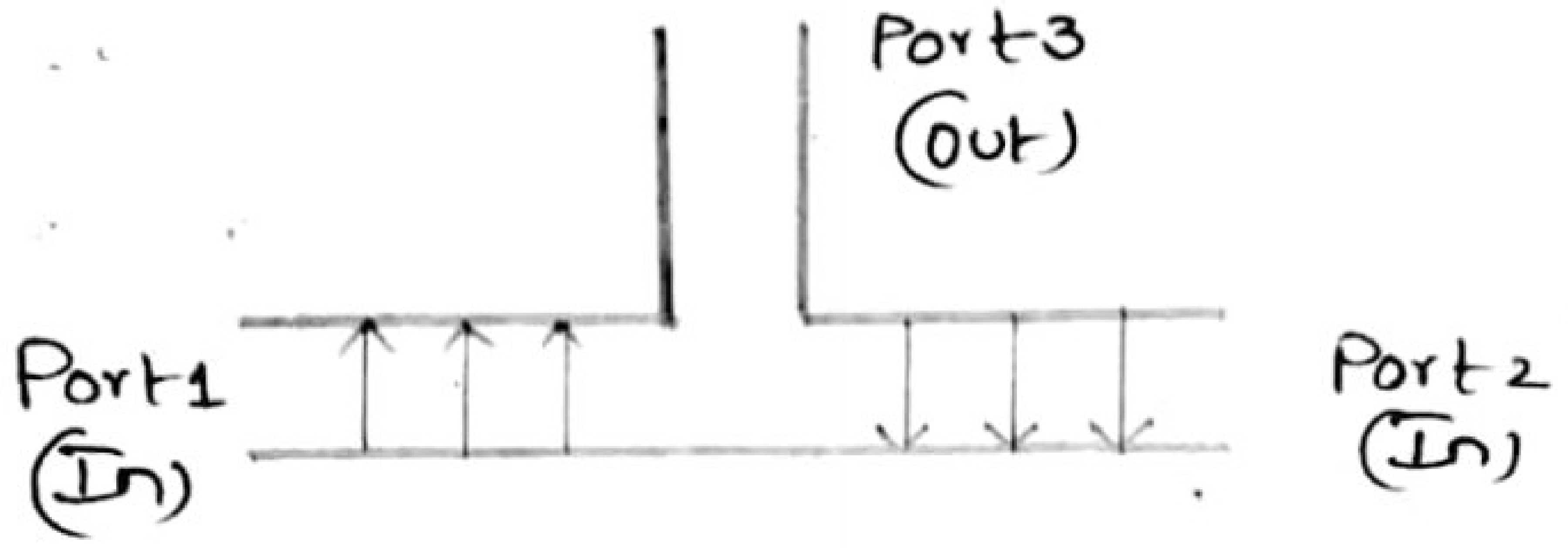


$a_3 = 0$

$a_1 = a_2 = a$

$$\therefore b_3 = \frac{a}{\sqrt{2}} + \frac{a}{\sqrt{2}} = \frac{2a}{\sqrt{2}} = \sqrt{2}\,a$$ (sum of the two i/p Ports)

(2) If the input is applied at Port 1 and Port 2 with same magnitude but out of phase, then the output at Port 3 is the difference of the two input ports.



$a_3 = 0$

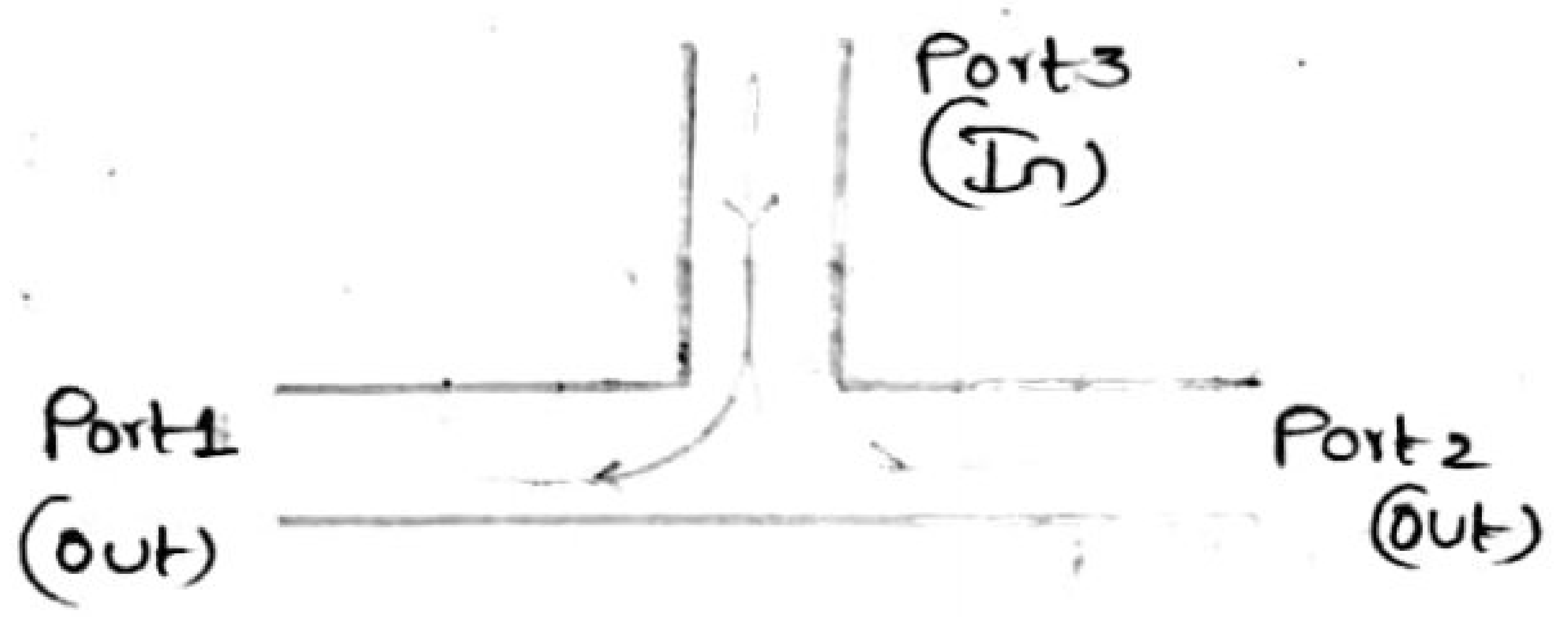
$a_1 = -a_2$

i.e., $a_1 = a$ and $a_2 = -a$

$$b_3 = \frac{a_1}{\sqrt{2}} + \frac{a_2}{\sqrt{2}} = \frac{a}{\sqrt{2}} + \left(-\frac{a}{\sqrt{2}}\right) = \frac{a}{\sqrt{2}} - \frac{a}{\sqrt{2}} = 0$$

$\therefore$ $b_3 = 0$ (difference of the two i/p ports)

(3) If input is applied at Port 3 then the output is considered at Ports 1 and 2 and it will be in phase.



$a_1 = 0$

$a_2 = 0$

$a_3 = a$

$$\therefore b_1 = \frac{a_1}{2} - \frac{a_2}{2} + \frac{a_3}{\sqrt{2}} = 0 - 0 + \frac{a_3}{\sqrt{2}} = \frac{a_3}{\sqrt{2}} = \frac{a}{\sqrt{2}}$$

$$b_2 = \frac{a_1}{2} + \frac{a_2}{2} + \frac{a_3}{\sqrt{2}} = 0 + 0 + \frac{a}{\sqrt{2}} = \frac{a}{\sqrt{2}}$$

$$b_3 = \frac{a_1}{\sqrt{2}} + \frac{a_2}{\sqrt{2}} = 0 + 0 = 0$$

$\left.\begin{array}{l} b_1 = a/\sqrt{2} \\ b_2 = a/\sqrt{2} \end{array}\right\}$ same magnitude & in phase

Magic T-junction (or) E-H Plane T-junction:-

→ The magic-T is a combination of the "E-type" and "H-type" junctions.

→ It consists of four Ports: two collinear Ports one E-arm and one H-arm.

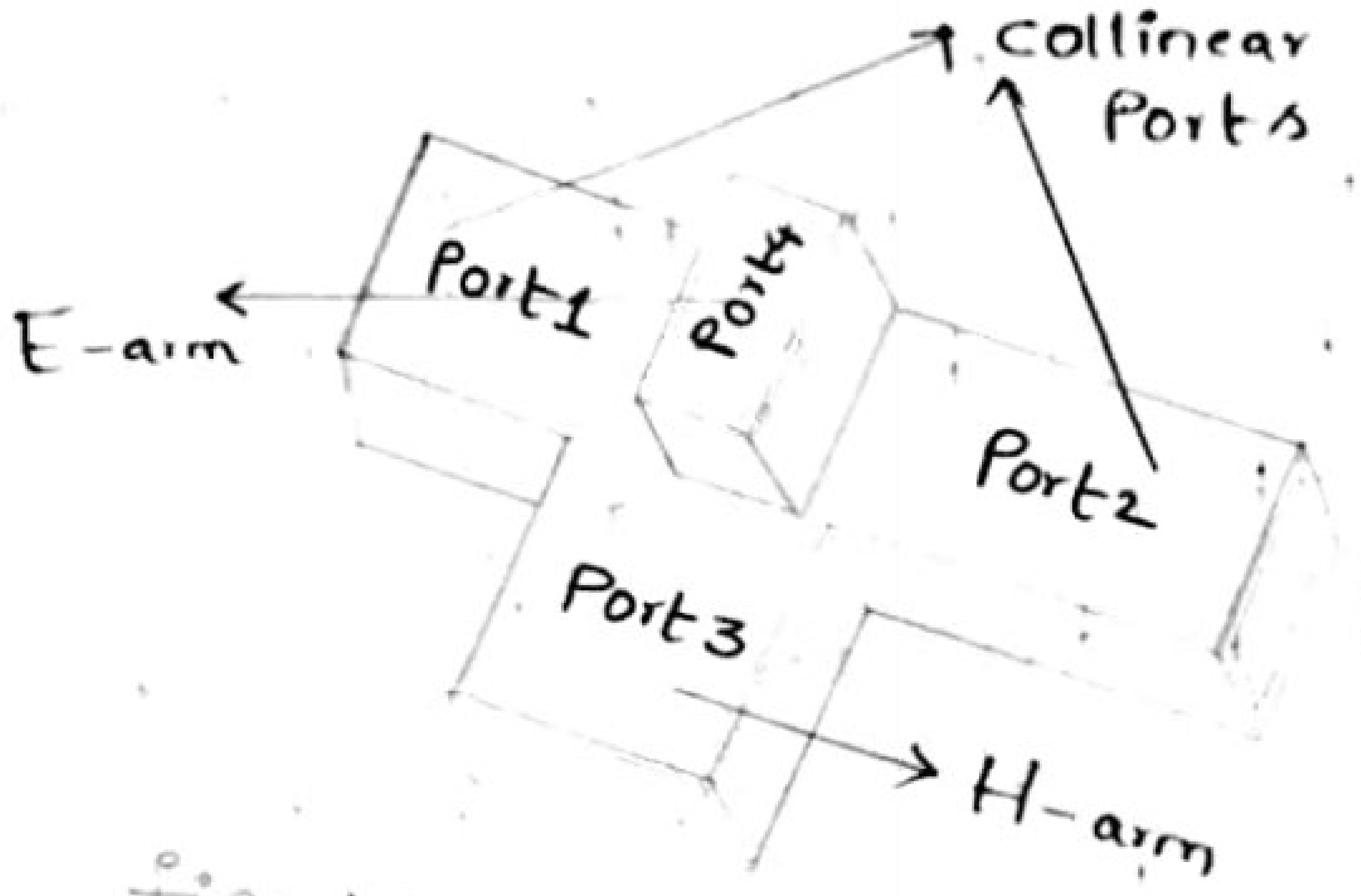


fig:- Magic T Waveguide junction

→ The diagram above depicts a simplified Version of the Magic T Waveguide junction With its four Ports.

Why it is called magic T-junction....?

→ The Magic T Waveguide junction consists of four Ports: Port1, Port2, Port3 & Port4.

→ 'Port 3 & Port 4' are considered to be 'input Ports' While 'Port 1 & Port 2' are considered to be 'output Ports.'

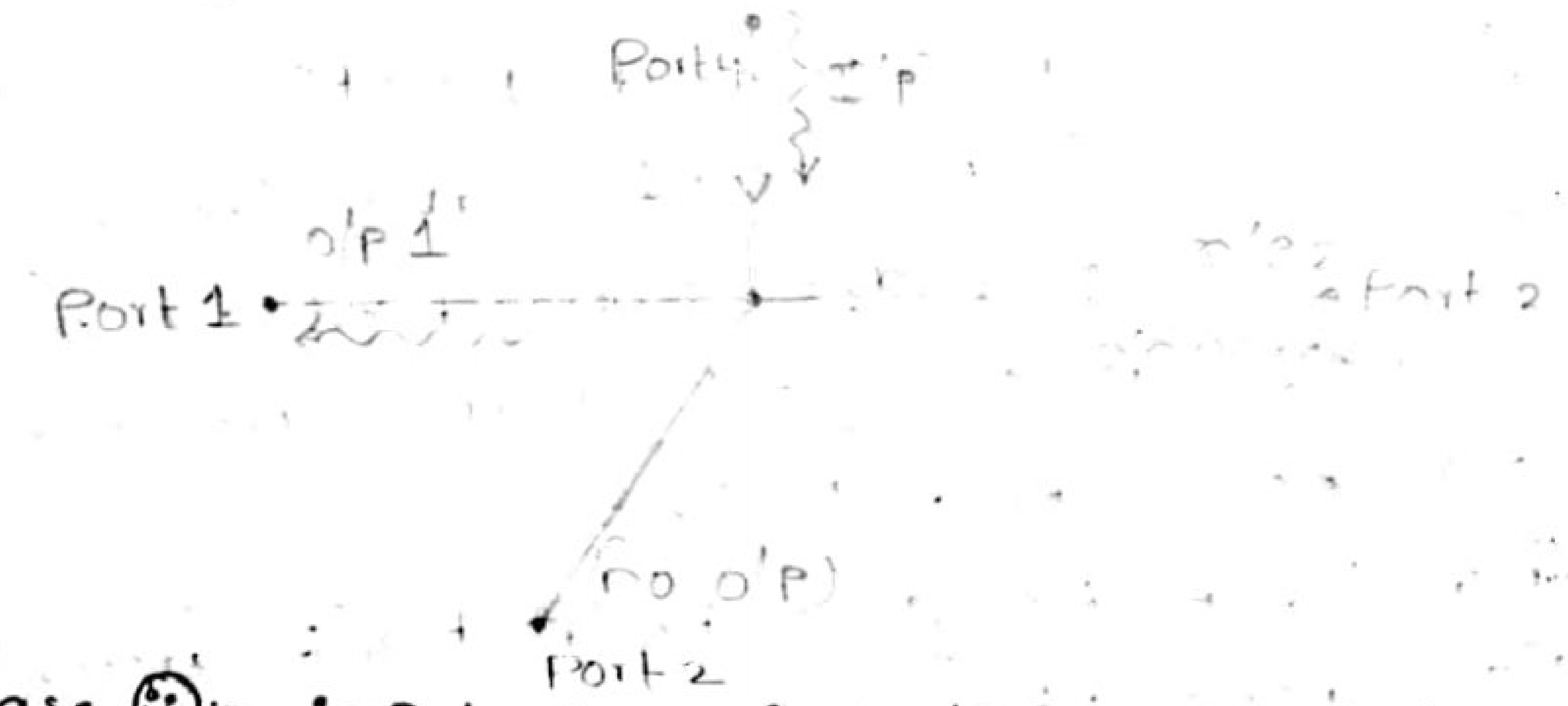
→ When input is given to Port4 (E-arm), in general We expect the output signal from the other 3 Ports since four of them together form a junction. But it will not happen so... Instead the output signal can be obtained only from Ports 1 and 2 alone and No signal Will reach Port3 (H-arm). In other words, electric field doesnot travel through H-arm. The o/p at Ports 1 & 2 will be of same magnitude but constitutes 180° phase shift.

→ Similarly, when input is given to Port 3 (H-arm) the output can be obtained only from Port 1 & Port 2, no signal will reach Port 4 (E-arm). In other words, magnetic field doesnot travel through E-arm. The o/p at Ports 1 & 2 will be of same magnitude and constitutes same phase.

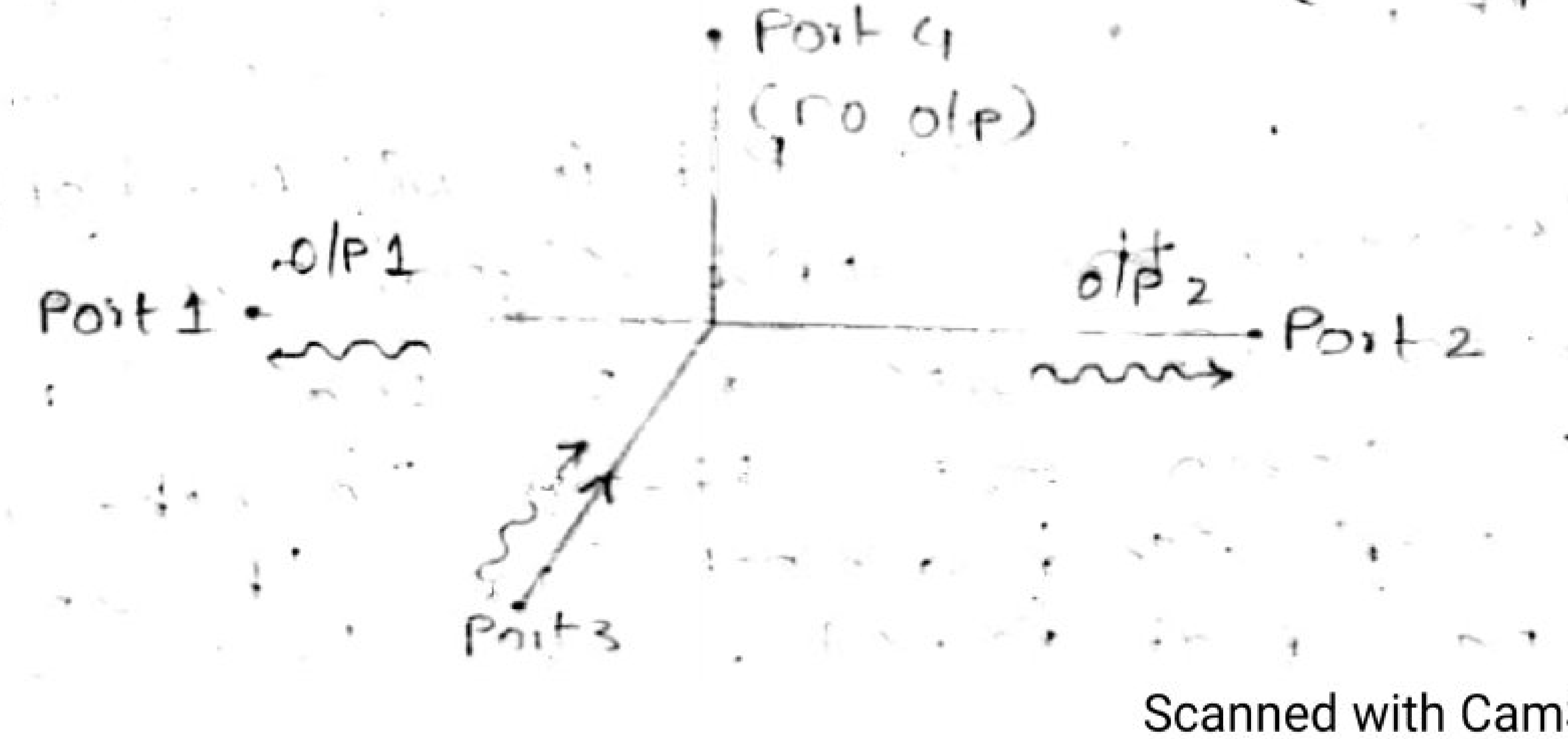
→ Though the 4 Ports together form a junction, they are just metal plates that are welded together insuch a way so as to form a junction. Hence, each and every Port behaves according to its own properties.

→ Due to this, E-H Plane T junction is also known as "Magic T-junction".

Case (i):- input is given to Port 4 (E-arm)



Case (ii):- input is given to Port 3 (H-arm)



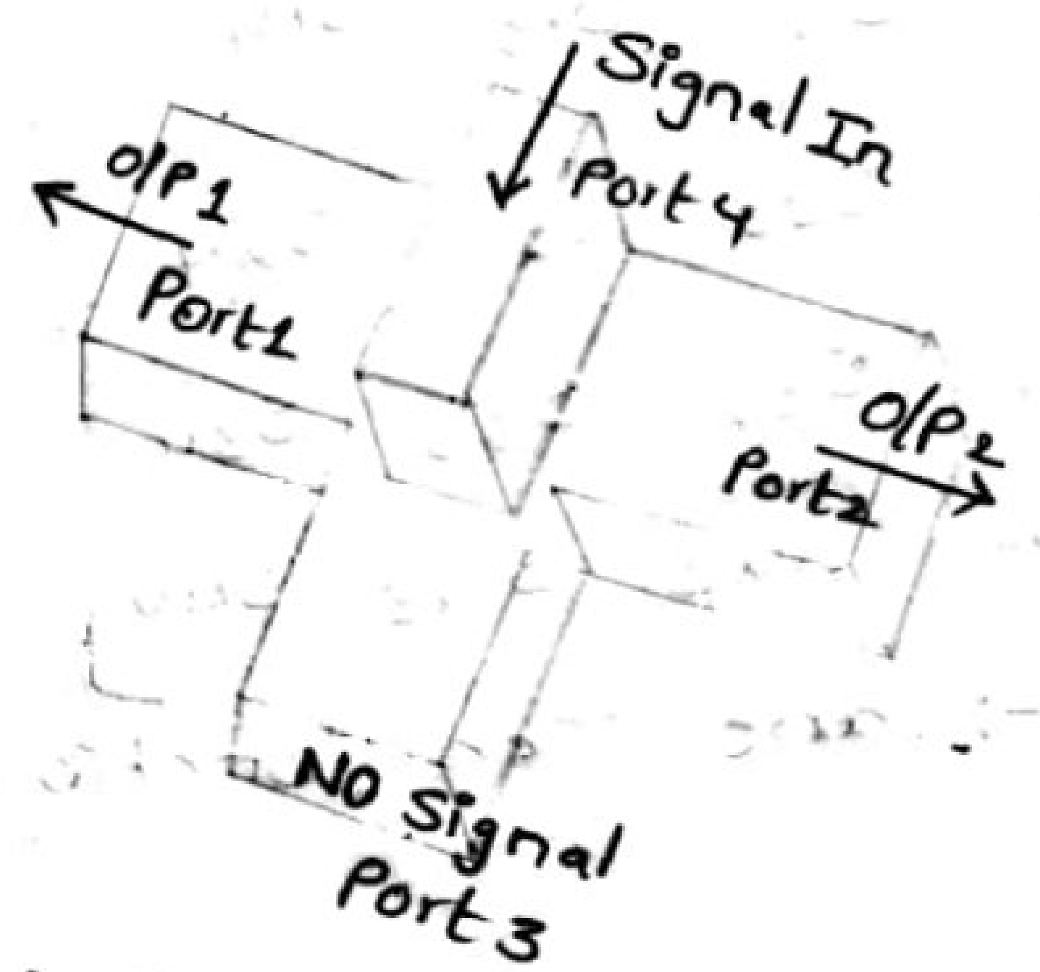
Operation:-

→ To look at the operation of the Magic T Waveguide junction, take the example of When a signal is applied into the "E Plane" arm. It will divide into two out of Phase components as it Passes into the leg consisting of the "a" and "b" arms. However, no signal will enter the "H Plane" arm as a result of the fact that a zero Potential exists there.

→ This occurs because of the signal conditions needed to create the signals in the "a" and "b" arms.

→ Similarly, when a signal is applied to the H-Plane arm, no signal appears at the "E-Plane" arm and the two signals appearing at the "a" and "b" arms are inphase With each other.

→ When a signal enters the 'a' or 'b' arm of the magic T Waveguide junction, then a signal appears at the E and H plane Ports, but not at the other 'b' or 'a' arm.



fig①:-

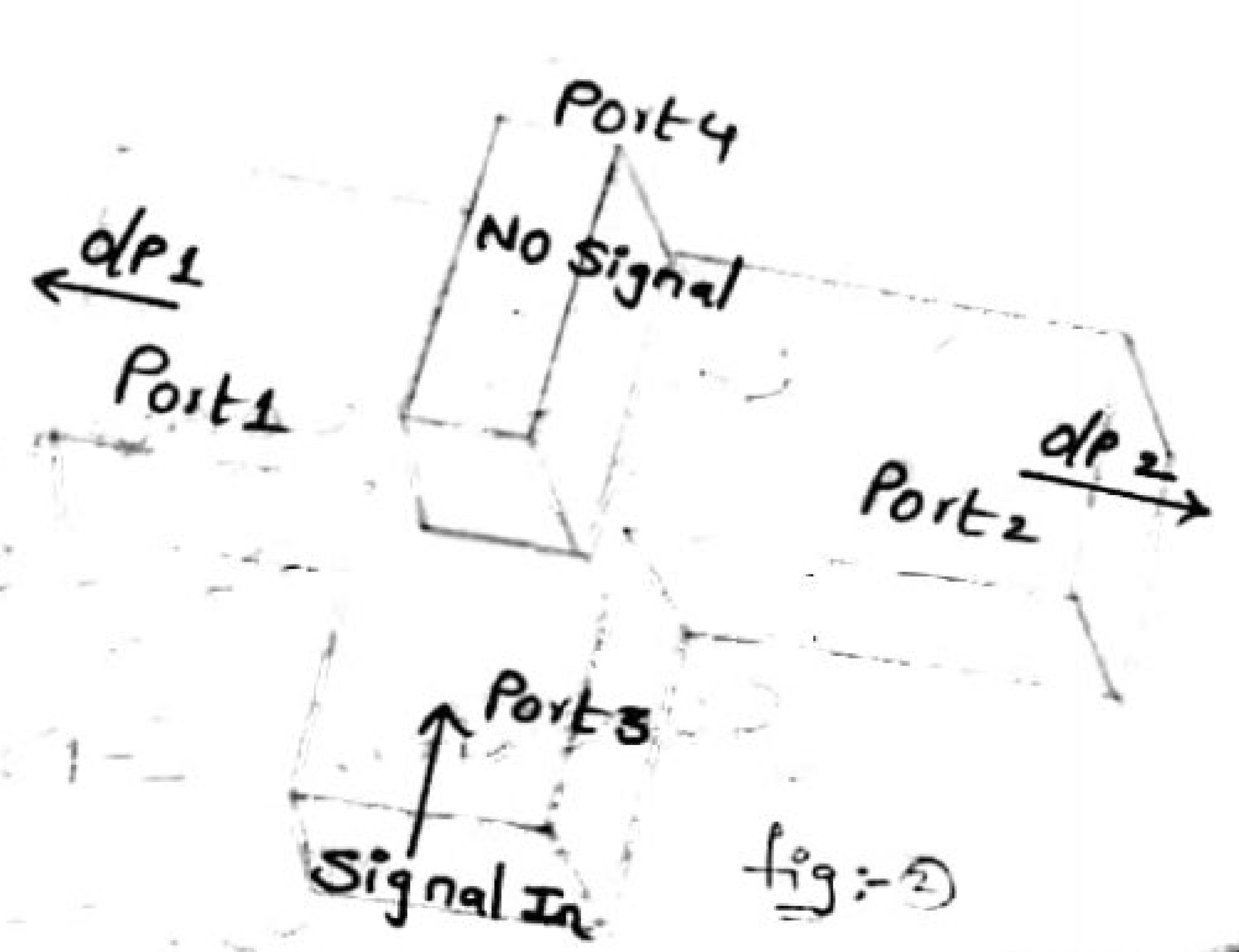


fig:-②

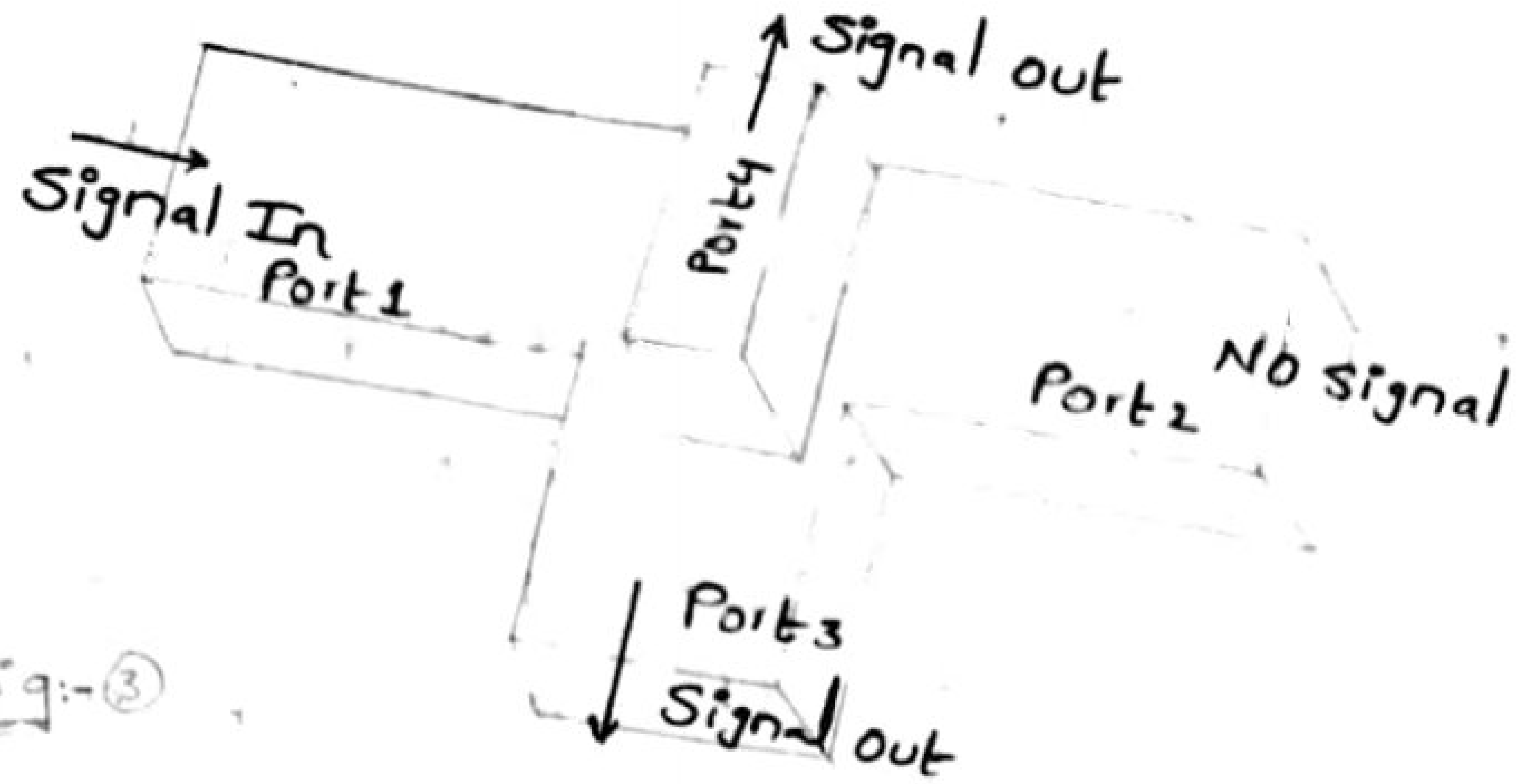


fig:-③

S-Matrix Calculations-(Magic T-junction):-

fig:- Magic Waveguide Tee junction

The scattering matrix of the Magic Tee, Can be used to describe its Properties.

① [S] is : 4×4 matrix, since there are 4 Ports

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix}$$

ⓘⓘ Let us consider, H-Plane Tee junction

$$\boxed{S_{23} = S_{13}}$$

Consider, E-Plane Tee junction

$$\boxed{S_{24} = -S_{14}}$$

Now, Port 3 and Port 4 are isolated to each other i.e., $\boxed{S_{34} = S_{43} = 0}$

(iii) If Port 3 and Port 4 are perfectly matched to the junction and there are no reflections at Port 3 and Port 4, then

$$\boxed{S_{33} = S_{44} = 0}$$

(iv) From the symmetric property, $S_{ij} = S_{ji}$

$$\therefore\ S_{12} = S_{21};\quad S_{13} = S_{31};\quad S_{23} = S_{32}$$
$$S_{34} = S_{43};\quad S_{24} = S_{42};\quad S_{41} = S_{14}$$

Now,
$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{13} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix}$$

(v) From unitary property,
$$[S][S]^* = [I]$$

$$\Rightarrow \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{13} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* & S_{14}^* \\ S_{12}^* & S_{22}^* & S_{13}^* & -S_{14}^* \\ S_{13}^* & S_{13}^* & 0 & 0 \\ S_{14}^* & -S_{14}^* & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\underline{R_1C_1}$:- $S_{11}\cdot S_{11}^* + S_{12}\cdot S_{12}^* + S_{13}\cdot S_{13}^* + S_{14}\cdot S_{14}^* = 1$

$$\Rightarrow |S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1$$

$\underline{R_2C_2}$:- $S_{12}\cdot S_{12}^* + S_{22}\cdot S_{22}^* + S_{13}\cdot S_{13}^* + (-S_{14})\cdot(-S_{14}^*) = 1$

$$\Rightarrow |S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1$$

$\underline{R_3C_3}$:- $S_{13}\cdot S_{13}^* + S_{13}\cdot S_{13}^* + 0 + 0 = 1$

$$\Rightarrow |S_{13}|^2 + |S_{13}|^2 = 1$$

Scanned with CamScanner

$$\Rightarrow 2|S_{13}|^2 = 1$$

$$\Rightarrow |S_{13}|^2 = 1/2$$

$$\Rightarrow S_{13} = 1/\sqrt{2}$$

$$\therefore \boxed{S_{13} = 1/\sqrt{2}}$$

<u>R_4C_4:-</u> $S_{14} \cdot S_{14}^* + S_{14} \cdot S_{14}^* + 0 + 0 = 1$

$$\Rightarrow |S_{14}|^2 + |S_{14}|^2 = 1$$

$$\Rightarrow 2|S_{14}|^2 = 1$$

$$\Rightarrow |S_{14}|^2 = 1/2$$

$$\Rightarrow S_{14} = 1/\sqrt{2}$$

$$\therefore \boxed{S_{14} = 1/\sqrt{2}}$$

Consider, $R_1C_1 = R_2C_2$

$$\Rightarrow |S_{11}|^2 + \cancel{|S_{12}|^2} + \cancel{|S_{13}|^2} + \cancel{|S_{14}|^2} = \cancel{|S_{12}|^2} + |S_{22}|^2 + \cancel{|S_{13}|^2} + \cancel{|S_{14}|^2}$$

$$\Rightarrow |S_{11}|^2 = |S_{22}|^2$$

$$\Rightarrow S_{11} = S_{22}$$

$$\therefore \boxed{S_{11} = S_{22}}$$

Consider, R_4C_1

$$\Rightarrow S_{14} \cdot S_{11}^* + -S_{14} \cdot S_{12}^* + 0 + 0 = 0$$

$$\Rightarrow S_{14} \cdot S_{11}^* - S_{14} \cdot S_{12}^* = 0$$

$$\Rightarrow S_{14}(S_{11}^* - S_{12}^*) = 0$$

$$\Rightarrow S_{11}^* - S_{12}^* = 0$$

$$\Rightarrow S_{11}^* = S_{12}^*$$

$$\Rightarrow S_{11} = S_{12} \qquad \therefore \boxed{S_{11} = S_{12}}$$

We have, $\boxed{S_{11} = S_{12} = S_{22}}$

Substitute R_1C_1 with $\boxed{S_{11} = S_{12}}$

<u>R_1C_1</u>:- $|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1$

$$\Rightarrow |S_{11}|^2 + |S_{11}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1$$

$$\Rightarrow 2|S_{11}|^2 + \left|\frac{1}{\sqrt{2}}\right|^2 + \left|\frac{1}{\sqrt{2}}\right|^2 + 0 = 1$$

$$\Rightarrow 2|S_{11}|^2 + \frac{1}{2} + \frac{1}{2} = 1$$

$$\Rightarrow 2|S_{11}|^2 + 1 = 1$$

$$\Rightarrow 2|S_{11}|^2 = 0$$

$$\Rightarrow |S_{11}|^2 = 0$$

$$\Rightarrow S_{11} = 0$$

$$\therefore \boxed{S_{11} = S_{12} = S_{22} = 0}$$

Finally, S matrix is given by,

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{13} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 & 0 \end{bmatrix}$$

We know that, $[b] = [S][a]$

$$\therefore \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}$$

$$b_1 = \frac{a_3}{\sqrt{2}} + \frac{a_4}{\sqrt{2}}$$

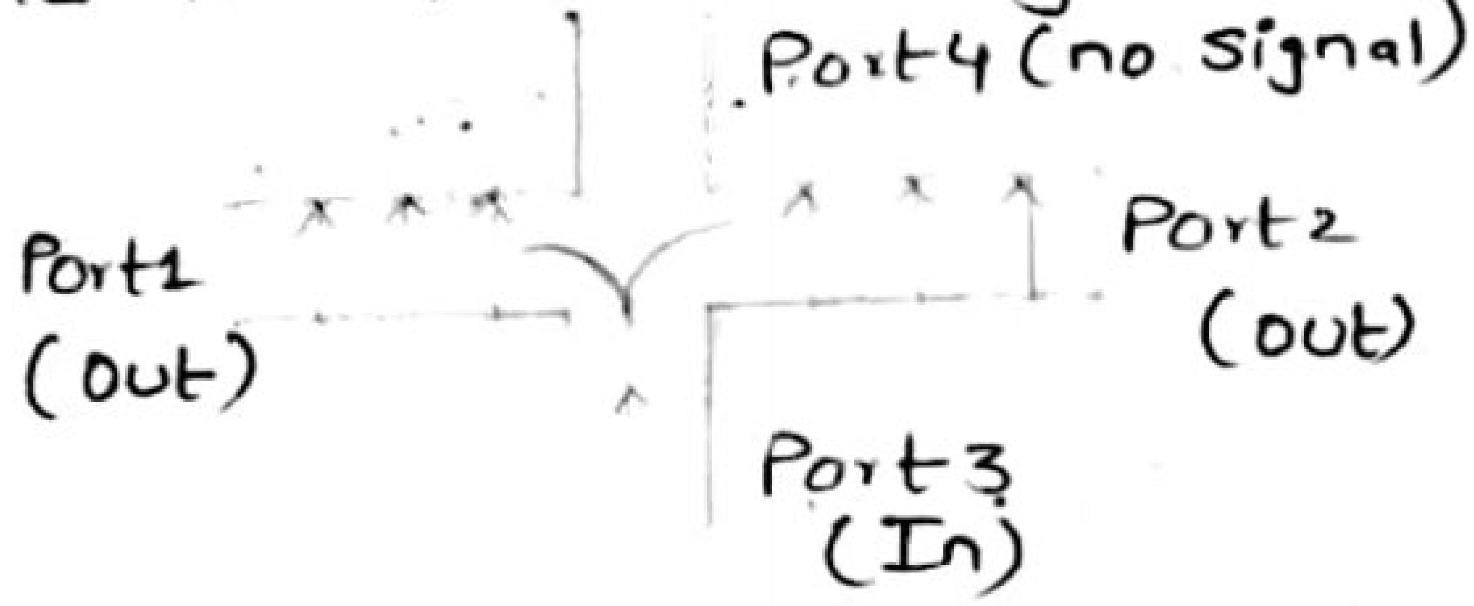
$$b_2 = \frac{a_3}{\sqrt{2}} - \frac{a_4}{\sqrt{2}}$$

$$b_3 = \frac{a_1}{\sqrt{2}} + \frac{a_2}{\sqrt{2}}$$

$$b_4 = \frac{a_1}{\sqrt{2}} - \frac{a_2}{\sqrt{2}}$$

Analysis:-

① If input is given to Port 3, then the signal doesnot propagate through Port 4 output is obtained at Port 1 and Port 2 and is same magnitude and same phase.



$a_3 = a$; $a_1 = a_2 = a_4 = 0$

$$\therefore \quad b_1 = \frac{a_3}{\sqrt{2}} + \frac{a_4}{\sqrt{2}} = \frac{a}{\sqrt{2}} + 0 = \frac{a}{\sqrt{2}}$$

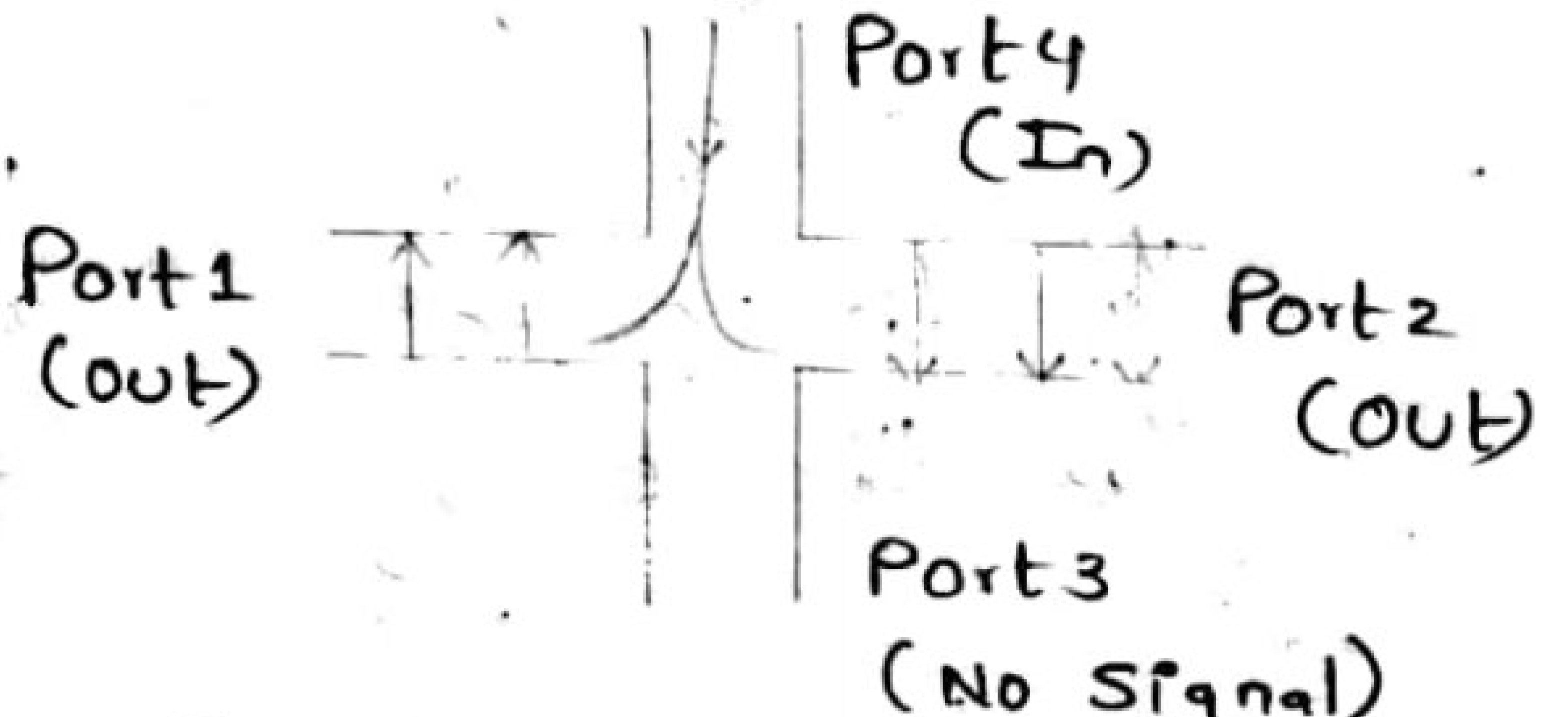
$$b_2 = \frac{a_3}{\sqrt{2}} - \frac{a_4}{\sqrt{2}} = \frac{a}{\sqrt{2}} - 0 = \frac{a}{\sqrt{2}}$$

Same magnitude and same phase

$$b_3 = \frac{a_1}{\sqrt{2}} + \frac{a_2}{\sqrt{2}} = 0 + 0 = 0$$

$$b_4 = \frac{a_1}{\sqrt{2}} - \frac{a_2}{\sqrt{2}} = 0 - 0 = 0$$

② If input is given to Port 4, then the signal doesnot propagate through Port 3 output is obtained at Port 1 and Port 2 with same magnitude but with a phase shift of 180°.



$a_4 = a$; $a_1 = a_2 = a_3 = 0$

$$\therefore \quad b_1 = \frac{a_3}{\sqrt{2}} + \frac{a_4}{\sqrt{2}} = 0 + \frac{a}{\sqrt{2}} = \frac{a}{\sqrt{2}}$$

$$b_2 = \frac{a_3}{\sqrt{2}} - \frac{a_4}{\sqrt{2}} = 0 - a/\sqrt{2} = -a/\sqrt{2}$$

Same magnitude but out of phase

(3). If input is given to Port 1 and Port 2 with same magnitude and same phase then the output at Port 3 is the summation of the two input ports whereas at Port 4 the output is the difference of the two i/p Ports.

i.e., $a_1 = a_2 = a;\ a_3 = a_4 = 0$

$$\therefore\ b_1 = \frac{a_3}{\sqrt{2}} + \frac{a_4}{\sqrt{2}} = 0 + 0 = 0$$

$$b_2 = \frac{a_3}{\sqrt{2}} - \frac{a_4}{\sqrt{2}} = 0 - 0 = 0$$

$$b_3 = \frac{a_1}{\sqrt{2}} + \frac{a_2}{\sqrt{2}} = \frac{a}{\sqrt{2}} + \frac{a}{\sqrt{2}} = \frac{2a}{\sqrt{2}} = \sqrt{2}a \text{ (summation of i/p Ports)}$$

$$b_4 = \frac{a_1}{\sqrt{2}} - \frac{a_2}{\sqrt{2}} = \frac{a}{\sqrt{2}} - \frac{a}{\sqrt{2}} = 0 \text{ (difference of i/p Ports)}$$

(4). If input is given to Port 1 and Port 2 with same magnitude and out of Phase, then the output at Port 3 is the difference of the two input Ports, whereas the output at Port 4 is the Summation of the two input Ports.

i.e., $a_1 = -a_2$

$a_1 = a$ and $a_2 = -a$

$a_3 = a_4 = 0$

$$b_1 = \frac{a_3}{\sqrt{2}} + \frac{a_4}{\sqrt{2}} = 0 + 0 = 0$$

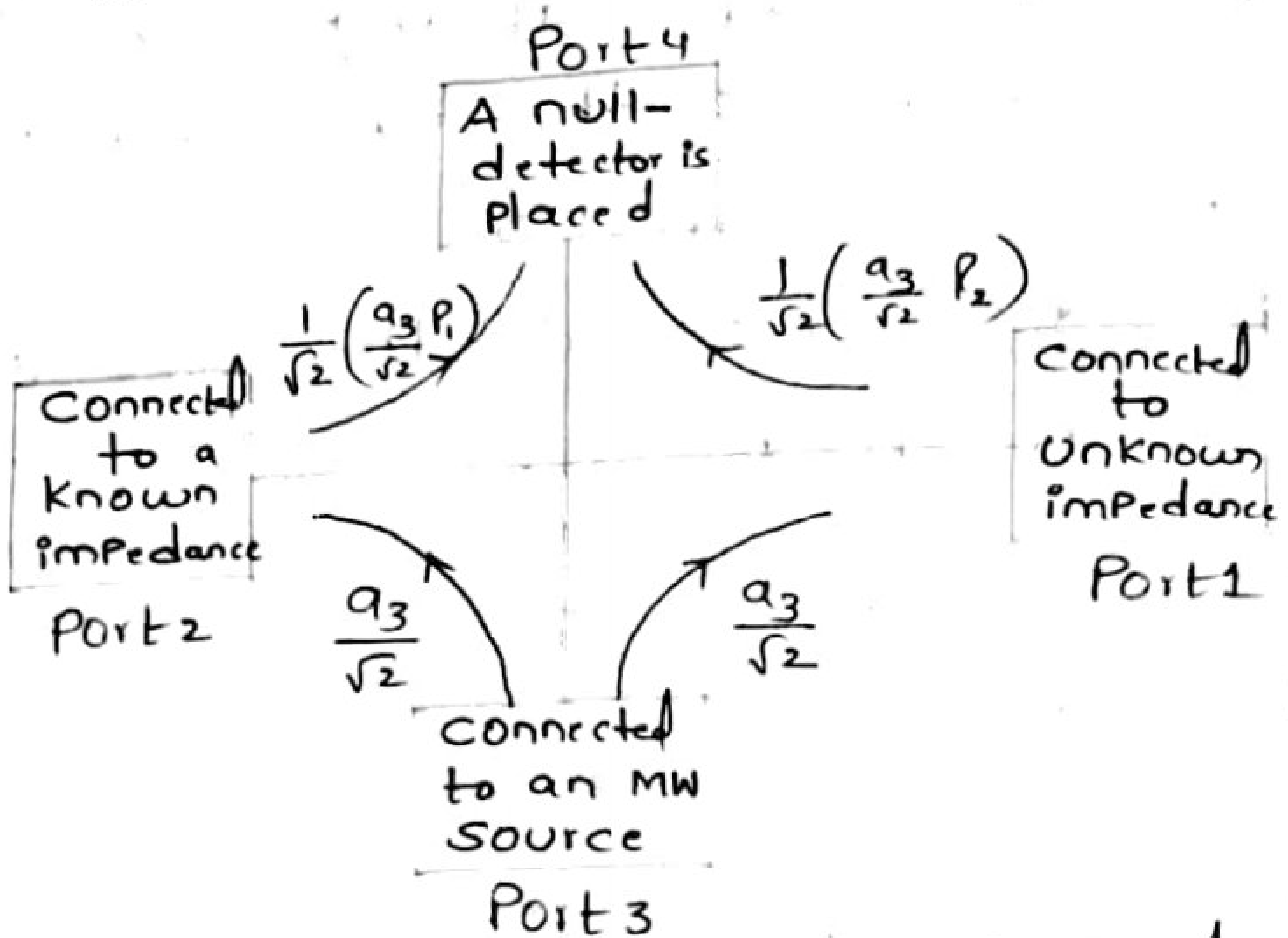
$$b_2 = \frac{a_3}{\sqrt{2}} - \frac{a_4}{\sqrt{2}} = 0 - 0 = 0$$

$$b_3 = \frac{a_3}{\sqrt{2}} + \frac{a_4}{\sqrt{2}} = \frac{a}{\sqrt{2}} - \frac{a}{\sqrt{2}} = 0 \text{ (difference of the i/p Ports)}$$

$$b_4 = \frac{a_3}{\sqrt{2}} + \frac{a_4}{\sqrt{2}} = \frac{a}{\sqrt{2}} + \frac{a}{\sqrt{2}} = \frac{2a}{\sqrt{2}} = \sqrt{2}a \text{ (Summation of the i/p Ports)}$$

Applications of Magic T junction:-

① . To measure the unknown impedance



→ When input is given to Port 3, half of the input power goes to Port 2 and the remaining half goes to Port 1.

→ There are reflections from Port 1 and Port 2 to Port 4. Let,

$\rho_1, \rho_2 \rightarrow$ reflection coefficients

o/p of null detector $\frac{1}{\sqrt{2}}\left(\frac{a_3}{\sqrt{2}}\rho_1\right) - \frac{1}{\sqrt{2}}\left(\frac{a_3}{\sqrt{2}}\rho_2\right) = 0$

$$\Rightarrow \frac{a_3}{\sqrt{2}}\rho_1 - \frac{a_3}{\sqrt{2}}\rho_2 = 0$$

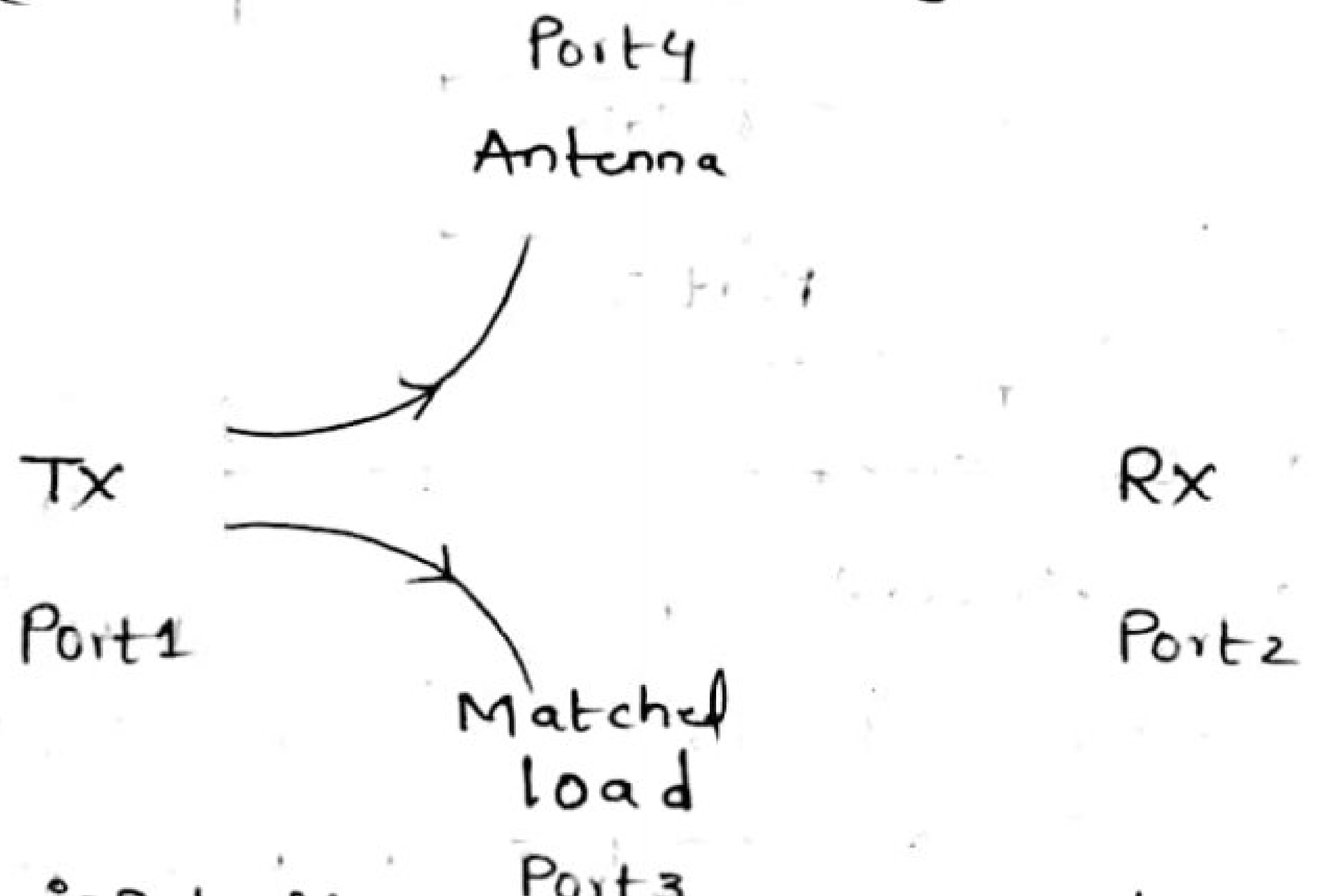
$$\Rightarrow (\rho_1 - \rho_2) = 0 \rightarrow \rho_1 = \rho_2 \quad \frac{Z_1 - Z_0}{Z_1 + Z_0} = \frac{Z_2 - Z_0}{Z_2 + Z_0} \quad \boxed{Z_1 = Z_2}$$

→ This is possible only when known impedance is equal to unknown impedance. To attain this condition, the known impedance value is changed until the null detector shows Zero value. Then known impedance becomes equal to unknown impedance. Hence there would be no reflections from Port 1 and Port 2 to Port 4. Therefore, $\boxed{P_1 - P_2 = 0 \Rightarrow P_1 = P_2}$,

→ The impedance at which null detector displays zero value, is referred to as unknown impedance.

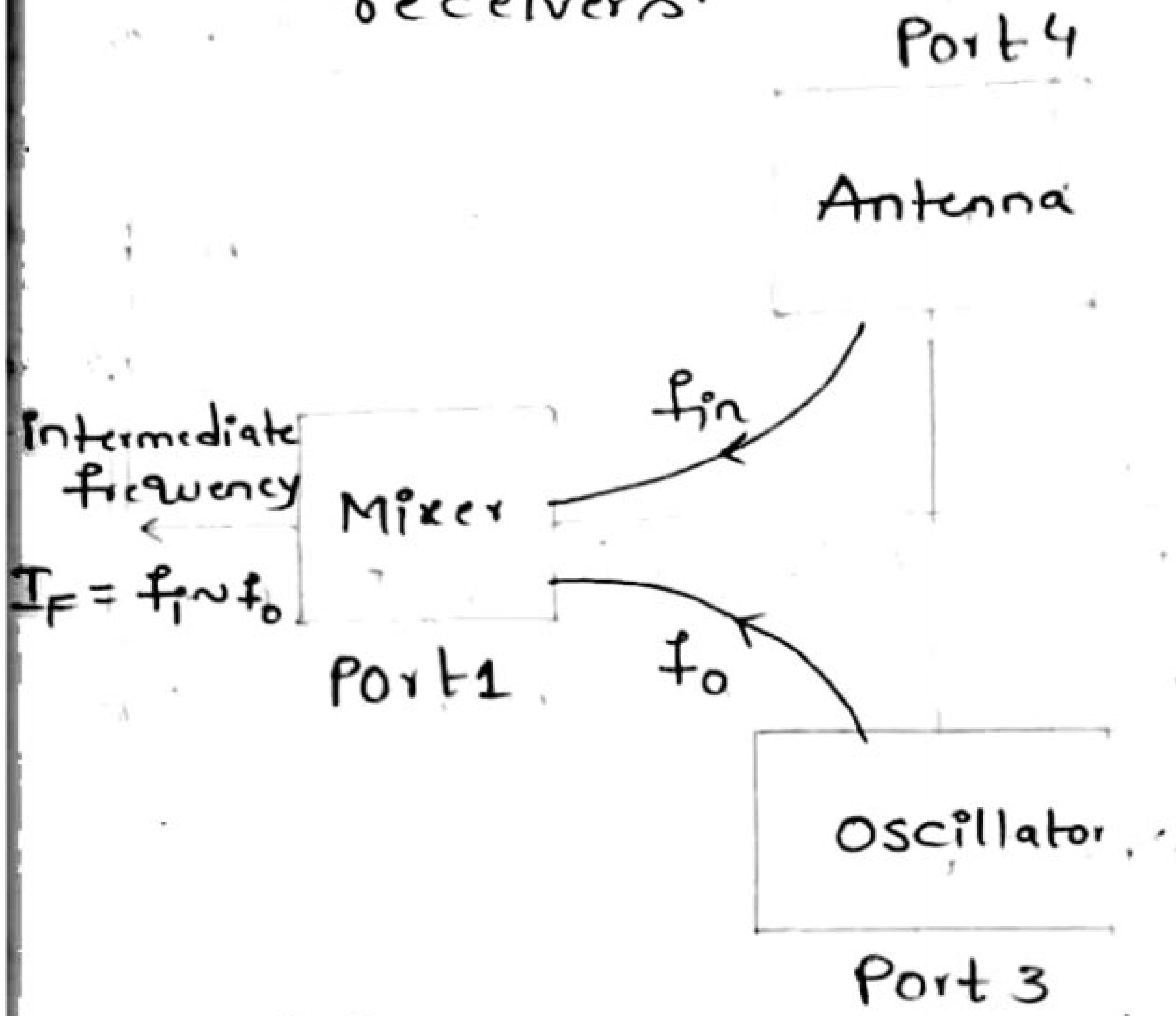
→ In this way, Magic T junction can be used to determine an unknown impedance value.

②. Magic T junction acts as a "Duplexer" (i.e., used for Two-way communication)



When input is given to Port 1, half the input power goes to Port 4 and half of the power goes to Port 3. Port 3 consists of a perfectly matched load and hence there would be no reflection of power from Port 3 to Port 1 (TX). Port 4 consists of an Antenna. A part of the input power transmitted from Port 1 to Port 4, is reflected back by the Antenna to Port 1. Now, there exists a possibility for two-way communication and in this way magic T junction can be used as a duplexer.

③. The most common application of this type of Waveguide junction is as the mixer section for microwave radar receivers.



→ Microwave mixers translate the frequency of electromagnetic signals.

→ This functionality is vital for an enormous number of applications such as military radar and surveillance, RF communications, radio astronomy and biological sensing.

→ A frequency mixer is a 3-Port RF electronic circuit. Two of the Ports are "input" Ports and the remaining port is an "output" Port. An ideal mixer "mixes" the two input signals in such a way that the output signal frequency is either the sum (or) difference of the inputs. In other words $\boxed{f_{out} = f_{in1} \pm f_{in2}}$

Drawback of Magic T-junction:-

→ one of the disadvantages of the Magic T function is, reflections arise from the impedance mismatches that naturally occurs within it.

→ These reflections not only give rise to Power loss but at Peak voltage Points, they can give rise to arcing when used with high Power transmitters.

→ The reflections Can be reduced by using matching techniques.

→ Normally, Posts (or) Screws are used within the E-Plane and H-Plane Ports.

→ These Solutions improve the impedance matches and hence the reflections, but there is a Power handling Capacity Penalty.

** Rat race T-junction (or) Hybrid Ring Waveguide junction:-

→ This form of Waveguide junction Overcomes the Power limitation of the magnetic T-waveguide junction.

→ A hybrid ring waveguide junction is a further development of the magic T.

→ The hybrid ring is used Primarily in high-Power radar and communication systems Where it acts as a duplexer- allowing the same antenna to be used for transmit and receive functions.

→ During the transmit Period, the junction Couples microwave energy from the transmitter to the antenna while blocking energy from the receiver input. Then as the receive cycle starts, the hybrid ring waveguide junction couples energy from the antenna to the receiver. During this period, it prevents energy from reaching the transmitter.

Construction:-

→ Rat Race T-junction / Hybrid ring waveguide junction is constructed from a circular ring of rectangular waveguide — a bit like an annulus.

→ The ports are then joined to the annulus at the required points.

→ Again, if the signal enters at one port, it does not appear at all the others.

→ The junction provides high levels of isolation although the exact values should be checked in the datasheets for the particular junction being considered.

→ For proper operation, the total circumference of the ring is maintained as $\boxed{\frac{6\lambda_g}{4} \approx 1.5\lambda_g}$

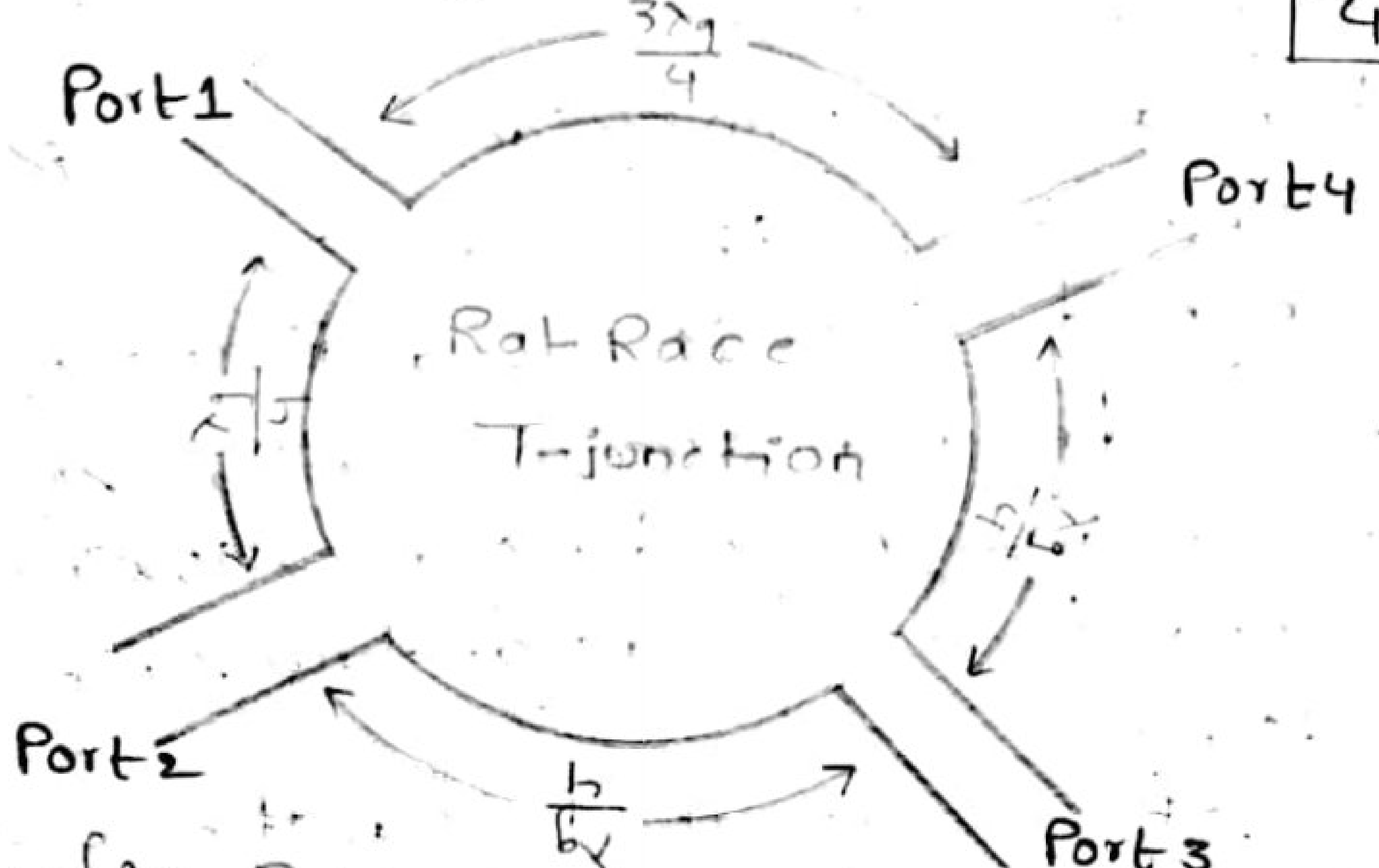


fig:- Rat Race Tee junction

Operation:-

→ If input is given to Port 1, it splits/is equally distributed equally to Ports 2 and 4 both in clock-wise and Anti-clockwise direction But the signal doesnot propagate through Port 3.

→ If input is given to Port 2 it is equally distributed to Ports 1 & 3, both in clock-wise and Anti-clockwise direction. But the signal doesnot propagate through Port 4.

→ If input is given to Port 3, it is equally distributed to Ports 2 & 4 both in clock-wise and Anti-clockwise direction. But the signal doesnot propagate through Port 1.

→ If input is given to Port 4, it is equally distributed to Ports 1 & 3, both in clock-wise and Anti-clockwise direction. But the signal doesnot propagate through Port 2.

Scattering matrix:-

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix} = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{14} & 0 & S_{34} & 0 \end{bmatrix}$$

Waveguide junctions are an essential type of configuration that enable power to be split and combined in a variety of ways. They considerably simplify many systems, and although many are quite expensive, they provide a high performance method of achieving their function.

** Directional Couplers:-

→ A directional coupler is a device that samples small amount of Microwave Power for measurement purposes. The power measurements include incident power, reflected power, VSWR values, etc...

→ It is a metallic pipe, that looks like a waveguide but acts as a coupler.

Construction:-

→ A directional coupler is formed by welding two rectangular waveguides - out of which one is a straight waveguide while the other is a bent waveguide together, in such a way that there exists a hollow spacing b/w them.

→ Directional coupler is a 4-port waveguide junction consisting of a "Primary main waveguide" and a "secondary auxiliary waveguide."

→ The following figure shows the image of a directional coupler.

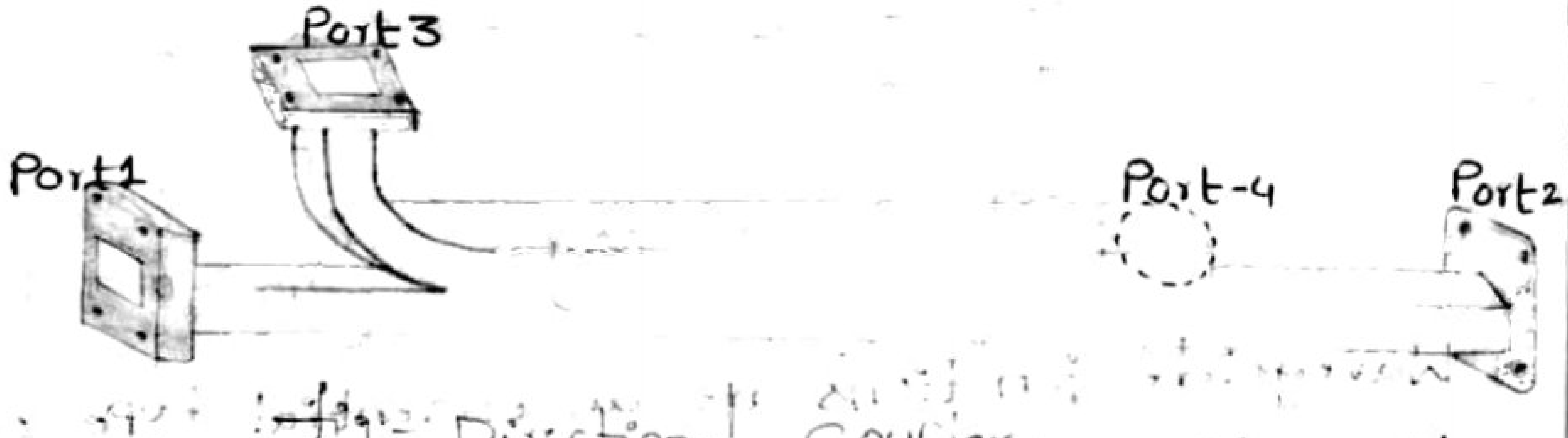


fig:- Directional Coupler

It can be shown symbolically as shown in the below figure:

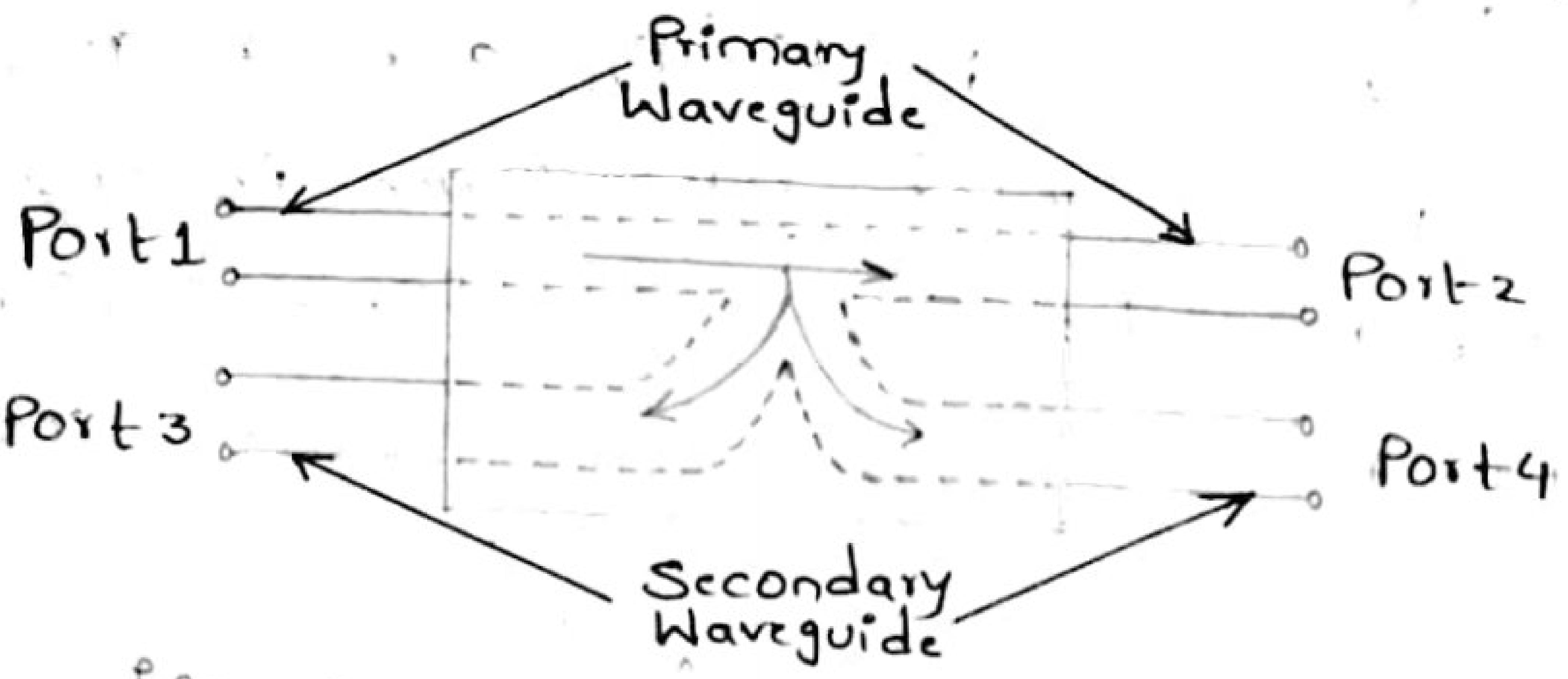


fig:- Directional Coupler

Directional coupler is used to couple the Microwave power which may be unidirectional or bi-directional.

Properties of Directional couplers:-

The properties of an ideal directional coupler are as follows:

→ All the terminations are matched to the ports.

→ When the power travels from Port 1 to Port 2, some of its portion is coupled to Port 4, but not to Port 3.

→ As it is also a bi-directional coupler, when the power travels from Port 2 to Port 1, some portion of it gets coupled to Port 3 but not to Port 4.

→ If the power is incident through Port 3, a portion of it is coupled to Port 2, but not to Port 1.

→ If the power is incident through Port 4, a portion of it is coupled to Port 1, but not to Port 2.

→ Port 1 and Port 3 are decoupled as Port 2 and Port 4.

Ideally, the output of Port3 should be zero. However, Practically, a small amount of power called back Power is observed at Port 3. The following figure indicates the power flow in a directional coupler.

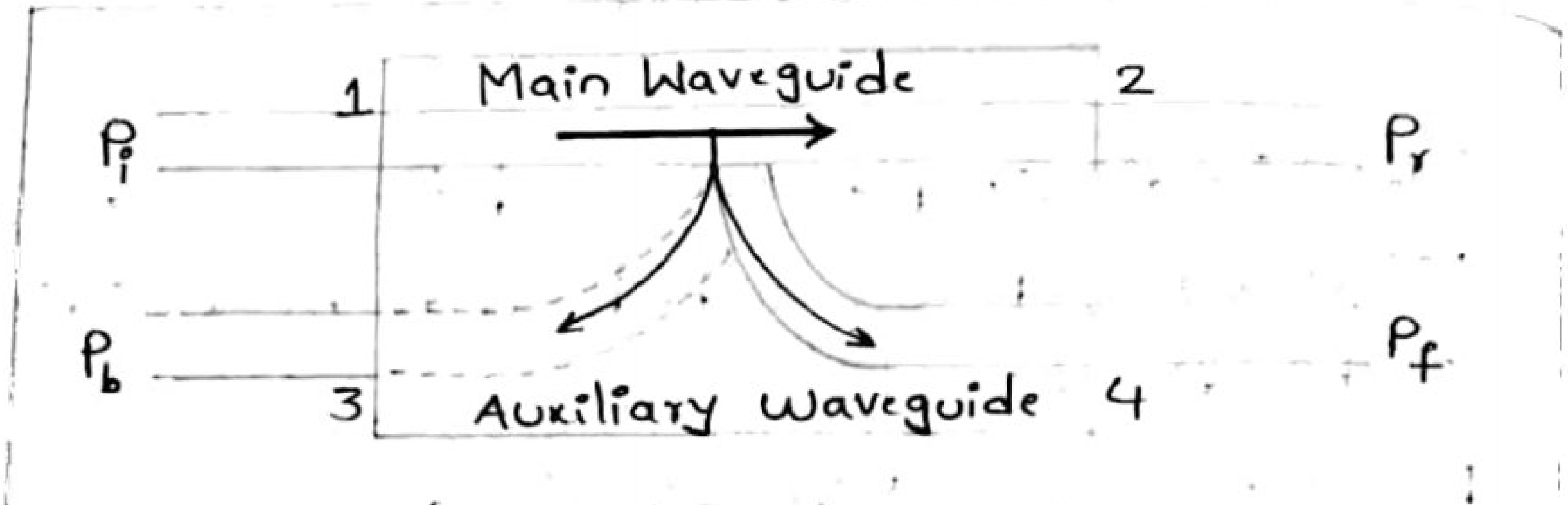


fig:- Directional Coupler indicating Powers

Where,

P_i = Incident Power at Port1

P_r = Received Power at Port2

P_f = Forward coupled Power at Port4

P_b = Backward/Back Power at Port3

Operation:-

→ Whenever a microwave signal is given as input to one of the four Ports, it is considered interms of power.

→ When input is given to Port1, a Portion of the input Power goes directly to Port2 and Some of Portion goes to Port 4. If at all there are any reflections in the input Power, they will be send to Port 3.

→ Now, Port1 is considered to be "incident Port", the power associated is referred to as incident Power at Port1 which is indicated as "Pi".

→ As a portion of the input power, is being received at Port2. Port2 is referred to as "Received Port" and the power associated is referred to as Received Power at Port2 which is indicated by "P_r".

→ While a portion of the input power, is taking diversion and propagating through Port4. Port4 is reffered to as "Forwarded Power Port" and the power associated, is referred to as forward power at Port4, which is indicated by "P_f".

→ Incase of any reflections in the input power, it will be reflected back to Port3. Hence, Port 3 is referred to as "back Port" and the power associated, is referred to as Back Power at Port3, which is indicated by "P_b".

→ Ingeneral, Back Power(P_b) of a directional coupler is very very small.

Case(i):-

fig:-I/P is given to Port1.

Port 1 → incident Port

Port 2 → Received Port

Port4 → forward Power coupling Port

Port 2 → Back Port

Case (ii):-

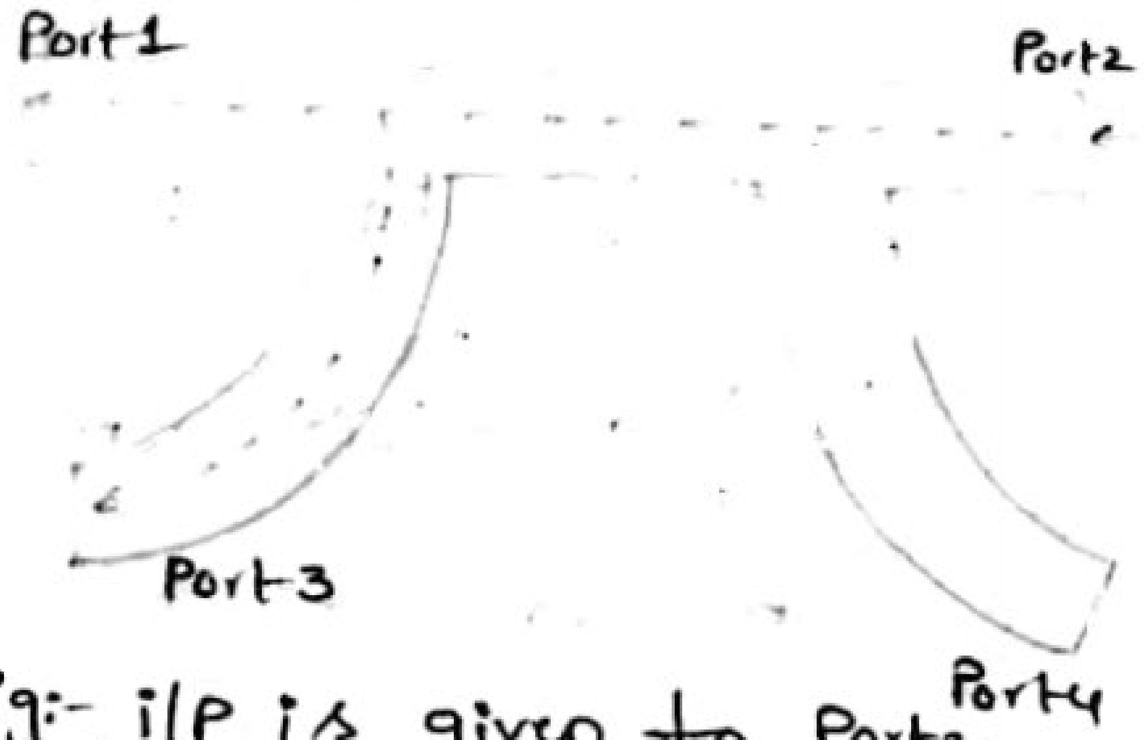


fig:- I/P is given to Port2

Port2 → incident Port

Port 1 → Received Port

Port3 → Forward Power coupling Port

Port4 → Back Port

Following are the Parameters used to define the Performance of a directional coupler: (i) coupling factor

(ii) Directivity

(iii) Isolation

(iv) Return loss

(i) Coupling factor (C):-

The coupling factor of a directional coupler is defined as "the ratio of incident power to the forward power".

It is measured in dB.

$$\boxed{C = 10\log_{10}(P_i/P_f) \quad (dB)}$$

Typically, for a directional coupler, $\boxed{C = 20\,dB}$

$$\therefore\ 20 = 10\log_{10}(P_i/P_f)$$

$$\Rightarrow 2 = \log_{10}(P_i/P_f)$$

$$\Rightarrow (10)^2 = \frac{P_i}{P_f}$$

$$\Rightarrow 100 = \frac{P_i}{P_f}$$

$$\Rightarrow \boxed{P_f = \frac{P_i}{100}}$$

(ii) Directivity (D):-

The directivity of a directional coupler is defined as "the ratio of forward power to the back power". It is measured in dB.

$$\boxed{D = 10\log_{10}(P_f/P_b) \quad (dB)}$$

Typically, for a directional coupler, $\boxed{D = 60\,dB}$

$$\therefore \; 60 = 10 \log_{10} (P_f/P_b)$$

$$\Rightarrow 6 = \log_{10} (P_f/P_b)$$

$$\Rightarrow (10)^6 = \frac{P_f}{P_b}$$

$$\Rightarrow P_b = \frac{P_f}{(10)^6}$$

$$\Rightarrow \boxed{P_b = \frac{P_i}{(10)^8}}$$

(iii) Isolation:-

It defines the directive properties of a directional coupler. It is defined as "the ratio of incident power to the back power". It is measured in dB.

$$\boxed{I = 10 \log_{10} (P_i/P_b) \; (dB)}$$

Isolation in dB = (Coupling factor) + (Directivity)

$$= 10 \log_{10} (P_i/P_f) + 10 \log_{10} (P_f/P_b)$$

(iv) Return loss:-

For signal transmission, return loss defines the actually transmitted power to the received power at the main guide. It is denoted by "R" and is given as:

$$\boxed{R = 10 \log_{10} (P_i/P_r) \; (dB)}$$

The noteworthy point over here is that all the parameters of the directional coupler are measured in dB.

Scattering matrix of Directional coupler:-

As directional couplers are 4 port devices, thus generally it is given as:

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix}$$

All the four ports of the directional coupler are matched perfectly. Thereby, ensuring that no power gets reflected back towards the port. Thus, the diagonal elements will be 0.

$$S_{11} = S_{22} = S_{33} = S_{44} = 0$$

$$\therefore \quad S = \begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{21} & 0 & S_{23} & S_{24} \\ S_{31} & S_{32} & 0 & S_{34} \\ S_{41} & S_{42} & S_{43} & 0 \end{bmatrix}$$

By the property of symmetry, $\boxed{S_{ij} = S_{ji}}$

Therefore,
$$S_{12} = S_{21}$$
$$S_{13} = S_{31}$$
$$S_{14} = S_{41}$$
$$S_{23} = S_{32}$$
$$S_{24} = S_{42}$$
$$S_{34} = S_{43}$$

So, the matrix will be given as,

$$S = \begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{12} & 0 & S_{23} & S_{24} \\ S_{13} & S_{23} & 0 & S_{34} \\ S_{14} & S_{24} & S_{34} & 0 \end{bmatrix}$$

Ideally, Port1, Port3 and Port2, Port4 are isolated, Withrespect to each other. So,

$S_{13} = S_{31} = 0$

$S_{24} = S_{42} = 0$

$$S = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{14} & 0 & S_{34} & 0 \end{bmatrix}$$

According to the identity Property,

$$[S][S^*] = [I]$$

$$\begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{14} & 0 & S_{34} & 0 \end{bmatrix} \begin{bmatrix} 0 & S_{12}^* & 0 & S_{14}^* \\ S_{12}^* & 0 & S_{23}^* & 0 \\ 0 & S_{23}^* & 0 & S_{34}^* \\ S_{14}^* & 0 & S_{34}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Further,

Forward Power: $S_{12} = S_{34} = P$

While the coupled Power: $S_{14} = S_{23} = Q$

Thus, the scattering matrix of the directional coupler will be given as,

(P.T.O)

$$S = \begin{bmatrix} 0 & P & 0 & Q \\ P & 0 & Q & 0 \\ 0 & Q & 0 & P \\ Q & 0 & P & 0 \end{bmatrix}$$

Applications:-

→ It is used to measure incident and reflected Power along with measuring voltage standing wave ratio values.

→ It also Provides the Path to the signal towards the receiver and used for the Purpose of unidirectional wave launching.

Types of Directional Coupler:-

Multi-Hole Directional coupler:-

→ It is a four Port waveguide junction consisting of Primary Wavelength and a secondary auxiliary waveguide.

→ They can sample a small amount of microwave Power for measurement Purpose.

→ They are designed to measure incident and reflected Power, SWR values, Provide a signal Path to a receiver (or) Perform other desirable operations.

→ The coupling is done through holes on the broad side of the Waveguide.

→ The diameter of no.of holes in a row and the no.of rows vary according to coupling sector required.

→ Scientific Microwave offers 3dB, 10dB and 20dB couplers to its customers with minimum VSWR.

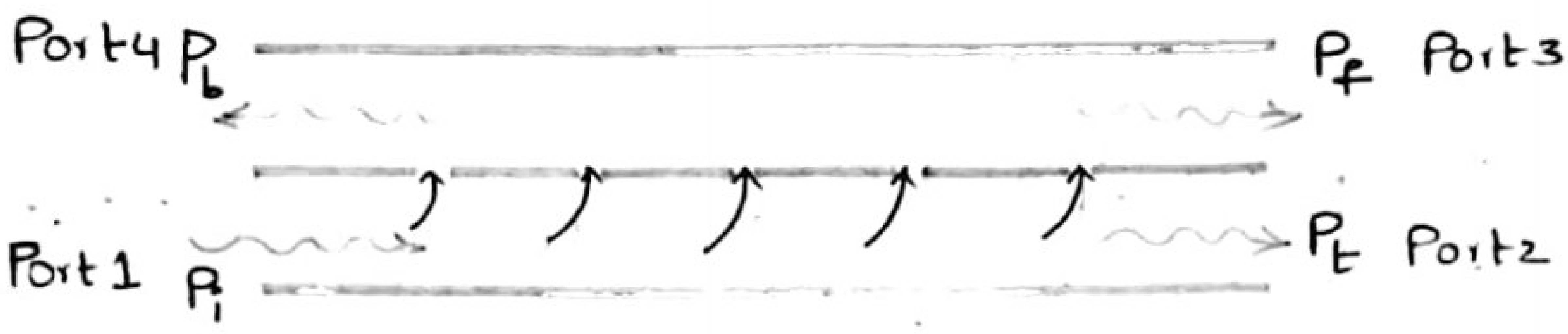


fig:- Multi-hole directional coupler

S-Matrix Calculations:-

(i) [S] is a square matrix of order 4x4, since there are 4 four ports.

$$\therefore [S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix}$$

(ii) [S] is a symmetric matrix i.e., $\boxed{S_{ij} = S_{ji}}$

$$\therefore \quad S_{12} = S_{21} \quad \Big| \quad S_{41} = S_{14}$$
$$S_{13} = S_{31} \quad \Big| \quad S_{42} = S_{24}$$
$$S_{32} = S_{23} \quad \Big| \quad S_{43} = S_{34}.$$

(iii) Consider, a perfectly matched directional coupler.

$$\therefore \boxed{S_{11} = S_{22} = S_{33} = S_{44} = 0}$$

(iv). Port 1, Port 3 and Port 2, Port 4 are isolated to each other

$$\therefore \boxed{\begin{matrix} S_{13} = S_{31} = 0 \\ S_{24} = S_{42} = 0 \end{matrix}}$$

Now, the updated S-matrix is given by,

$$[S] = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{14} & 0 & S_{34} & 0 \end{bmatrix}$$

(v) From the unitary property, $[S][S^*] = [I]$

$$\Rightarrow \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{14} & 0 & S_{34} & 0 \end{bmatrix} \begin{bmatrix} 0 & S_{12}^* & 0 & S_{14}^* \\ S_{12}^* & 0 & S_{23}^* & 0 \\ 0 & S_{23}^* & 0 & S_{34}^* \\ S_{14}^* & 0 & S_{34}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\underline{R_1 C_1}$:- $0 + S_{12} \cdot S_{12}^* + 0 + S_{14} \cdot S_{14}^* = 1$

$$\Rightarrow |S_{12}|^2 + |S_{14}|^2 = 1$$

$\underline{R_2 C_2}$:- $S_{12} \cdot S_{12}^* + 0 + S_{23} \cdot S_{23}^* + 0 = 1$

$$\Rightarrow |S_{12}|^2 + |S_{23}|^2 = 1$$

$\underline{R_3 C_3}$:- $0 + S_{23} \cdot S_{23}^* + 0 + S_{34} \cdot S_{34}^* = 1$

$$\Rightarrow |S_{23}|^2 + |S_{34}|^2 = 1$$

$\underline{R_4 C_4}$:- $S_{14} \cdot S_{14}^* + 0 + S_{34} \cdot S_{34}^* + 0 = 1$

$$\Rightarrow |S_{14}|^2 + |S_{34}|^2 = 1$$

$\underline{R_1 C_3 = 0}$:-

$$S_{12} \cdot S_{23}^* + S_{14} \cdot S_{34}^* = 0$$

consider, $R_1 C_1 = R_2 C_2$

$\Rightarrow |S_{12}|^2 + |S_{14}|^2 = |S_{12}|^2 + |S_{23}|^2$

$\Rightarrow |S_{14}|^2 = |S_{23}|^2$

$\Rightarrow S_{14} = S_{23}$

$\therefore \boxed{S_{14} = S_{23}}$

Similarly, consider $R_2C_2 = R_3C_3$

$\Rightarrow |S_{12}|^2 + \cancel{|S_{23}|^2} = \cancel{|S_{23}|^2} + |S_{34}|^2$

$\Rightarrow |S_{12}|^2 = |S_{34}|^2$

$\Rightarrow S_{12} = S_{34}$

$\therefore \boxed{S_{12} = S_{34}}$

Let, $S_{12} = S_{34} = P = S_{34}^*$ ($P \rightarrow$ some Real value)

We have, R_1C_3 as

$S_{12} \cdot S_{23}^* + S_{14} \cdot S_{34}^* = 0$

$\Rightarrow P S_{23}^* + S_{23} \cdot P = 0$

$\Rightarrow P(S_{23}^* + S_{23}) = 0$

$\Rightarrow S_{23}^* + S_{23} = 0$

$\Rightarrow \boxed{S_{23}^* = -S_{23}}$

Let $S_{23} = jP$

$\therefore$ The scattering matrix is given by,

$$[S] = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{14} & 0 & S_{34} & 0 \end{bmatrix} = \begin{bmatrix} 0 & P & 0 & jP \\ P & 0 & jP & 0 \\ 0 & jP & 0 & P \\ jP & 0 & P & 0 \end{bmatrix}$$

Single-hole directional coupler
(or)
Bethe-hole directional coupler

→ Bethe-hole is a Waveguide directional coupler, using a single hole, and it works over a narrow band.

→ The Bethe-hole is a reverse coupler, as opposed to most Waveguide couplers that use multi-hole and are foward couplers.

→ The origin of the name comes from a paper published by H.A. Bethe titled "Theory of Diffraction by small holes," Published in the Physical Review, back in 1942.

→ The following figure represents a single-hole / Bethe-hole directional coupler.

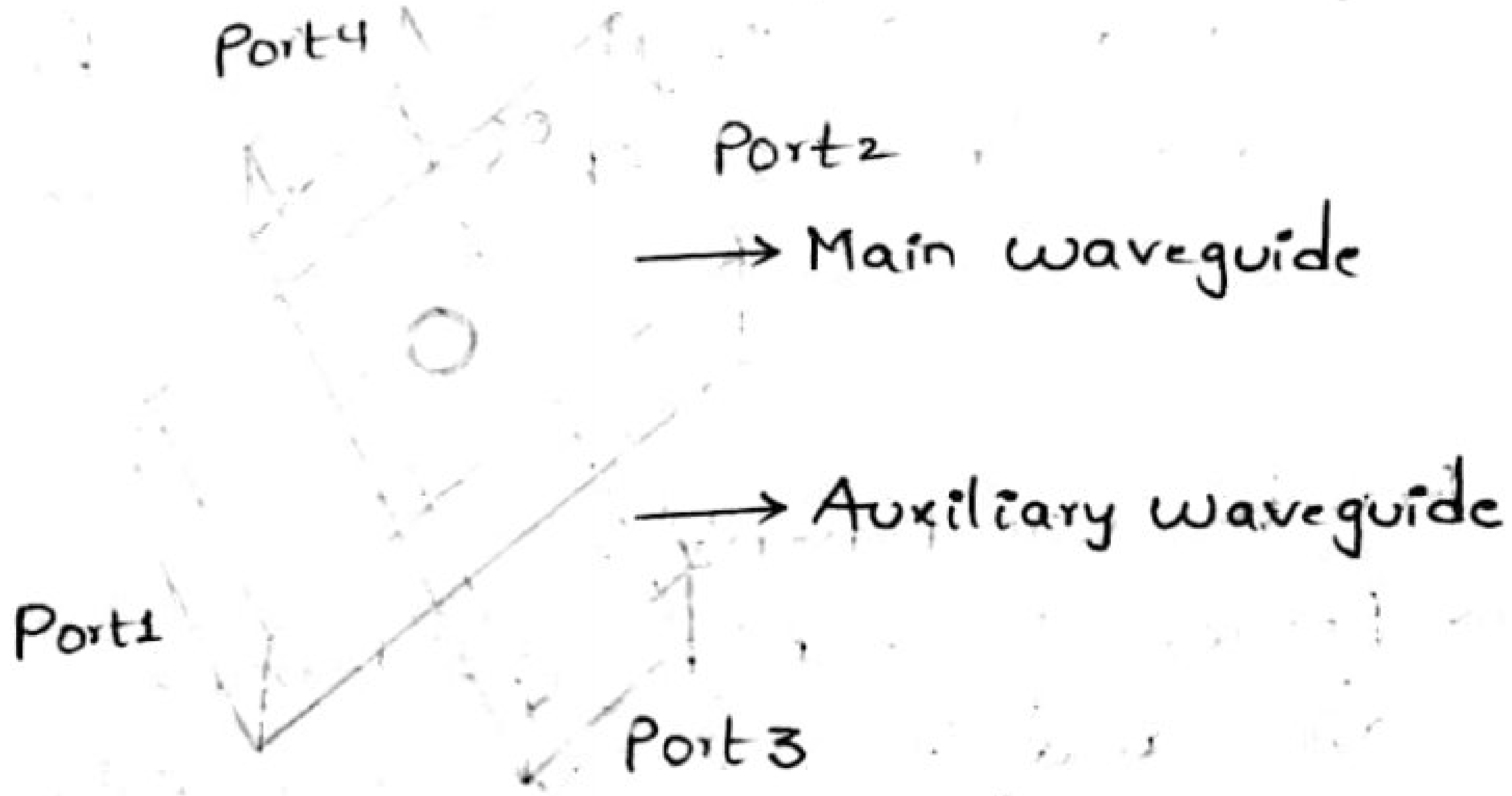


fig: Bethe-hole directional coupler

→ It is a single-hole directional coupler, the hole is located at the center of the broadwall waveguide.

→ It consists of a Primary/main waveguide and a secondary/auxiliary waveguide which are placed at 'θ' inclination.

Scanned with CamScanner

→ when the Power travels from Port 1 to Port 2, through the Bethe-hole, the hole also known as aperture, acts as an <u>electric dipole</u>. And its size is less than the wavelength of the wave.

→ Therefore, it radiates Power to the secondary waveguide and this entire phenomenon is referred to as "<u>dipole radiation</u>".

→ In other words, the coupling to the auxiliary waveguide is due to the radiation radiated by the electric dipole (aperture).

→ Finally, the Power entering the two waveguides (Port 4 & Port 3) can be controlled by varying 'θ' and wavelength (λ) of the microwave signal through the waveguide.

(Note:- S-Matrix Calculations are same for the both types of directional couplers)

Ferrite Components:-

→ Ferrite is a high resistance magnetic material and it consists of mainly ferrite oxide along with one (or) more other metals.

→ Ferrite material is extremely useful at microwave frequencies.

→ Electromagnetic wave Passes through ferrites with negligible attenuation.

→ Electromagnetic wave Propagation undergoes Phase shift due to ferrites, which can be influenced by the applied DC magnetic fields.

Properties of ferrite components:-

Ferrite components constitute peculiar properties as listed below:

(i) Ferrites are "non-metallic materials" with resistivity (ρ) nearly 10^4 times greater than metals and with dielectric constant (ϵ_r) around 10–15 and relative permeabilities in the order of 1000.

(ii) They have magnetic properties similar to those of "ferrous materials".

(iii) They are oxide based compounds having general composition of the form $\boxed{MeO\,Fe_2O_3}$ i.e., a mixture of metal oxide & Ferric oxide. Here 'MeO' represents divalent metallic oxides such as MnO, ZnO, CdO, NiO (or) a mixture of these.

(iv) These are obtained by firing powdered oxides of materials at 1100°C (or) more & pressing them into different shapes.

(v) Ferrites have atoms with large no. of spinning electrons, which result in strong magnetic properties. The magnetic properties are due to dipole moment associated with the electron spin.

(vi) Due to high resistivity, they can be used upto 100 GHz.

(vii) Ferrites have one more peculiar property, which is used at microwave frequencies. The property is known as "Non-reciprocal property".

This property states that when two circularly polarised waves, out of which one is rotating in clockwise direction while the other is rotating in anti-clockwise direction, are made to propagate through a ferrite material the material reacts differently to the two rotating fields, thereby presenting different medium constants to both the waves. i.e., $\varepsilon_{r1}, \mu_{r1}, \rho_1$ for "left-circularly polarized" wave and $\varepsilon_{r2}, \mu_{r2}, \rho_2$ for "right-circularly polarized" wave. This property is used in "Faradays rotation".

(viii) The specific resistivity of ferrites for use at microwave frequency is on the order of 10^{12} ohm-cm.

(ix) Typical relative's permittivities of ferrites lie in the range of 5-20.

Applications:-

→ Because of above properties, ferrites find application in a no. of microwave devices "to reduce reflected power" for "modulation purposes" & in "switching circuits".

→ The ferrites are popularly used in microwave "isolators", "circulators" & "switches".

→ They are used at RF frequencies in inductors as core material.

→ They are also used in TV (cathode ray tube) deflection yokes.

→ Also used in phase shifters, variable attenuat

Faraday rotation in ferrites:-

→ Consider a three-dimensional coordinate system with X, Y and Z axes respectively.

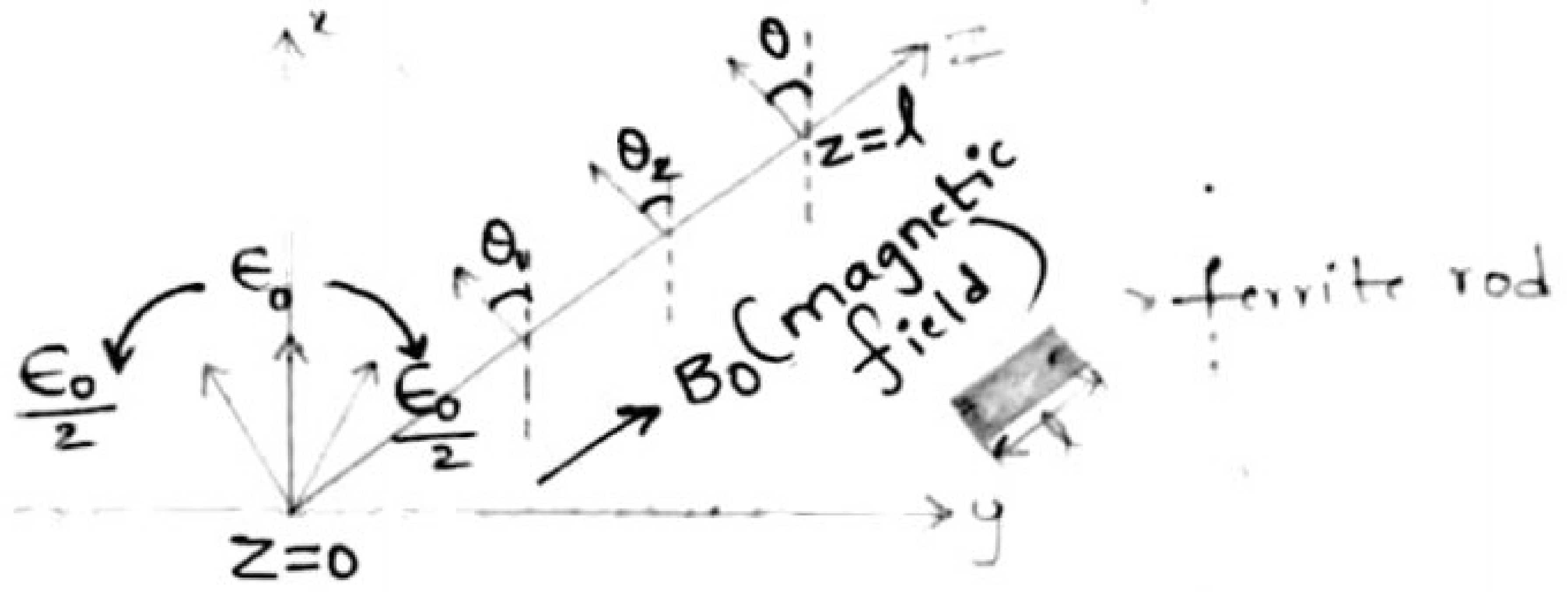


fig:- Faraday's Rotation

→ consider an infinite lossless medium. A static magnetic field $\underline{B_0}$ is applied along Z-direction. A Plane TEM wave that is vertically Pola-rised along Z-axis, at Z=0 is made to Propagate through the ferrite material in the Z-direction.

→ The Plane of Polarization of this wave, will rotate with distance and this Phenomenon is referred to as Faraday Rotation.

→ Any linearly Polarized wave can be resolved into two components:

(i) Left circularly Polarized

(ii) Right circularly Polarized

Hence, it can be regarded as the vector sum of two counter rotating circularly Polarized waves.

→ When the wave Propagates through the ferrite rod of length "l" which is Placed along z-axis, the plane of Polarization changes with distance by θ_1. When it further travels, the Plane of Polarization changes by θ_2 and finally at $z=l$, the Plane of Polarization changes by θ.

→ The ferrite material offers different characteristics to these waves, with the result that the phase change for one wave is larger than the other wave resulting in rotation 'θ' of the linearly polarized wave at $z=l$.

→ If the direction of propagation is reversed the plane of polarization continues to rotate in the same direction i.e., from $z=l$ to $z=0$. The wave will come back at $z=0$, polarized at an angle of <u>2θ</u>, relative to x-axis.

→ The angle of rotation 'θ' is given by

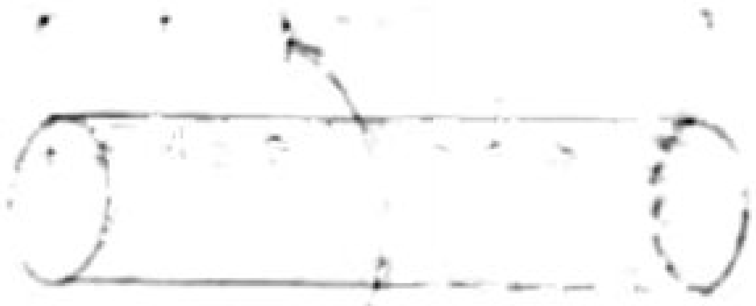
$$\boxed{\theta = \frac{l}{2}(\beta^{+} - \beta^{-})}$$

Where, l = length of ferrite rod

β^{+} = phase shift of right circularly polarized wave

β^{-} = Phase shift of left circularly polarized wave.

→ A two port ferrite device is shown below:

Port 1  Port 2

(a) When a wave is transmitted from Port ① to Port ②, it undergoes rotation in <u>anti-clockwise direction</u>, as shown above.

(b) If the same wave is allowed to propagate from port ② to Port ①, it will undergo a rotation in the same direction (Anti-clockwise direction).

→ The Principle of Faraday Rotation is used in Gyrator, Circulator and Isolator.

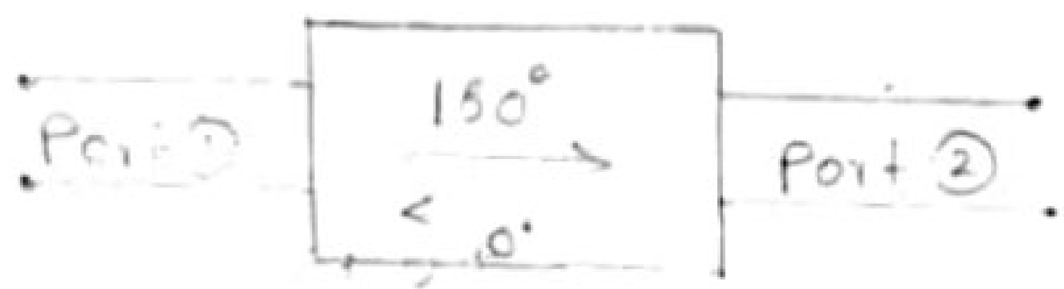
Gyrator:-

→ Gyrator is a "non-reciprocal ferrite device".

→ It is a two port device that has a relative phase shift of 180° in the forward direction and zero phase shift in reverse direction.

→ When signal is transmitted from Port1 to Port2 it offers a phase shift of 180° (π radians) and when the signal is fed to Port2 it offers 0° phase shift to the signal.

→ Hence it is also known as "differential phase shift device".



Construction:-

→ Gyrator filter consists of a circular to rectangular waveguide transition both at dominant mode.

→ A twin circular ferrite rod tappered at both ends is located inside the circular waveguide, surrounded by permanent magnets which generate D.C. magnetic field, for proper operation of the ferrite.

(Or)

A thin circular ferrite rod tapered at both ends is located inside the circular waveguide, supported by polyfoam and surrounded by permanent magnets. These will generate DC magnetic field, for proper operation of the ferrite rod.

→ A rectangular waveguide twisted by 90° is connected to input.

→ The ferrite rod is tapered at both ends to reduce attenuation and also for smooth rotation of polarised wave.

→ The schematic diagram of gyrator is shown in the figure below:

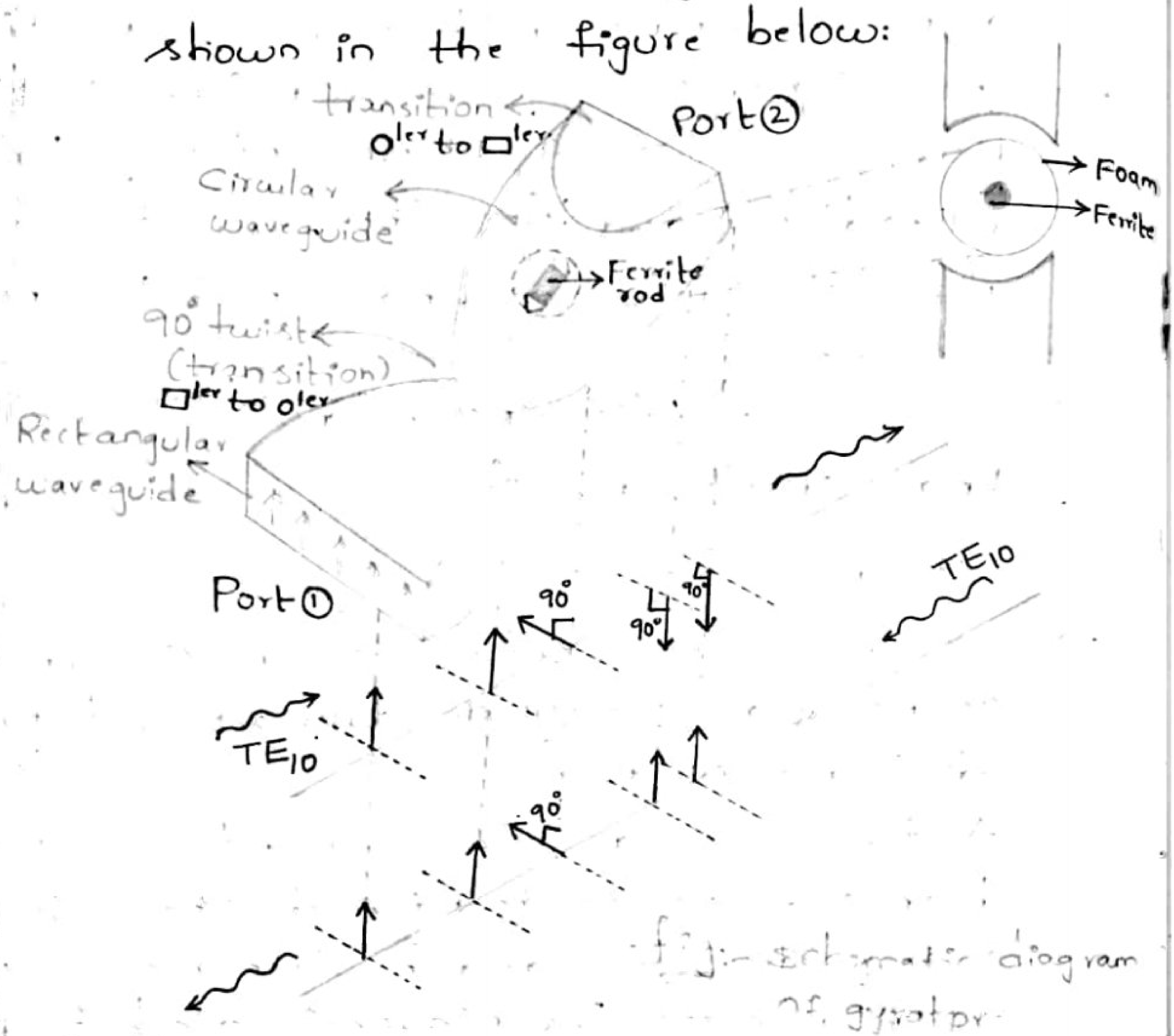


fig:- schematic diagram of gyrator

Operation:-

→ When a wave enters Port①, its plane of polarization, rotates by 90°, because of twist in the waveguide.

→ It again undergoes Faraday rotation through 90°, because of ferrite rod and the wave coming out of Port②, will have a phase shift of 180° compared to wave entering Port①.

→ When the same wave (TE10 mode Signal) enters Port②, it undergoes Faraday Rotation through 90°, in the anti-clockwise direction.

→ Because of twist, the wave gets rotated back by 90°, comes out of Port①, with 0° Phase shift as shown in the figure.

→ Hence, the wave from Port① to Port② undergoes a Phase shift of π radians, but the same wave from Port② to Port① does not change its Phase in gyrator.

Gyrator and Transformer:-

→ A gyrator is linear, lossless, Passive and memory-less two Port device which is similar to an ideal transformer.

→ However, a transformer couples the voltage on Port 1 to the voltage on Port 2 and current on Port1 to current on Port 2, the gyrator cross couples the voltage to current and current to voltage.

→ 2 gyrators cascaded together gives us a voltage to voltage coupling similar to an ideal transformer.

Gyrator- S Matrix Parameters:-

(i) S is a 2X2 matrix, since there are two Ports.

$$\therefore [S] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

(ii) If the the Ports are Perfectly matched and there are no reflections, then

$$\boxed{S_{11} = S_{22} = 0}$$

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We know that

$$[b] = [S][a]$$

Hence, $b_2 = S_{21} a_1$

$b_2 = -a_1 \Leftrightarrow S_{21} = -1$

$\therefore \boxed{S_{21} = -1}$

Similarly, $b_1 \Rightarrow b_1 = S_{12}\, a_2$

$\Rightarrow b_1 = a_2 \Leftrightarrow S_{12} = 1$

$\therefore \boxed{S_{12} = 1}$

$$\therefore [S] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

Gyrator Applications:-

* A gyrator can be used to transform load into an inductance. At very low frequencies & low power the behaviour of the gyrator can be reproduced by small OP-Amp circuit. It can be done by producing a small inductive element in a small electronic circuit. Before the transistor came into existence coils of wire with large inductance might be used in electronic filters. An inductor can be replaced by smaller assembly containing a capacitor, OP-amp/transistor and resistor. This is used in "integrated circuit technology".
* gyrator as an inductor – the main application of gyrator is to reduce the size and cost of a system by removing the heavy, bulky and expensive inductors.

for example. RLC bandpass filter characteristics can be realized with capacitors, OP-amps, resistors without using inductors. Graphic equilization is possible using gyrators. There are two types of gyrators one is passive gyrator and other is active gyrator.

** Isolator:-

→ An isolator is a "non-reciprocal transmission device" that is used to isolate one component from reflections of other components in the Transmission line.

→ An ideal isolator completely absorbs the power for propagation in one direction and provides lossless transmission in the opposite direction. Thus, the isolator is usually called "Uniline".

→ An isolator is a two-port device, which provides a very small amount of attenuation for transmission from port ① to port ②, but provides maximum attenuation for transmission from port ② to port ①.

MW Source $\xrightarrow{}$ Isolator $\xrightarrow{}$ Load

MW Source $\underset{=0}{\longleftarrow}$ Isolator $\longleftarrow$ Load

→ The mismatch of generator o/p. to the load, results in a reflected wave from load. But, these reflected waves should not be allowed to reach the microwave generator, which will cause "amplitude and frequency instability" of microwave generator.

→ When isolator is inserted between generator and load, the generator O/P is coupled to the load with zero attenuation and reflections if any from the load are completely absorbed by the isolator without affecting the generator o/p. Hence generator appears to be matched for all loads in the presence of isolator.

Construction:-

→ Isolators can be constructed in many ways. They can be made by terminating Port3 and Port4, of a circular (four-Port) with matched loads.

→ on the other hand, isolators can be made by inserting a ferrite rod along the axis of a rectangular waveguide. Now, Let us see the construction of a "Faraday Rotation isolator".

→ The construction of Faraday Rotation isolator is similar to gyrator, except that an isolator makes use of "45° twisted rectangular waveguide" (instead of 90° RWG) and "45° clock-wise rotation ferrite rod" (instead of 90° anti-clockwise ferrite rod used in gyrator).

→ "Resistive cards" are placed along the larger dimensions of the waveguide, so as to absorb any wave whose plane of polarization is parallel to the plane of resistive card.

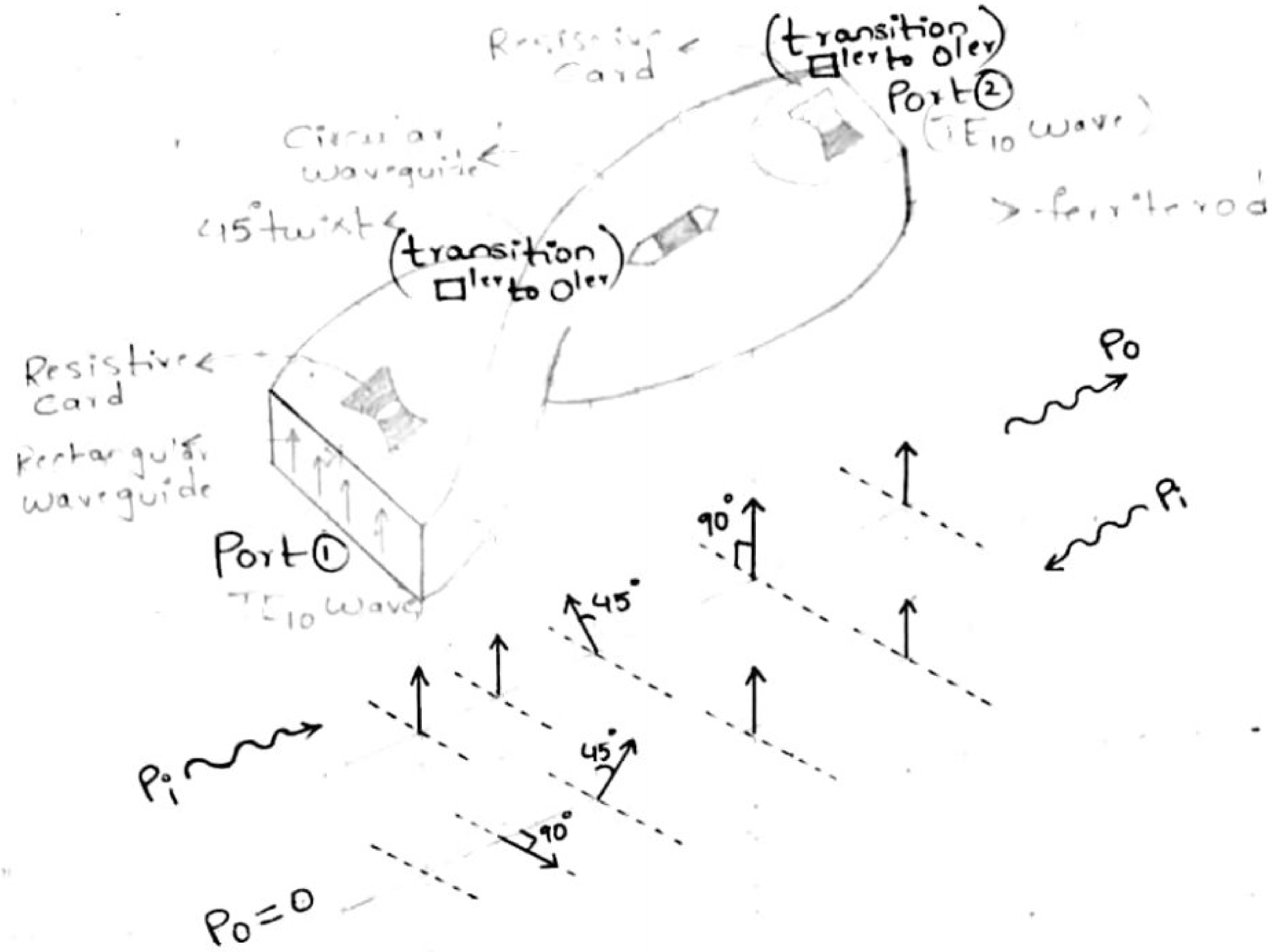


fig:- Schematic diagram of isolator.

→ The resistive cards will not absorb any wave, whose plane of polarization is perpendicular to its own plane.

→ It provides 20 to 30dB isolation from Port② to Port①.

Operation:-

→ A vertically polarised TE_{10} wave passing from Port① through the resistive card is not absorbed.

→ After coming out of the card, the wave gets shifted 45°, because of twist in anti-clock wise direction and then by another 45° in clock-wise direction because of ferrite rod and comes out of Port② with same polarization as that of Port① without any attenuation.

→ But a TE_{10} wave fed from Port ② gets a Pass from resistive card placed near Port ② since the Plane of Polarization of wave is Parallel to the Plane of resistive card.

→ Then the wave gets rotated by 45° due to faraday-rotation in clock-wise direction and further gets rotated by 45° in clock-wise direction due to twist in the waveguide.

→ Now the Plane of Polarization of wave is Parallel with that of resistive card and hence the wave will be completely absorbed by the resistive card and therefore the o/p at Port① will be zero. The Power in the card gets dissipated as heat.

Isolator- S Matrix Parameters:-

(i) S is a 2×2 matrix, Since there are two Ports.

i.e., $$[S] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

(ii) If the Ports are Perfectly matched and there are no reflections, then

$$\boxed{S_{11} = S_{22} = 0}$$

We know that

$$[b] = [S][a]$$

Hence, $b_2 = S_{21} a_1$

$\Rightarrow b_2 = a_1 \Leftrightarrow S_{21} = 1$

$\therefore \boxed{S_{21} = 1}$

Similarly, $b_1 = S_{12}\, a_2$

$$\Rightarrow b_1 = 0 \Leftrightarrow S_{12} = 0$$

$$\therefore \boxed{S_{12} = 0}$$

$$\therefore [S] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

Isolator Applications:-

→ Isolators are generally used to improve the frequency stability of microwave generators, in which the reflection from the load affects the generating frequency. In such cases, the isolator placed between the generator and load prevents the reflected power from the unmatched load from returning to the generator. As a result the isolator maintains the frequency stability of the generator.

→ The applications of isolators involve high voltage devices such as transformers.

→ These are protected with a locking system on the external (or) with a lock to stop accidental usage.

→ Isolator in substation:- When a fault occurs in a substation, then the isolator cuts out a portion of substation.

This is all about an overview of the electrical isolator. The characteristics of this isolator include it is an offload device operated manually, De-energize the circuit, entire isolation for secure maintanance includes a Padlock, etc...

** Circulator:-

→ A circulator is a "multi-Port ferrite device".

→ There is no restriction on number of Ports. "Four Port" microwave Circulator is most common.

→ It has a "Peculiar Property" that each terminal is connected only to the next clockwise terminal i.e., Port ① is connected to Port ② only and not to Port ③ & Port ④. Similarly, Port ② is connected to Port ③ but not to Port ④ & Port ①, and so on.

→ Wave can flow from one Port to another Port in one direction.

→ Circulators are useful in "Parametric amplifiers," "tunnel diode amplifiers and as "duplexer in radars".

Construction:-

↳ The schematic diagram of a Circulator is shown in the below figure.

→ The arrows within the circulator signify the direction of the magnetic field when the signal is applied to one of the Ports of these devices.

→ If a signal is applied at Port-A, and Port B is well-suited, then the applied signal will exit from Port B with 0.4dB loss.

→ If there is a difference at Port-B, the signal can be reproduced from Port-B, that will be directed toward Port c.

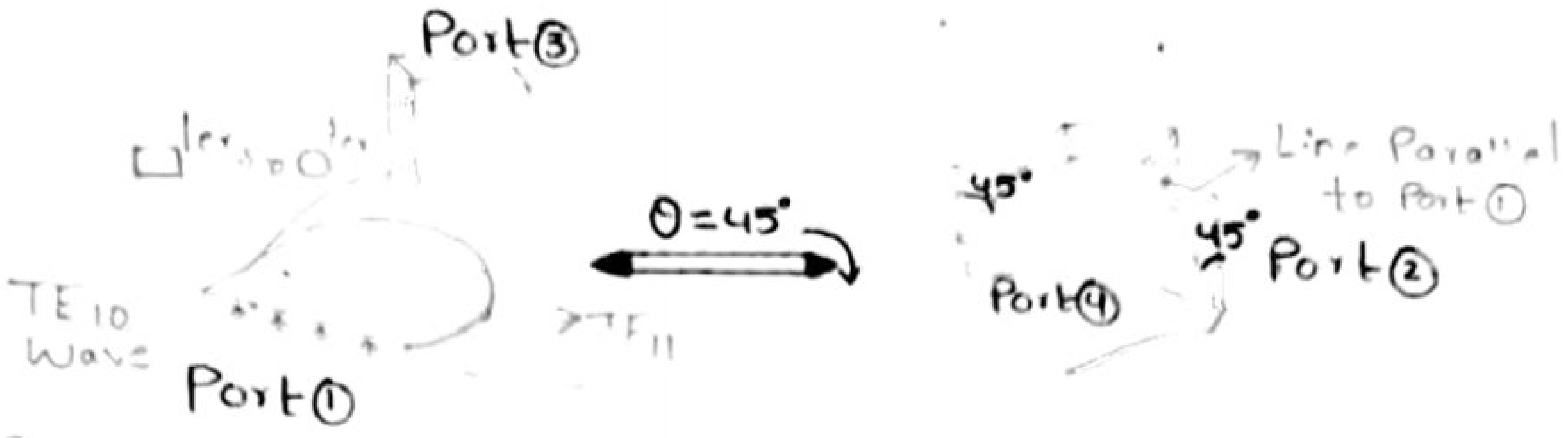


fig:- Schematic diagram of four-Port circulator

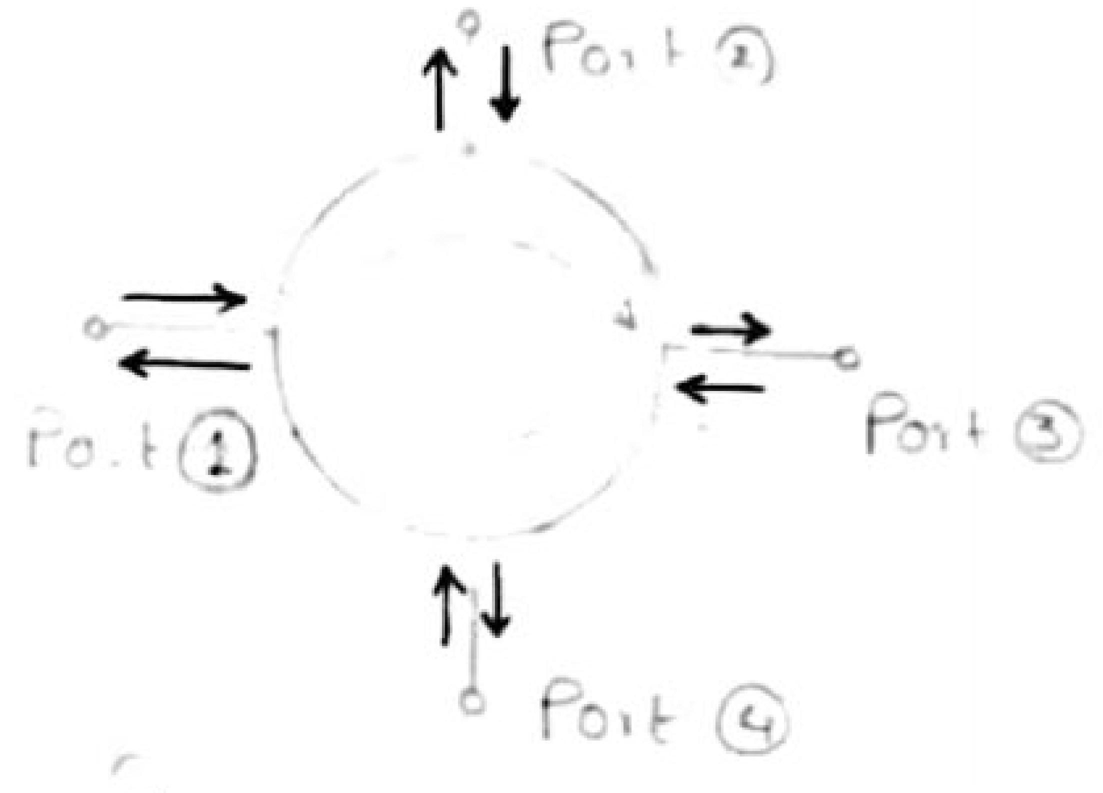


fig:- 4-Port circulator symbolic representation

Operation:-

→ The wave entering Port① is TE_{10} mode and is converted to TE_{11} mode because of rectangular to circular transition.

→ This waves passes Port③, unaffected since the electric field is not significantly cut and is rotated through 45° in clockwise due to ferrite rod, passes Port④, unaffected (for the same reason as it passes Port ③). Finally, the wave emerges out of Port ②.

→ The wave entering Port② will have plane of polarization already tilted 45° with respect to Port①. This wave passes Port④ unaffected because the electric field is not significantly cut. This wave gets rotated another 45°, due to ferrite rod in clockwise direction.

→ This wave whose plane of polarization tilted by 90°, finds Port③ suitably alinged and emerges out of it.

→ Similarly, Port ③ is coupled to Port ④ and Port ④ is coupled to Port ①.

<u>Circulator - S Matrix Parameters:-</u>

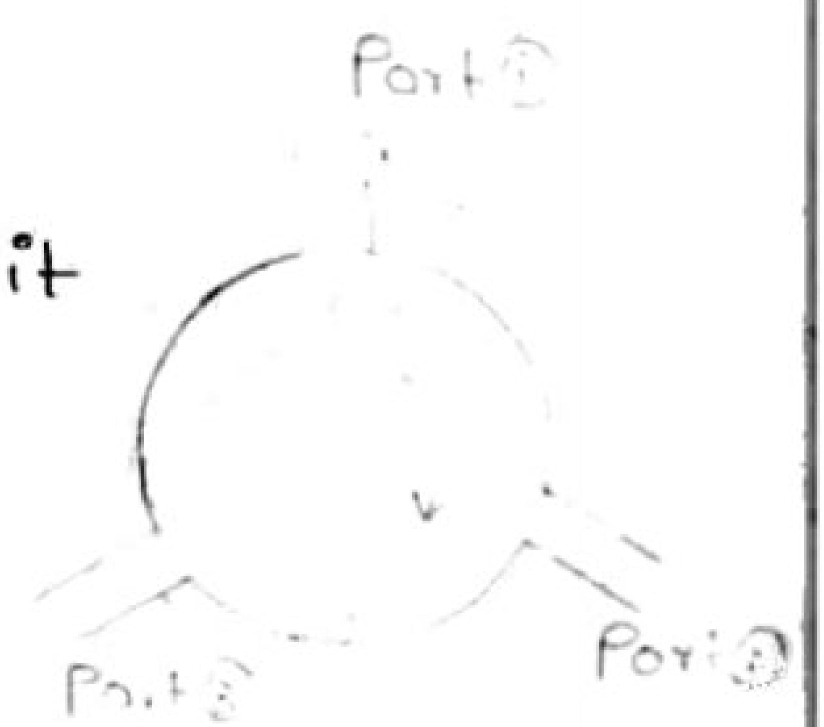
Consider a 3-Port circulator.

(i) S is a <u>3×3</u> matrix, since there are 3 Ports

i.e., $[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$

(ii) If all the 3 Ports are perfectly matched and if there are no reflections from the 3 Ports, then

$$\boxed{S_{11} = S_{22} = S_{33} = 0}$$



(iii) When i/p is given to Port ②, it will not come to Port ③.

i.e., $S_{12} = 0$

When i/p is given to Port ③, it will not come to Port ②.

i.e., $S_{23} = 0$

Similarly, when i/p is given to Port ①, it will not come to Port ③.

i.e., $S_{31} = 0$

$$\therefore \boxed{\begin{array}{l} S_{12} = 0 \\ S_{23} = 0 \\ S_{31} = 0 \end{array}}$$

(iv) When i/p is given to Port ①, it will flow to Port ②

i.e., $S_{21} = 1$

When i/p is given to Port ②, it will flow to Port ③.

i.e., $S_{32} = 1$

When i/p is given to Port ③, it will flow to Port ①.

i.e., $S_{13} = 1$

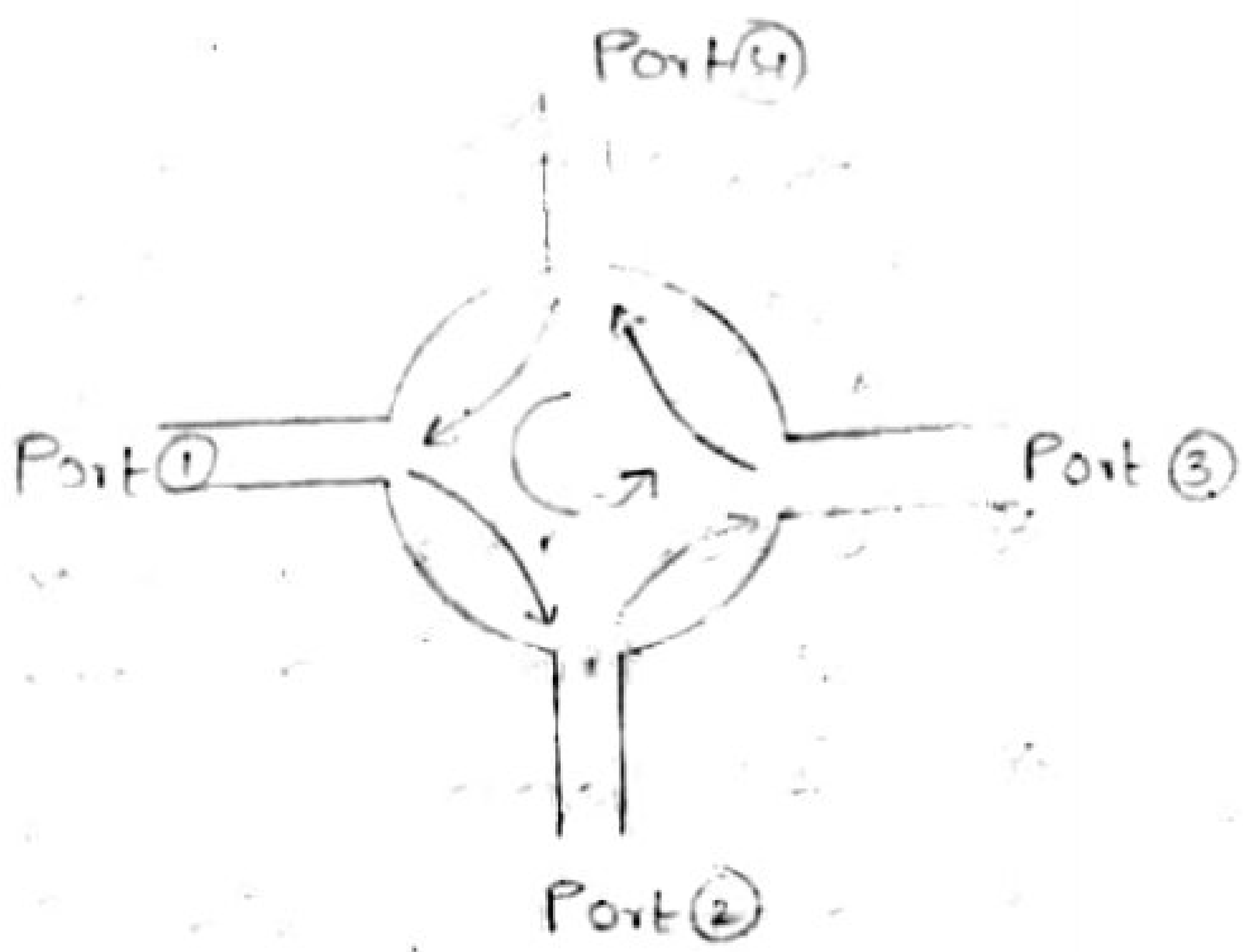
$$\therefore \boxed{S_{21} = S_{32} = S_{13} = 1}$$

$$\therefore [S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Similarly, for a 4-Port Circulator, S-Matrix is given by,

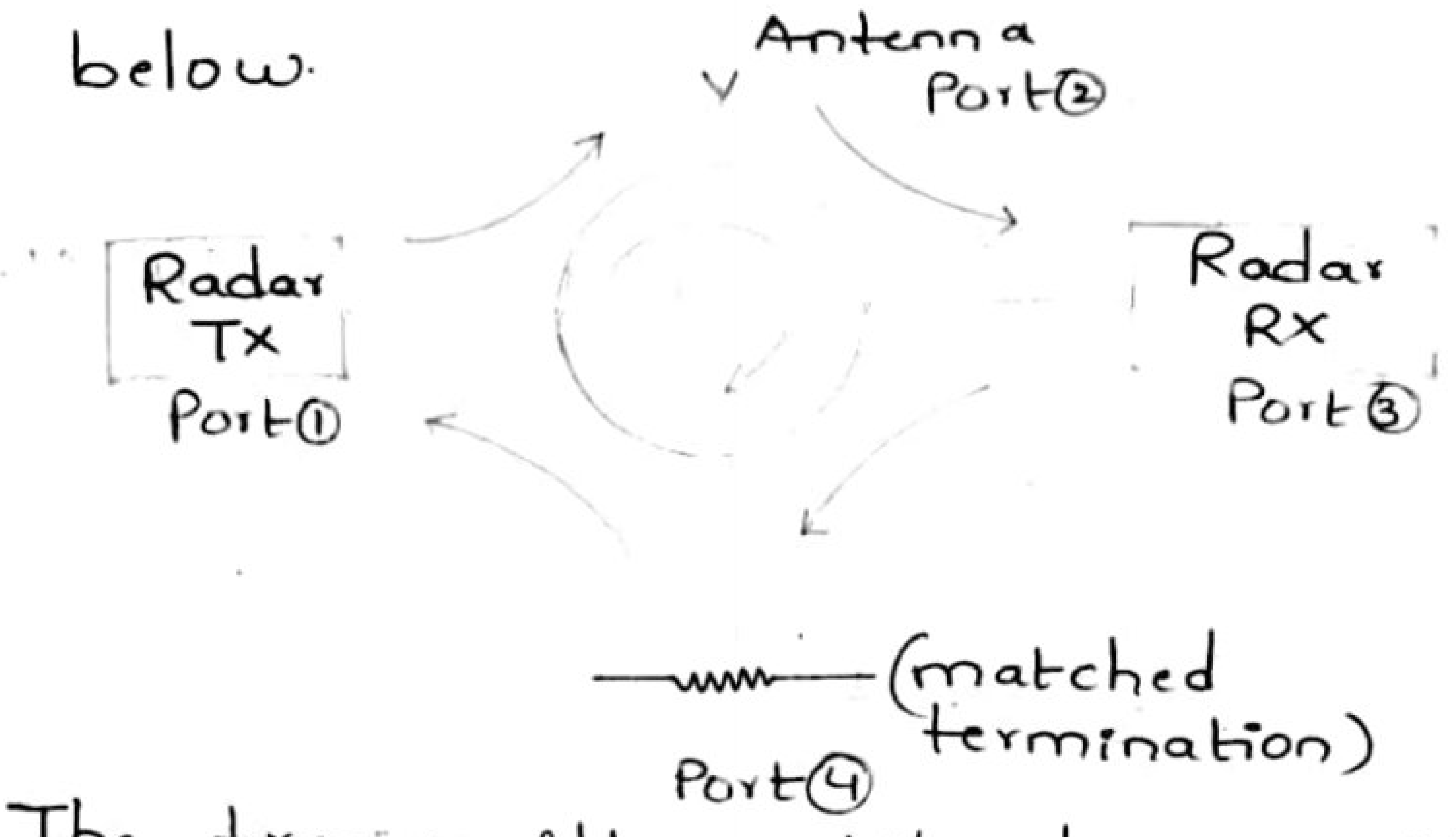
$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix} = \begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{21} & 0 & S_{23} & S_{24} \\ S_{31} & S_{32} & 0 & S_{34} \\ S_{41} & S_{42} & S_{43} & 0 \end{bmatrix}$$

$$\Rightarrow S = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$



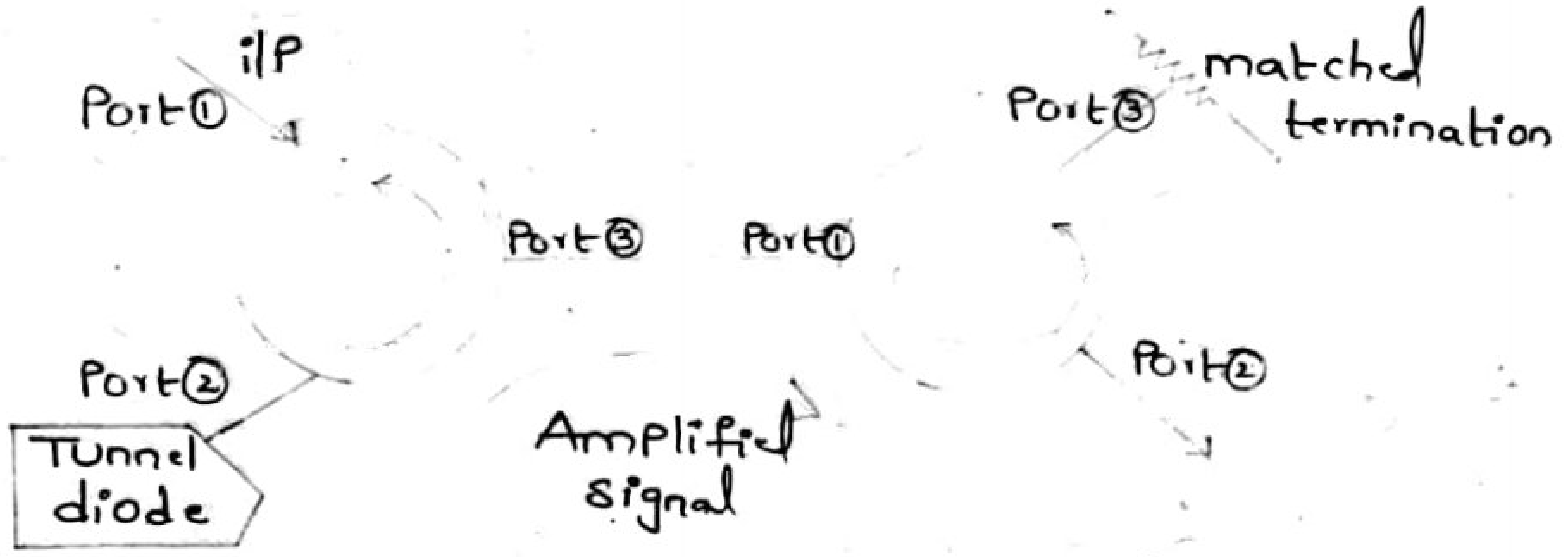
Applications of Circulator:-

* A circulator can be used as a duplexer for radar antenna system as shown below.



The transmitter while transmitting, is connected to the antenna, when antenna receives a signal, it directs it to the Receiver. The same antenna can be used for Transmission & Reception purpose. Hence, the circulator performs the function of a "Duplexer".

* A 3-Port Circulator can be used in "tunnel diode" (or) "Parametric amplifier".



* Circulators can be used as low power devices as they can handle Low Power only.
* Isolator
* Reflection Amplifier
* Radar systems

* Amplifier systems
* Antenna transmitting (or) receiving

Circulator characteristics:-

The characteristics of Circulator include the following.

→ Insertion loss is < 1 dB.

→ Isolation range is approximately from 30 dB to 40 dB.

→ VSWR (voltage Standing Wave Ratio) is < 1.5

Thus, this is all about circulators. The selection of circulator can be done using features like frequency, isolation, Power & insertion loss.

Difference between isolator & circulator:-

→ RF Circulator is a 3-Port device and isolator is a 2-Port device. (branch transmission system)

→ Both allow signal to flow in any one direction and Prevents signal going into the other direction as Per design.

→ RF circulator being having 3 Ports, there are two main types clockwise and anti-clockwise.

→ If Ports are say P_1, P_2 and P_3 then isolator can Pass signal from P_1 to P_2, P_2 to P_3 and from P_3 to P_1 and not in other direction, if designed so otherwise it will Pass signal from P_3 to P_2 and P_2 to P_1 and from P_1 to P_3.

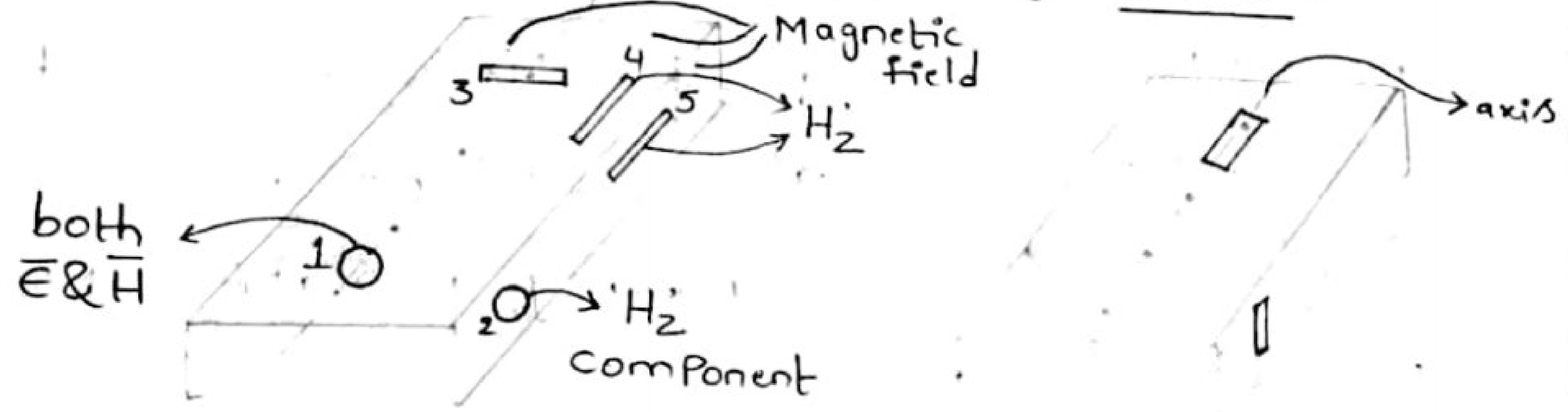
→ The uni-directional transmission feature can be used to isolate the effects of load changes on the signal source.

Waveguide Apertures:-

→ Techniques used for coupling microwave energy include inductive loops, capacitive probes etc...

→ The most common method is to use Apertures in the Waveguide walls, usually in the form of circular holes (or) thin slots.

→ The theory of radiation through small apertures was developed by Bethe.



fig(a):- Radiating Apertures

fig(b):- Non-radiating Apertures

fig:- Radiating & non radiating apertures in rectangular waveguide (TE_{10} mode propagation)

→ Assuming TE_{10} mode of Propagation, the ones in fig(a), radiate a portion of wave energy & therefore find use in Antenna arrays and directional couplers.

→ The figures (c) & (d) shows field patterns for a circular hole centrally located in the broad wall of a waveguide (aperture 1).

→ The electric field "E_y" in the main waveguide extends into the Auxiliary/secondary waveguide in fig(c), whereas "H_x" couples into secondary waveguide in fig(d).

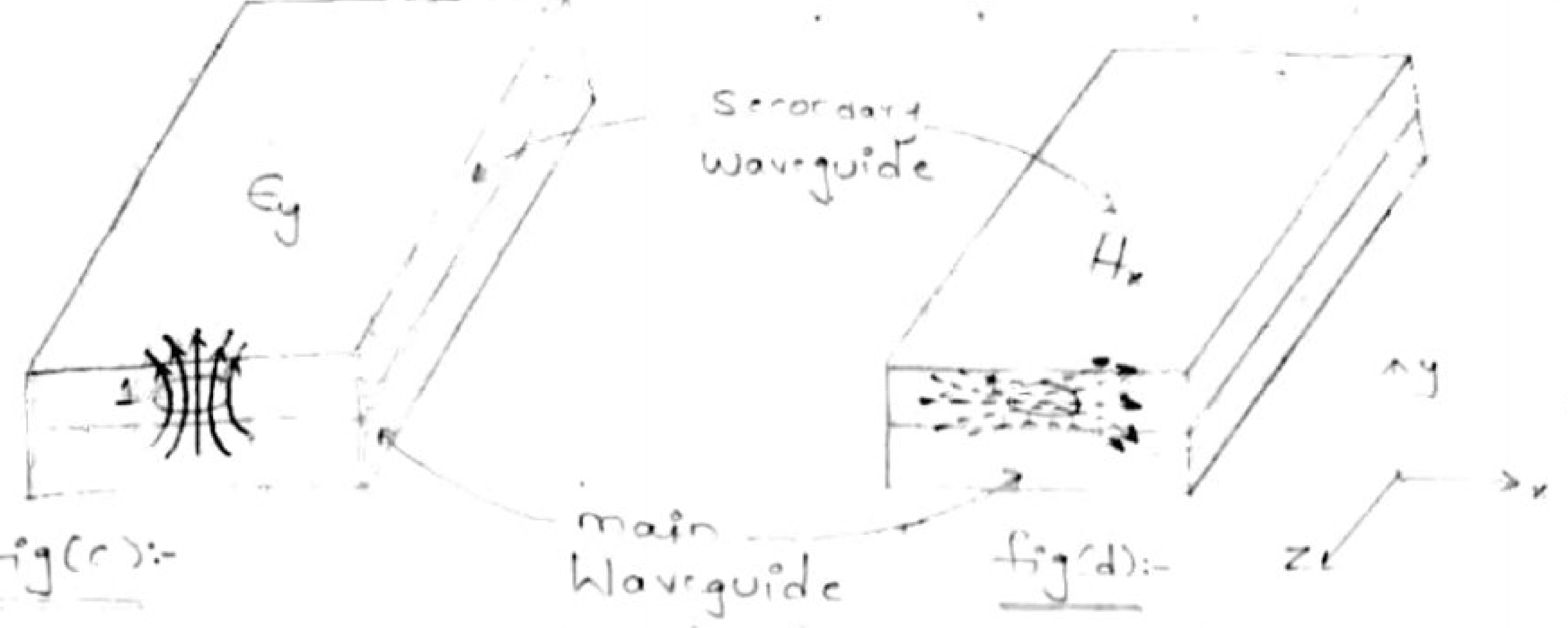


fig:- Examples of electric & magnetic coupling through a Waveguide Aperture

→ The Coupling through Circular hole in narrow wall (aperture 2) is magnetic, since only 'H_z' component exists at that aperture.

→ For the case of thin slots, the coupling is essentially magnetic. Only the magnetic field component parallel to long dimension of the slot, couples through the aperture. (Apertures 3, 4, 5)

→ Aperture ③ radiates energy, since the longitudinal conduction current (J_z) is interrupted, thus creating a displacement current which produces a coupled magnetic field. (H_x in this case)

→ Apertures ④ & ⑤ interrupt transverse conduction currents (J_x and J_y respectively), there by producing magnetic coupling via "H_z" component

→ The thin slots shown in fig(b), are non-radiating apertures, since they donot interrupt any conduction currents.

→ If the thin slot on the broader wall is offset from the center line, it becomes a radiating slot.

** Waveguide discontinuities - Waveguide Irises, Tuning Screws & Posts, Matched loads

→ Any interruption in the uniformity of a transmission line leads to impedance mismatch and is known as "impedance discontinuity" (or) "Waveguide discontinuities".

→ Due to mismatch of load, reflections will occur.

→ To minimize these reflections, "lumped elements" (or) "stubs" are used

→ In a Waveguide system, when there is a mismatch, reflections will occur. Any susceptance appearing across the guide causing mismatch, needs to be cancelled out by introducing another susceptance of same magnitude but of opposite value.

→ The waveguide Irises are used for this purpose.

Waveguide Irises:-

→ Fixed (or) adjustable projections from the walls of waveguides are used for impedance matching purposes, and these are known as "Windows" (or) "Irises".

→ An Iris is a metal Plate that contains an opening through which the waves may pass.

→ It is located in the transverse plane of either a magnetic field (or) an electric field.

→ Irises are classified according to the sign of the imaginary part of the impedance.

→ If the reactance of the impedance is positive (or) if the susceptance of the admittance is negative we have an <u>inductive Iris</u>. If the reactance is negative (or) if the susceptance is positive we have a <u>Capacitive Iris</u>.

Inductive Iris:-

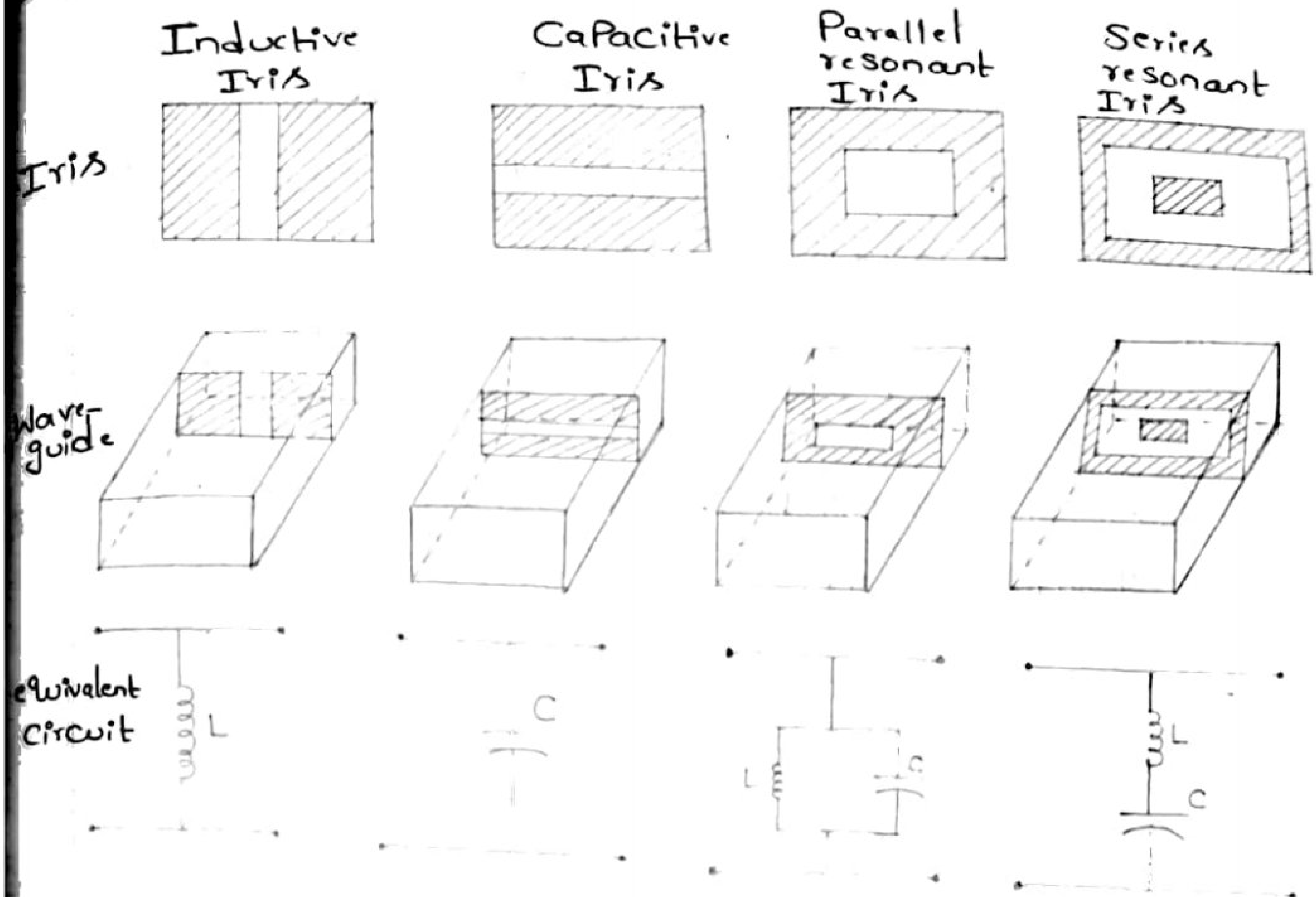
- Usually, inductive irises are used as <u>coupling networks</u> between half-wavelength cavities in rectangular waveguides.
- Generally, an inductive iris is placed, where either magnetic field is strong (or) electric field is weak.
- The plane of polarization of the electric-field becomes parallel to the plane of inductive iris.
- This causes a current flow, which sets up a magnetic field. Then the energy is stored in the magnetic field.
- Hence, inductance will increase at that point of the waveguide.

Capacitive Iris:-

- A capacitive iris is also known as capacitive window. It extends from the top and bottom walls into the waveguide
- The capacitive iris has to be placed in strong electric field.
- This capacitive iris creates the effect of capacitive susceptance which is in parallel to that point of waveguide where the electric field is strong.

Parallel resonant Iris:-

- If the inductive and capacitive irises are combined suitably (correctly shaped and positioned), the inductive and capacitive reactances introduced will be equal and the iris will become a parallel resonant circuit.
- For the dominant mode, the iris presents a "high impedance" and the "shunting effect" of this mode will be negligible.
- Other modes are completely attenuated and the resonant iris acts as a "Band-Pass filter" to suppress unwanted modes.
- A series resonant circuit that is supported by a non-metallic material and is transparent to the flow of microwave energy.



Tuning Screws and Posts:-

→ Posts and screws made from conductive material can be used for impedance-changing devices in waveguides.

→ A post (or) screw can also serve as a reactive element. The only significant difference between posts and screws is that "posts are fixed" and "screws are adjustable".

→ A post (or screw) that only penetrates partially into the waveguide acts as a shunt capacitive reactance.

→ When a post extends completely through the waveguide, making contact with the top and bottom walls, it acts as an inductive reactance. The screw acts as an LC-tuned circuit in such cases.

Screws:-

- A screw is generally inserted into the top (or) bottom walls of the waveguide, parallel to the electric-field lines.
- It can give a variable amount of susceptance depending on the depth of penetration.
- A screw with an insertion distance (screw depth) less than $\lambda/4$ produces capacitive susceptance.
- When the distance is greater than $\lambda/4$, it produces inductive susceptance as shown in the figure below:

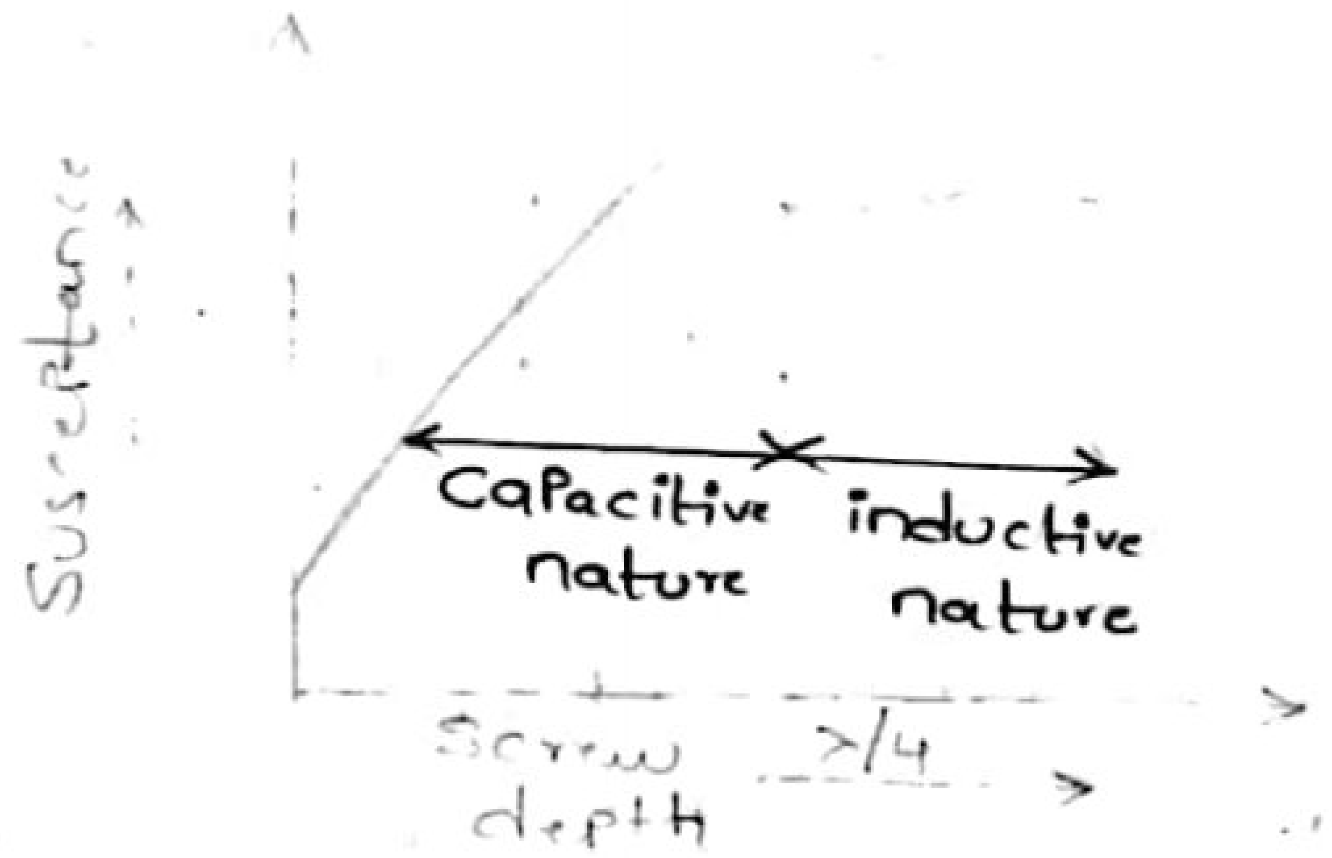


fig:- Susceptance nature at different screw depths

→ The adjustable waveguide screw is shown in the figure below. The capacitive setting is show in the first figure and the inductive setting is shown in the second figure.

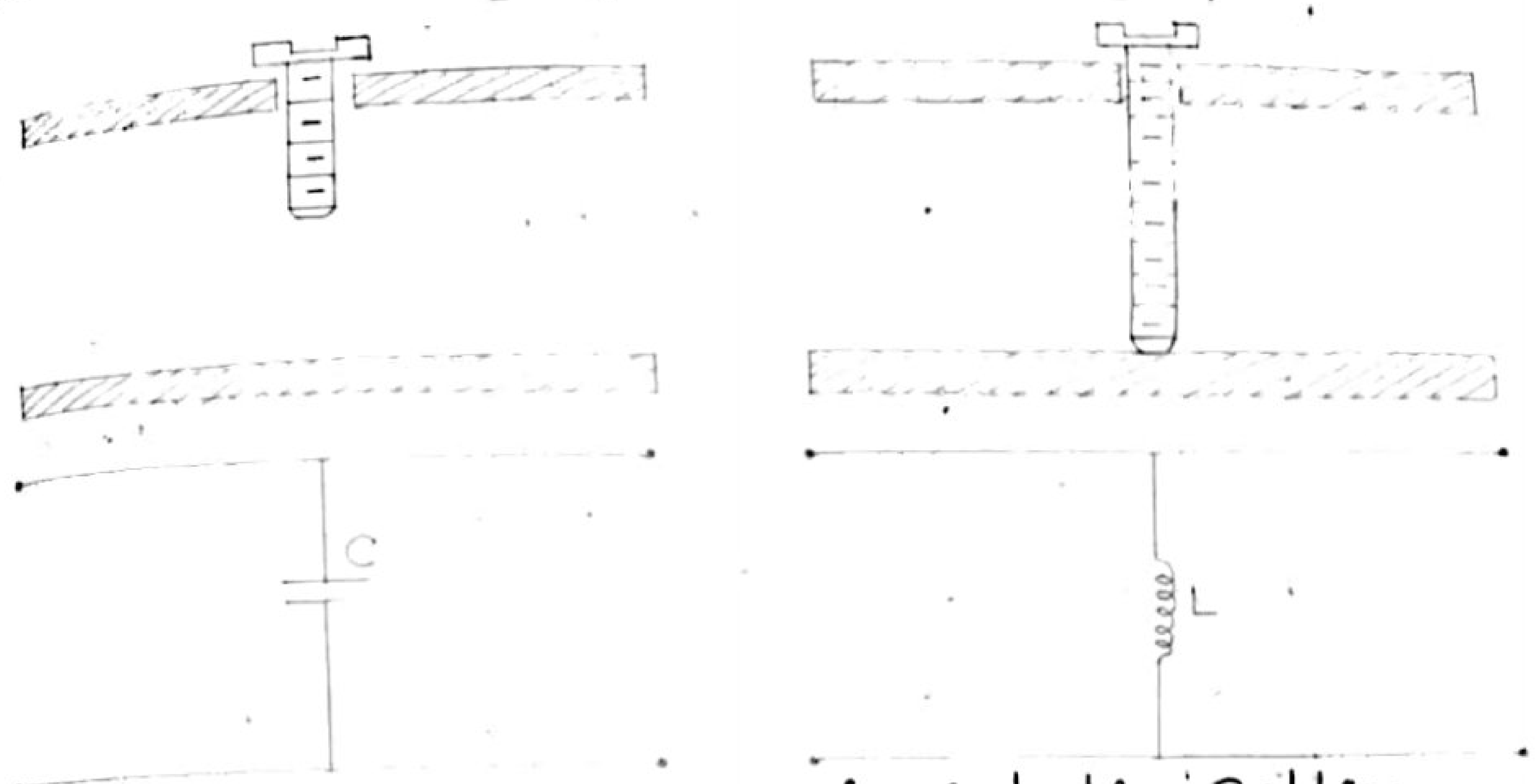


fig:- Capacitive setting

fig:- inductive setting

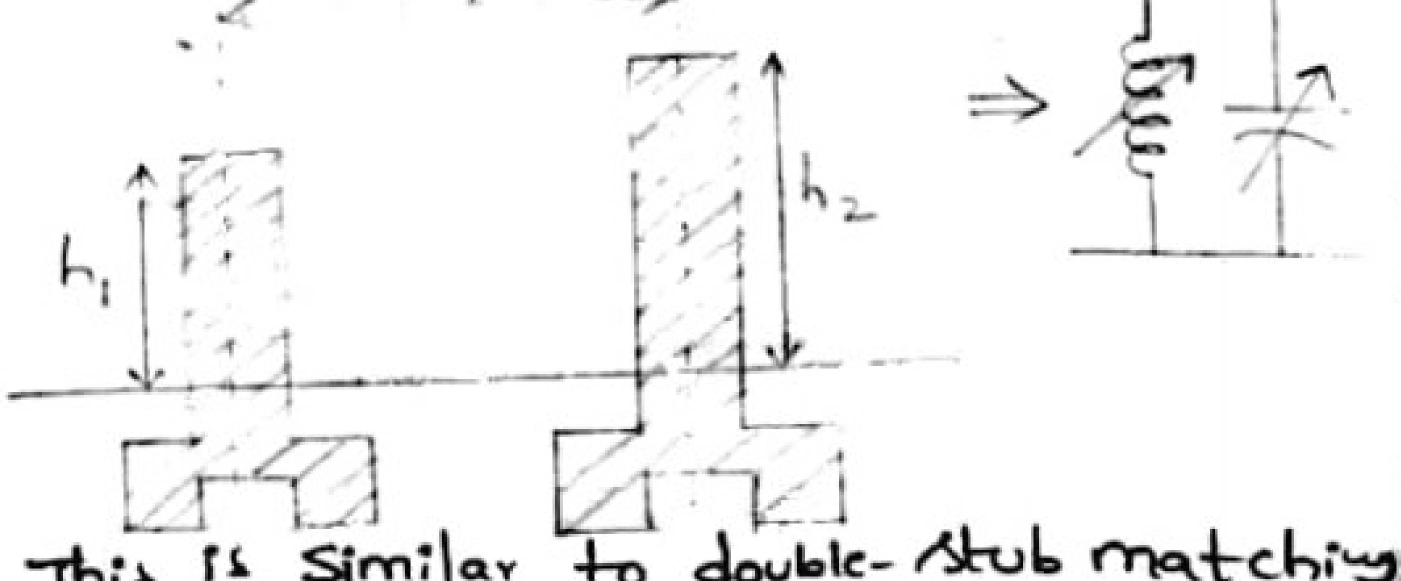
fig- Adjustable Waveguide components

→ The most direct method of impedance matching with a matched screw involves using a single screw that is adjustable in both length and position along the Waveguide. However, it requires a slot in the waveguide.

→ An alternative arrangement is to use double (or) triple screws units with a spacing of $\lambda/8$ (or) $\lambda/4$.

→ A combination of two screws which are $\frac{\lambda_g}{4}$ apart, can be used to match a waveguide to its load similar to use of two fixed stubs in a Transmission line.

→ A very effective waveguide matches can be realized when two tuning screws are placed in close proximity separated by $\frac{3\lambda_g}{8}$ as shown in the figure:



This is similar to double-stub matching in Tx lines.

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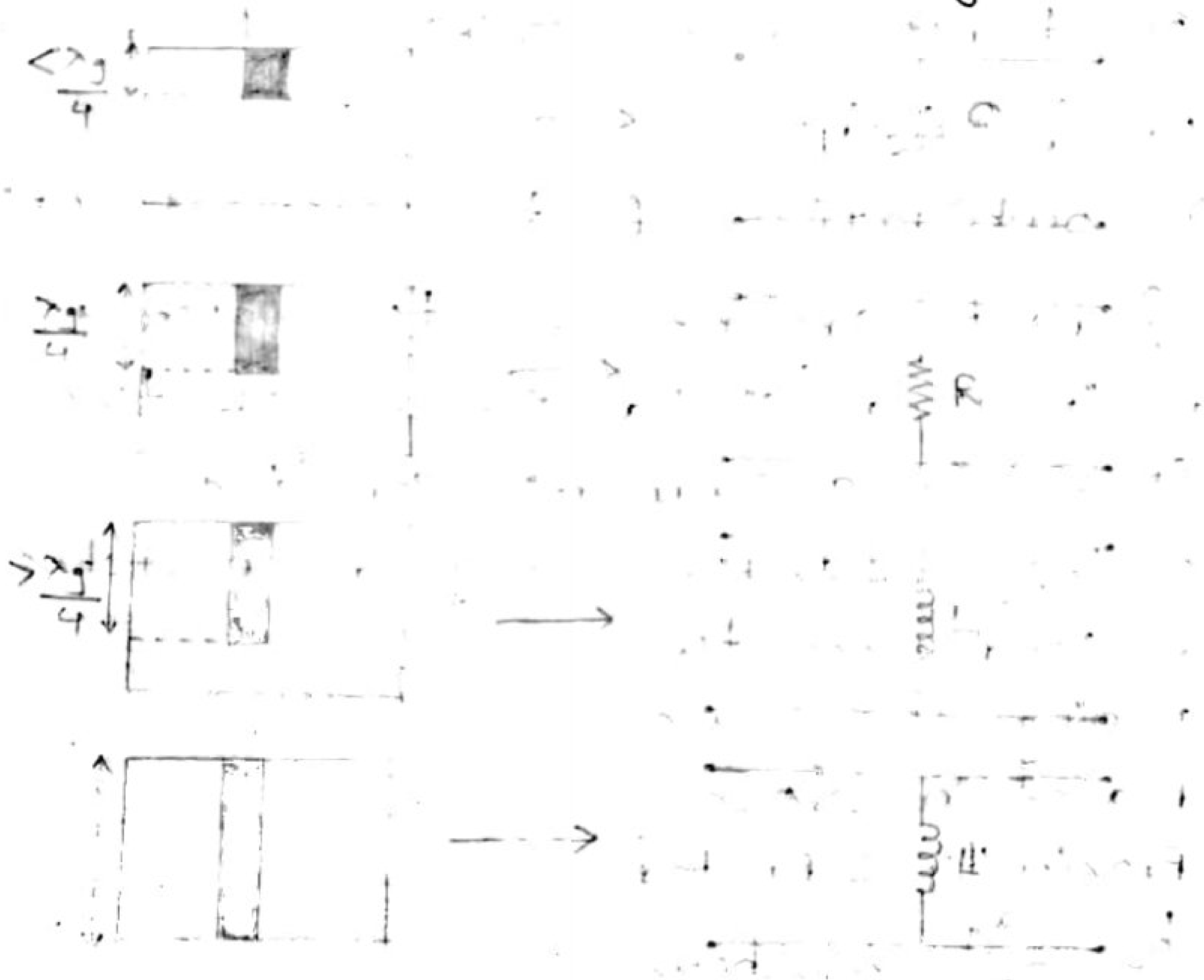
Posts:-

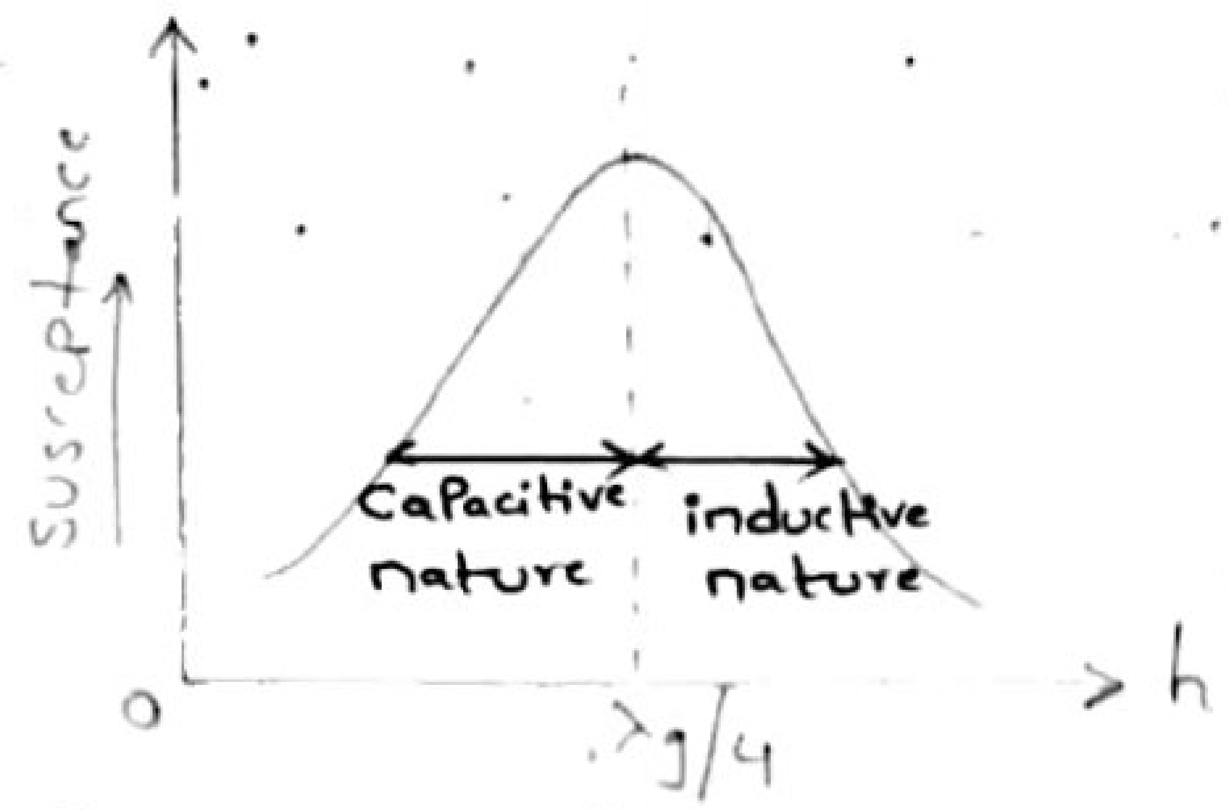
→ A cylindrical post is introduced into the broader side of the Waveguide; it produces a similar effect as an iris in providing lumped capacitive/inductive reactance at that point.

→ When a metal post extends completely across the Waveguide parallel to an electric field, it adds an inductive susceptance that is parallel to the waveguide.

→ A post extending across the waveguide at right angles to the electric field produces an effective capacitive susceptance that is in shunt with the waveguide at the position of the post.

→ The advantage of such posts over irises is the flexibility they provide, which results in ease of matching.





→ If the Post extends by $<\frac{\lambda g}{4}$, into the waveguide it behaves capacitively and this susceptance increases with depth of penetration.

→ If the depth of Post is equal to $\frac{\lambda g}{4}$, it acts as a series-resonant circuit.

→ If the depth of Post is $>\frac{\lambda g}{4}$, it behaves inductively and this inductive susceptance decreases as depth of Post increases.

→ when the Post is completely extended, the Post becomes inductive.

→ If the Post is made thicker, the effective 'Q' will be lowered, the Post acts as a Band-Pass filter similar to an iris.

Matched Loads:-

→ The most commonly used waveguide terminations are the matched loads. Whenever the load impedance and characteristic impedance of the transmission line are not matched/equal, reflections exist.

→ These reflections would cause "frequency instability" to the source.

→ Matched Loads are used for minimizing the reflections by placing a material in the waveguide parallel to the electric field to absorb the incident power completely.

→ One of the methods involved in the matched load is to place a resistive card in the waveguide parallel to the electric field. The front Portion of the card is tapered to avoid discontinuity of the signal and it almost absorbs the incident field.

*** Waveguide Attenuators:-

→ An attenuator is a passive device that is used to reduce the strength (or) amplitude of a signal.

→ At microwave frequencies, the attenuators were not only meant to do this, but also meant to maintain the characteristic impedance (Z_0) of the system.

→ If the Z_0 of the transmission line is not maintained, the attenuator would be seen as impedance discontinuity which causes reflections.

→ Usually, a microwave attenuator controls the flow of microwave power by absorbing it.

→ Attenuation in dB of a device is ten times the logarithmic ratio of power flowing into the device (P_i) to the power flowing out of the device (P_o) when both the input and output circuits are matched.

$$\boxed{\text{Attenuation in dB} = 10\log_{10}(P_i/P_o)}$$

Principle of Waveguide Attenuator:-

In a microwave transmission system, the microwave power transferring from one section to another section can be controlled by a device known as microwave

Attenuator". These Attenuators operate on the principle of interfering with electric (or) magnetic (or) both the fields. A resistive material placed in parallel to electric field lines, will induce a current in the material, which will result in I^2R Loss. Thus, attenuation occurs by heating of the resistive element.

Attenuators may be of three types:-

1. Fixed
2. Mechanically (or) electronically variable.
3. Series of fixed steps

1. Fixed Attenuators:-

→ Fixed Attenuators are used where a fixed amount of attenuation is needed. They are also called "Pads".

→ In this type of attenuator, tapering is provided by placing a short section of a waveguide with an attached tapered plug of absorbing material at the end.

→ The purpose of tapering is for the gradual transition of microwave power, from the waveguide medium to the absorbing medium

→ Because of the absorbing medium, reflections at the media interface will be minimized.

→ The pad is placed in such a way that the plane is parallel to the electric field. For this, two thin metal rods are used.

→ The figure below represents a Fixed Attenuator.

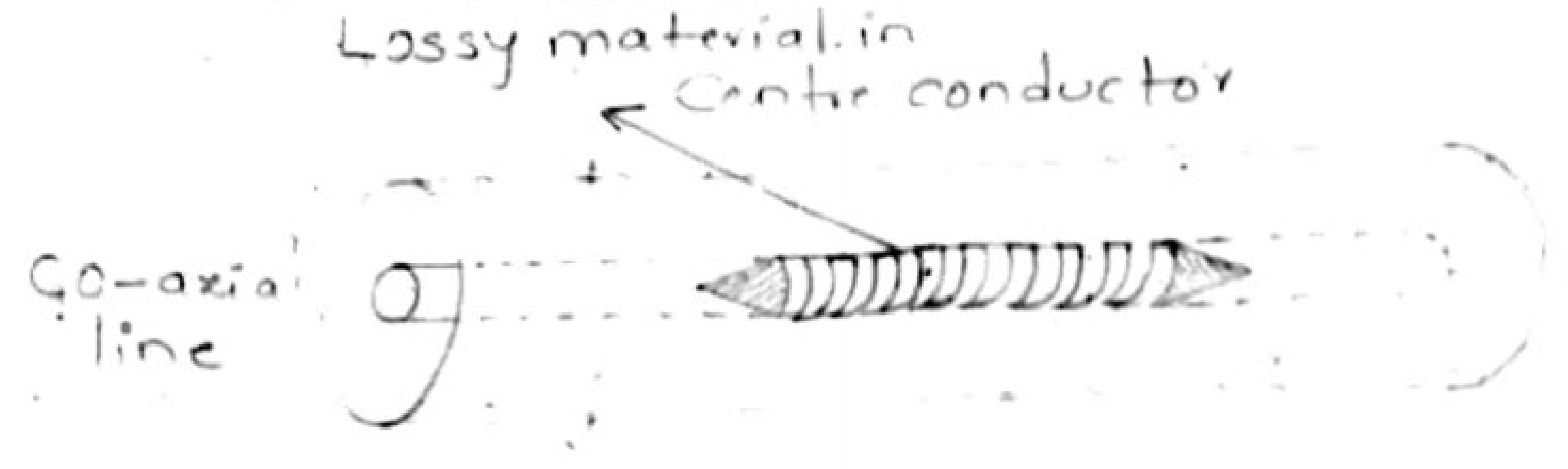


fig:- Co-axial line fixed attenuator

→ The amount of attenuation Provided by the fixed attenuator depends on

* Strength of the dielectric material
* the location and area of the Pad
* type of material used for the Pad Within waveguide
* frequency of operation

② Variable Attenuators:-

→ For Providing continuous (or) step-wise attenuation, variable attenuators are used.

→ The Provided attenuation depends on the insertion depth of the absorbing Plate into the Waveguide.

→ The maximum attenuation will be achieved When the Pad extends totally into the Waveguide.

→ This type of variable attenuation is Provided by knob & gear assembly, which can be Properly Calibrated.

→ The Power transmitted to the load can be varied manually (or) electronically from nearly the full Power of the source to as little as a millionth of a Percent of the source Power depending on the frequency of operation. The types of Variable attenuators are,

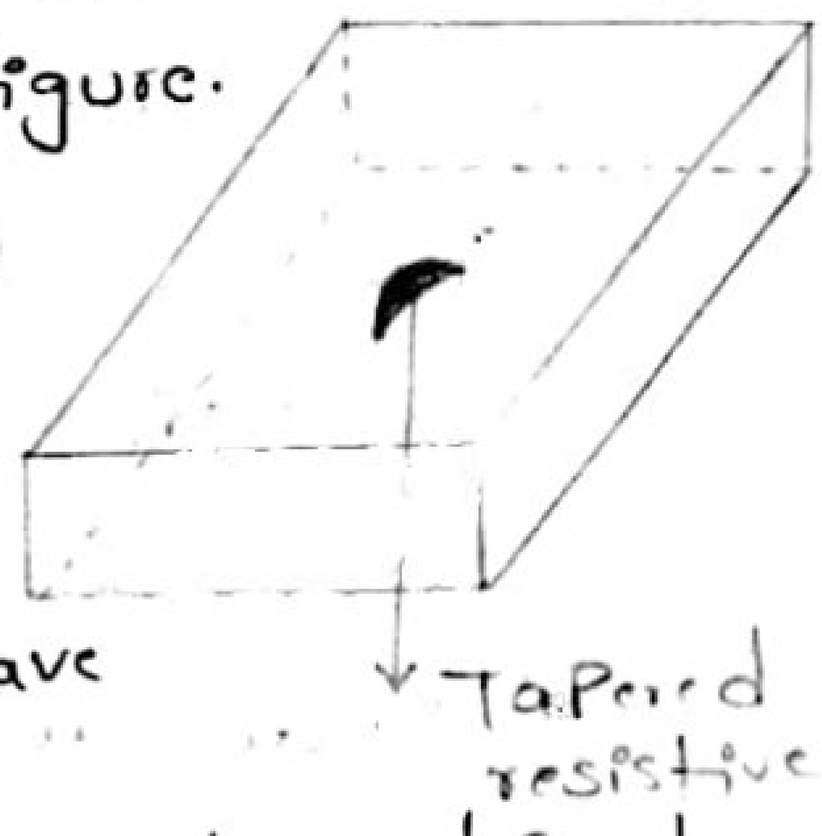
1. Flap (or) resistive-card type attenuators
2. Slide vane attenuators
3. Rotary vane attenuators

(1). Resistive card (flap type) attenuator:-

- Mechanically Variable attenuators are stepwise Variable attenuators. Examples are flap type, slide vane type attenuators.
- In contrast, electronically variable attenuators are continuously Variable attenuators.
- They are used for Various applications like requiring automatic signal leveling and control, amplitude modulation, remote signal control and so on.
- The resistive card attenuator may be either fixed (or) variable.

Fixed Resistive card:-

- In fixed version, the card is bonded to the Waveguide as shown in the figure.
- The card is tapered at both ends inorder to maintain a Low i/p and o/p standing wave Ratio (SWR) over useful microwave band.



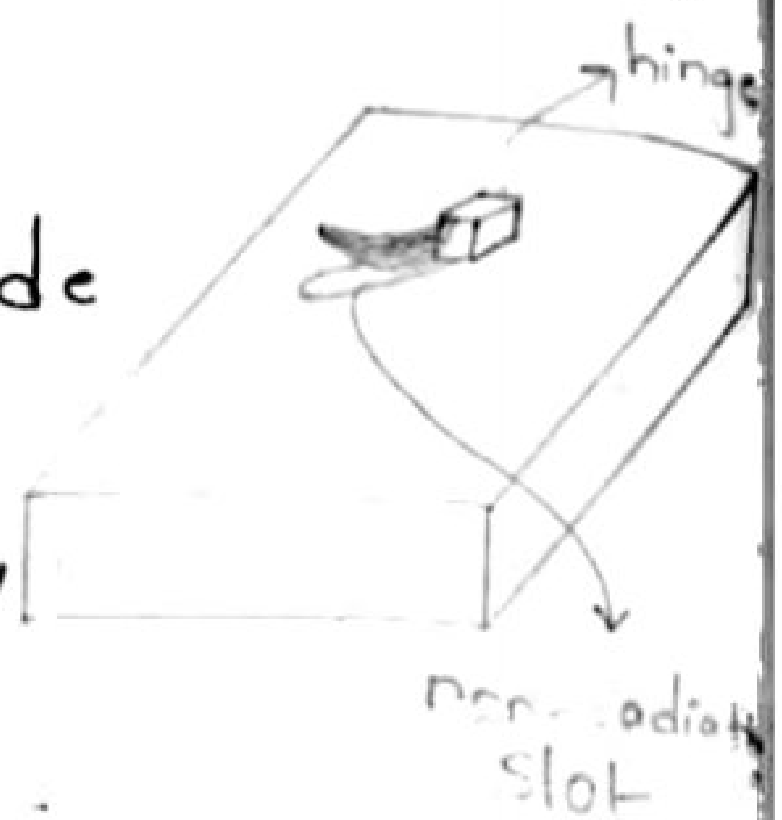
- The maximum attenuation is achieved by having the card parallel to electric field and at the centre of the waveguide where the electric field is maximum.
- The conductivity and size of the card are adjusted by trail & error, to obtain desired

attenuation value

- In high power versions, ceramic type absorbing materials are used instead of resistive card.

Variable Resistive Card:-

- A variable version of this attenuation is known as Flap attenuation as shown in the figure.



- The card enters the waveguide through the non-radiating slot in broad wall, and thereby interscepting and absorbing a portion of the TE_{10} wave.
- The hinge arrangement allows the card penetration and hence attenuation in the range of (0-30) dB can be achieved with longitudinal slot.
- None of the TE_{10} wave is radiated through the slot.

Disadvantages:-

The attenuation is frequency attenuation sensitive, which makes it inconvinient to use as a "calibrated attenuator".

(2) Slide vane (or) adjustable disk attenuator:-

- In this attenuator, the vane is positioned at the center of the waveguide and can be moved laterally from the center, where it provides maximum attenuation to the edges.
- However, the attenuation is reduced at the

edges, as the electric field lines are always concentrated at the center of the Waveguide.

- The vane is tapered at both ends for matching the attenuator with the waveguide.
- An adequate match is obtained, if the taper length is made equal to $\lambda/2$.
- The biggest disadvantage with these attenuators is that their attenuation is frequency sensitive and also the phase of the output signal is a function of attenuation.
- The slide vane (or) adjustable disk attenuator is shown in the figure below.

Slide vane or Adjustable disk

③ Rotary Vane Attenuators:-

- The most satisfactory precision attenuator is the rotary vane attenuator.
- The structure of this attenuator is shown in the figure below.
- It consists of two rectangular to circular waveguide tapered transitions along with an intermediate section of a circular waveguide that is free to rotate. All the three sections contain thin resistive cards.
- The input signal passes the first card with a negligible attenuation, because the electric field of the TE_{10} wave mode is

is Perpendicular to the card.

- Then, the wave enters through a transition to the circular waveguide.
- The attenuation is adjusted by rotating the circular waveguide section and the resistive card within it.
- The field of the TE_{11} wave mode can be divided into two components: one perpendicular to the card and the other parallel to it.
- The latter component is absorbed by the card; the former component enters the output of the waveguide, in which again its component parallel to the resistive card is absorbed.

fig:- Rotary vane attenuator

- The Plates are usually thin with $\varepsilon_r > 1$, $\mu_r = 1$ and conductivity (σ) of a finite non-zero value.
- The Plates attenuates the wave that is travelling, and the amount of attenuation is dependent on the properties of the material from which the plate is made,

the dimensions of the slab and the angle between the electric field at the input and the plane of the resistive card in the circular section. The attenuation in decibles is given by,

$$\boxed{\text{Attenuation in dB} = -40 \log(\cos\theta)\ \text{dB}}$$

Where θ is the angle between the electric field at the input and the plane of the resistive card in the circular section. Hence, the attenuation is controlled by the rotation of the center section. Minimum attenuation at $\theta = 0°$ and maximum attenuation at $\theta = 90°$. The attenuation provided by this device depends only on the rotation angle 'θ' but not on the frequency. This device is very accurate, and is, hence, being used as a calibration standard. Its accuracy is limited only by imperfect matching and by misalignment of the resistive cards.

** Waveguide Phaseshifters :-

→ A phase shifter is a two-Port component that provides a fixed (or) Variable change in the phase of the Wave.

→ An ideal phase shifter is lossless and matched. It only shifts the phase of the output wave.

→ for example, phase shifters are used in phased antenna arrays.

→ Electrically controlled phase shifters are much faster than mechanical phase shifters. They are often based on "PIN diodes" (or) "FETs".

→ The phase delay due to a Waveguide section of length 'l' is given by,

$$\boxed{\beta l = \frac{2\pi}{\lambda_g} \cdot l}$$ Where λ_g = Guided Wavelength

→ The phase delay can be adjusted by varying guided Wavelength (λ_g). This can be accomplished by varying either "ε_r" or "guide width" (a) as shown below.

$$\lambda_g = \frac{\lambda}{\sqrt{1-(f_c/f)^2}}$$

$$\Rightarrow \lambda_g = \frac{c}{f\sqrt{1-(f_c/f)^2}} \qquad (\because c = f\lambda)$$

$$\Rightarrow \lambda_g = \frac{c}{f\sqrt{\varepsilon_r}\sqrt{1-(f_c/f)^2}}$$

$\because c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$ for free space

$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0 \varepsilon_r}}$ for any medium other than freespace

$$\therefore \boxed{\lambda_g = \frac{c}{f\sqrt{\varepsilon_r}\sqrt{1-(f_c/f)^2}}}$$

if $\sqrt{\varepsilon_r}$ and a ↑, λ_g ↓ and therefore β ↑.

- Phase shifters
  - Fixed Phase shiters
  - Variable Phase shifters
    - Dielectric Phase-shifters
    - Rotary Phase-shifters

Fixed Phase shifters:-

- Fixed Phase shifters are usually extra transmission line sections of a certain length that are meant to shift the Phase with regard to the reference line.
- Therefore depending on the bias current, the wave travelling along the transmission line will have an additional travelling Path.
- Since these Phase shifters are binary switches only discrete Phase shifts are Possible.

Variable Phase shifters:-

- The variable phase shifters use mechanical (or) electronic means to achieve a dynamic range of Phase difference.
- The mechanically tuned Phase shifter usually consists of variable short-circuits that are used with hybrids, or in the case of Waveguide components, a dielectric slab with a variable Position in the guide.
- Step motors move the slab across the guide (from its center toward the outer walls), thereby accomplishing a maximum (or) minimum Phase shift.
- Another method for obtaining the desired mechanically tuned phase shifter involves combining variable short circuits and hybrid circuits.
- The movement of the short-circuit along a transmission line results in the phase-shift, thus making it appear shorter (or) longer.

Dielectric Phase Shifters:-

- The variable type of dielectric Phase shifters employs a low-loss dielectric insertion in the air-filled guide at a Point of the maximum electric field to increase its effective dielectric constant.
- This causes the guide wavelength (λ_g) to decrease.
- Thus, the insertion of the dielectric increases the Phase shift in the wave Passing through the fixed length of the waveguide section.
- Tapering of the dielectric slab has resorted inorder to reduce the reflections.

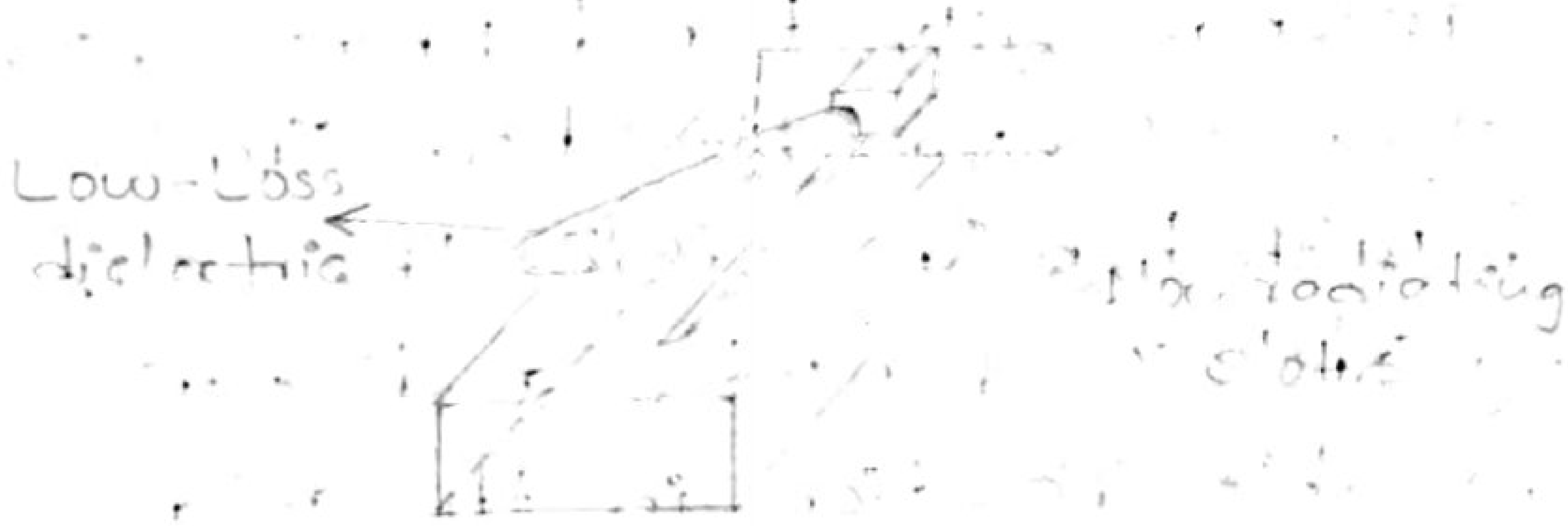
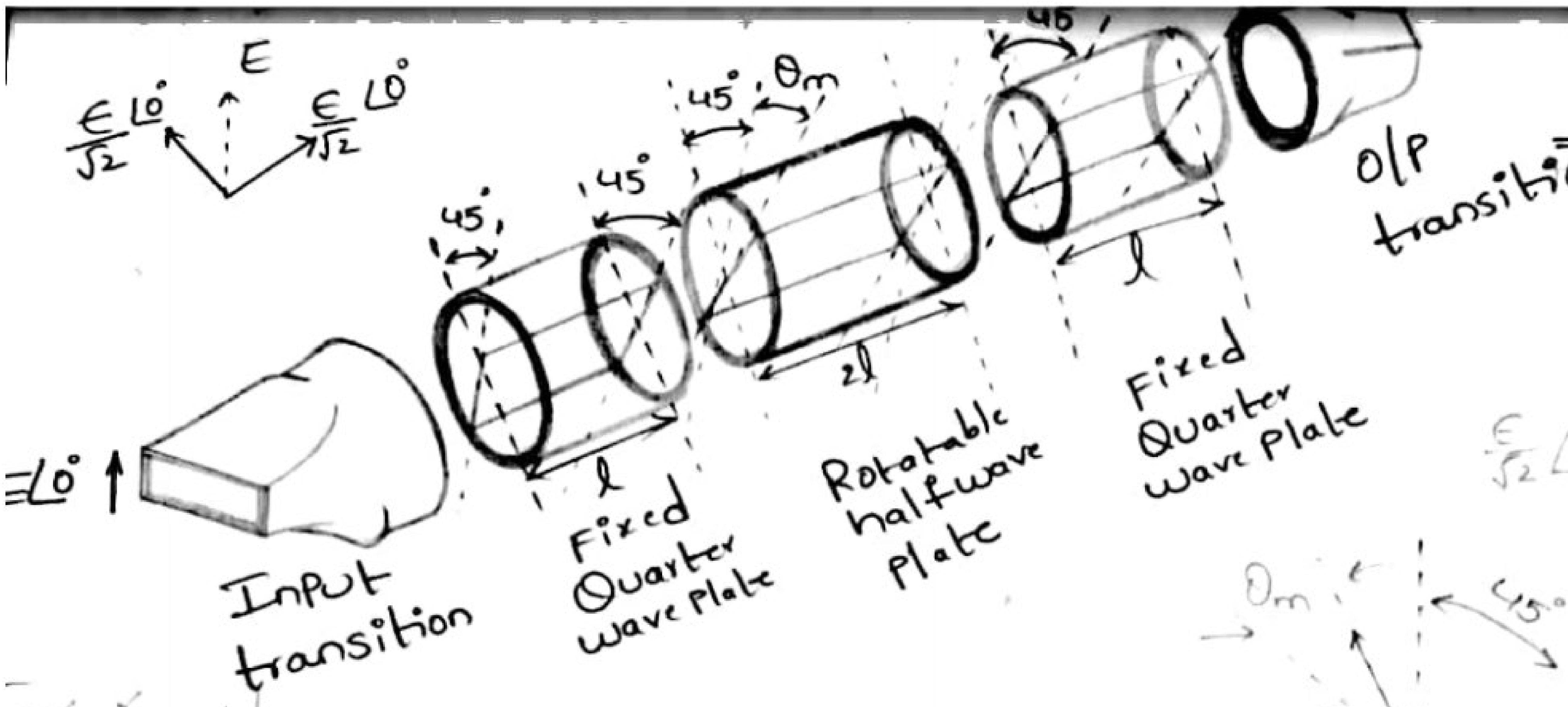
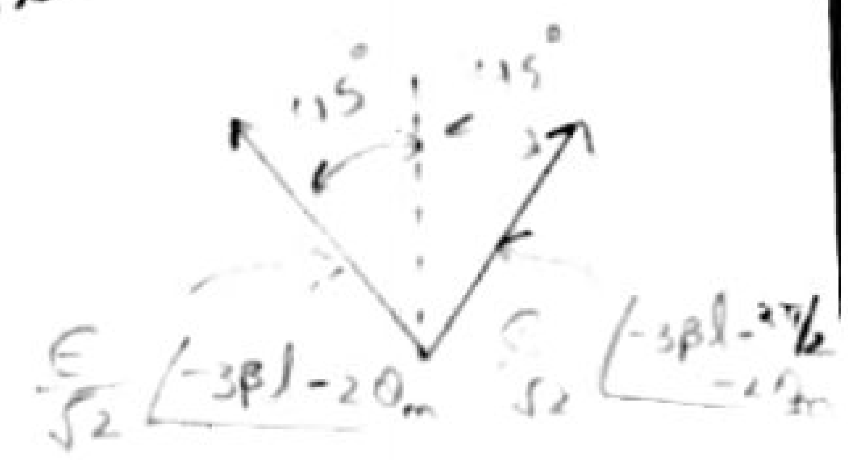
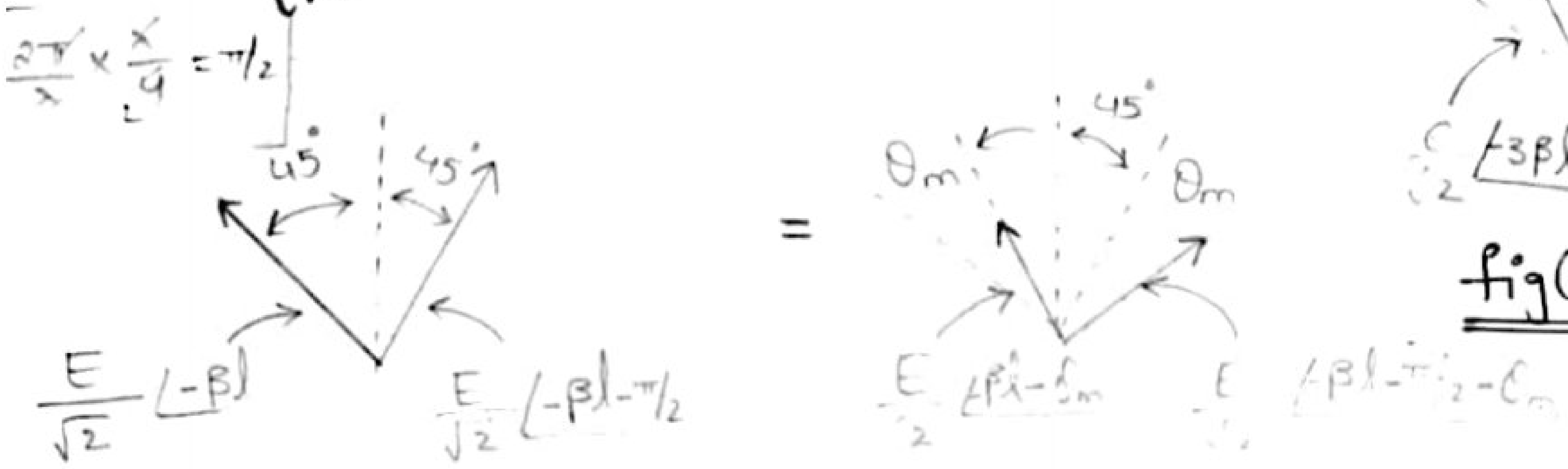


fig:- Dielectric type Variable Phase shifters

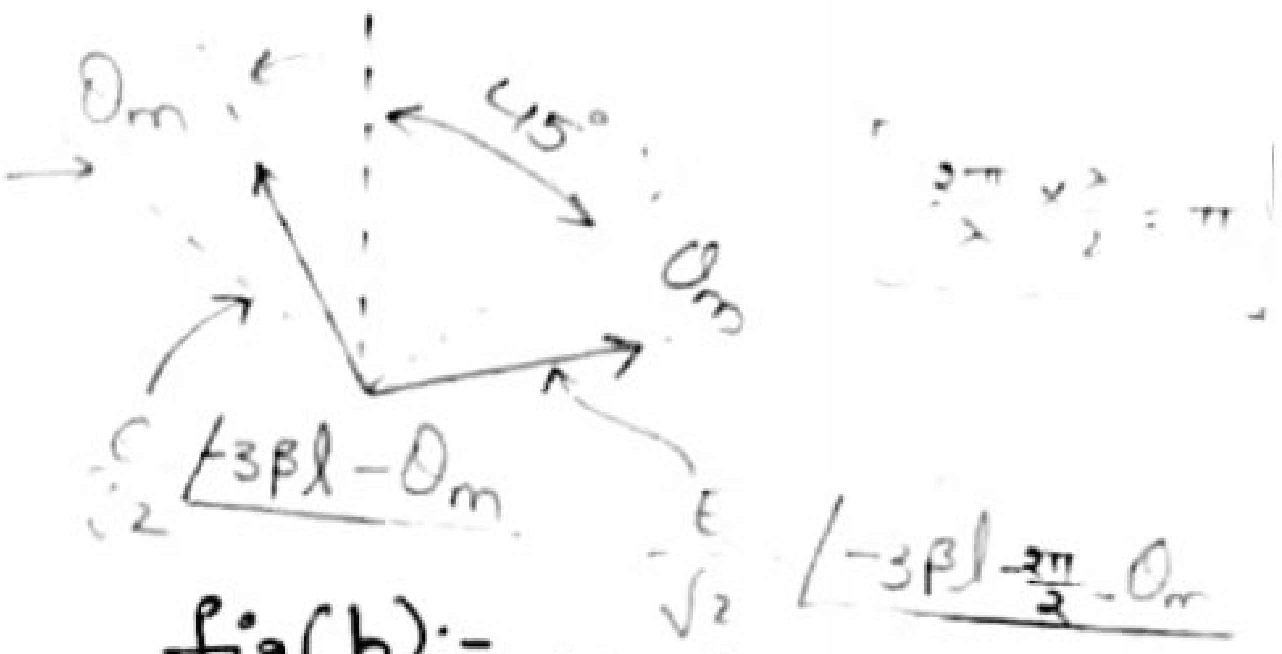
Rotary Phase shifter







fig(a):- vector phase components at the i/p to the rotable system



fig(b):- Vector Phase components at i/p rotatable Section

Operation:-

→ It consists of three circular waveguide sections, two fixed and one rotatable. Two fixed sections are Quarter wave plate, while rotatable one is half-wave plate.

→ The vector Phasor $E\angle 0^\circ$, represents the vertically polarized i/p wave. It may be decomposed into two components, Parallel & Perpendicular to the dielectric slab of the i/p Quarterwave plate. The value of each component is $\frac{E}{\sqrt{2}}\angle 0^\circ$.

→ The effect of i/p quarter wave plate is to delay the ⊥er component by "βl" and //el component by "$\beta l+\pi/2$", which results in a clockwise circularly polarized wave at the i/p of rotatable section.

→ with the length of half plate is equal to "$2l$",
the ⊥er and //el components are further delayed by "$2\beta l$" and "$2\beta l + \pi$" respectively as shown in the fig(b).

→ The o/p quarter wave plate delays these components by an additional "βl" and "$\beta l + \pi/2$" respectively. As a result the o/p components are "$\frac{E}{\sqrt{2}} \angle -4\beta l - 2\theta_m$" and "$\frac{E}{\sqrt{2}} \angle -4\beta l - 2\pi - 2\theta_m$" with these two components are in phase, the vector addition results in a vertically polarized o/p wave of value $E \angle -4\beta l - 2\theta_m$.

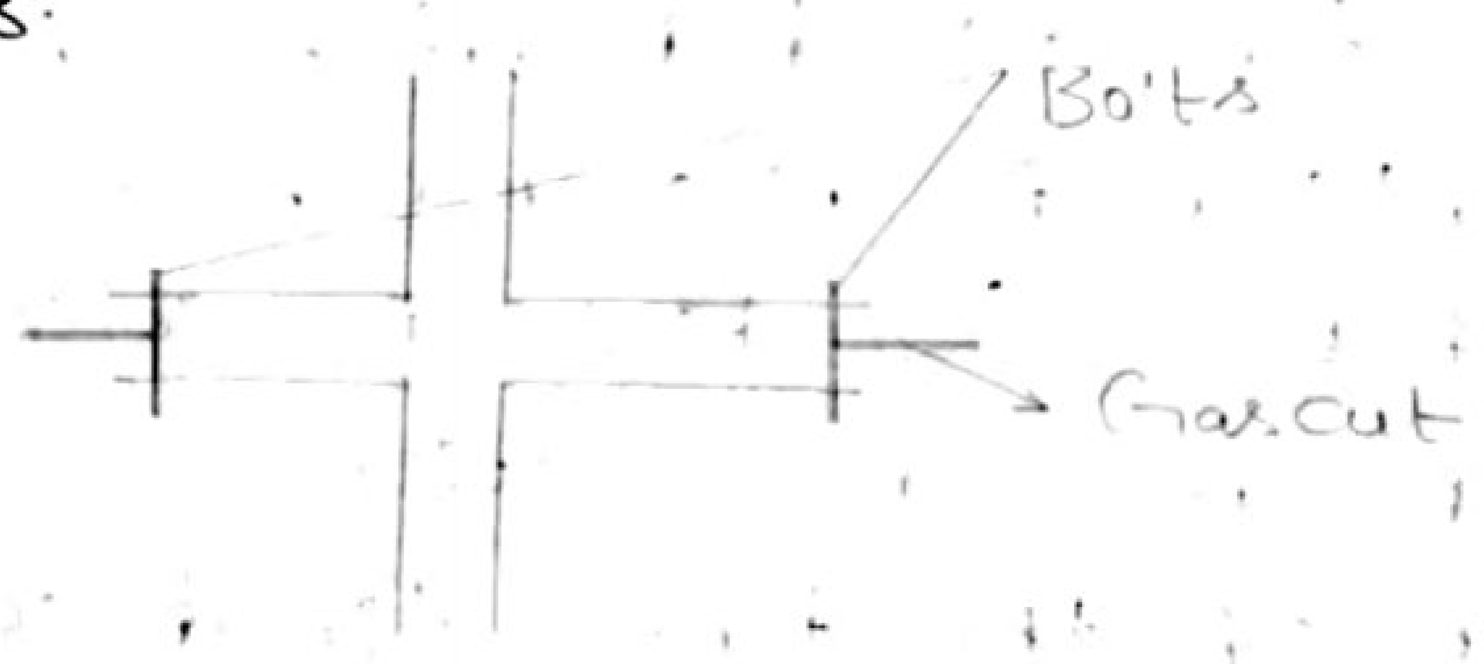
→ Because of accuracy, the rotary phase shifters are used as a calibration standard in microwave laboratories.

** Waveguide Joints:-

① Permanent Joints :- These joints are made by company itself, it doesnot need any maintainance.

② Semi-Permanent Joints :-

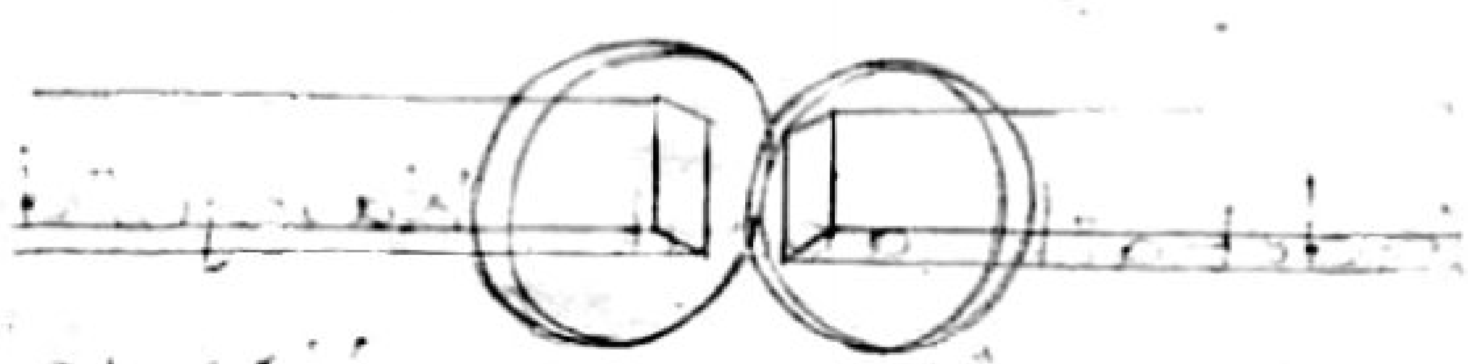
(i) Bolted Joints :- Gascut reduces the moisture (or) air. Here, two rectangular waveguides are connected by using bolts.



(ii) $\lambda g/4$ Joint:- Here, we use $\lambda g/4$, because it acts as inverter.

(iii) chock Joints:-

(iv) Rotatable Joints:-



Waveguide Bends:-

→ Waveguide is normally rigid, except for flexible Waveguide and therefore it is often necessary to direct the waveguide in a Particular direction.

→ Using Waveguide bends, and twists it is Possible to arrange the waveguide into the Positions required.

→ When using waveguide bends and waveguide twists, it is necessary to ensure the bending and twisting is accomplished in the correct manner otherwise the electric and magnetic fields will be unduly distorted and the signal will not Propagate in the manner required causing loss & reflections.

→ Accordingly, waveguide bend and waveguide twist sections are manufactured specifically to allow the waveguide direction to be alerted without unduly destroying the field Patterns and introducing loss.

Types of Waveguide bend :-

→ There are several ways in which waveguide bends can be accomplished. They may be used according to the applications and the requirements.

- Waveguide E bend
- Waveguide H bend
- Waveguide sharp E bend
- Waveguide sharp H bend

→ Each type of bend is achieved in a way that enables the signal to Propagate correctly and with the minimum of disruption to the fields and hence to the overall signal.

→ Ideally the waveguide should be bent very gradually, but this is normally not viable and therefore specific waveguide bends are used.

→ Most proprietary waveguide bends are common angles – 90° waveguide bends are the most common by far.

Waveguide E bend:-

→ This form of waveguide bend is called an E bend because it distorts (or) changes the electric field to enable the waveguide to be bent in the required direction.

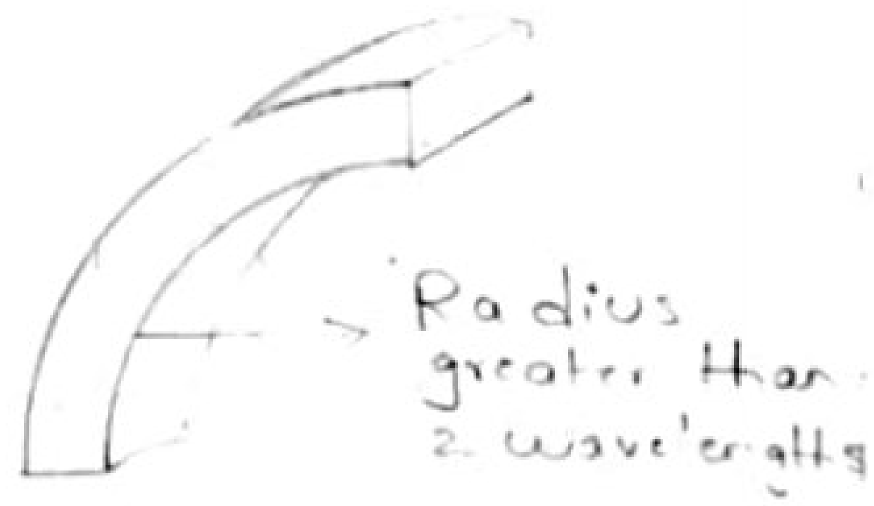


fig:- Waveguide bend

→ To prevent reflections, this waveguide bend must have a radius greater than two wavelengths.

Waveguide H bend:-

→ This form of waveguide bend is very similar to the E bend, except that it distorts the H (or) magnetic field. It creates the bend around the thinner side of the waveguide.

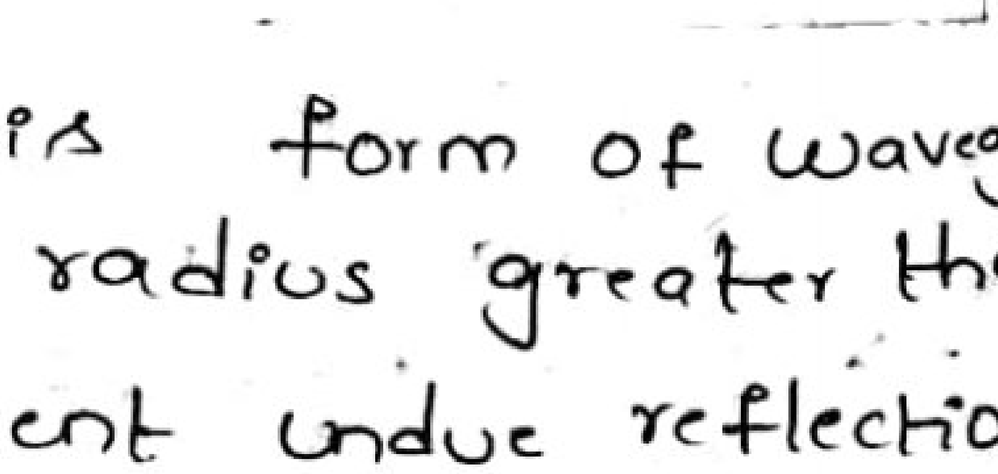


fig:- Waveguide H bend

→ As with the E bend, this form of waveguide bend must also have a radius greater than 2 wavelengths to prevent undue reflections and disturbance of the field.

→ This is inbrief about E and H bends.

Waveguide sharp E bend:-

→ In some circumstances, a much shorter (or) sharper bend may be required.

→ This can be accomplished in a slightly different manner.

→ The techniques is to use a 45° bend in the waveguide. Effectively, the signal is reflected, and using a 45° surface, the reflections occur in such a way that the fields are left undisturbed, although the phase is inverted and in some applications this may need accounting for (or) correcting.

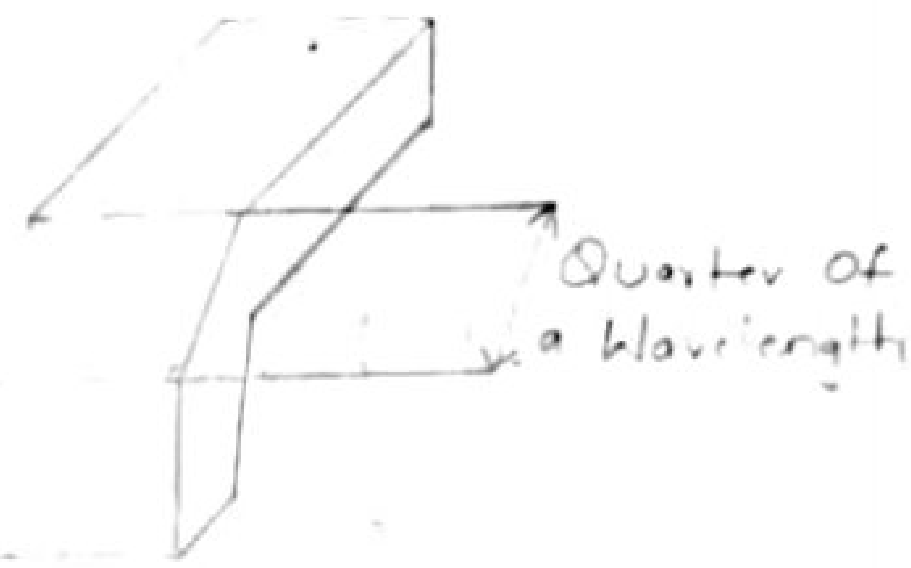


fig:- Waveguide sharp E bend

Waveguide sharp H bend:-

→ This form of Waveguide bend is the same as the sharp E field bend, except that the waveguide bend effects the H-field rather than the E-field.

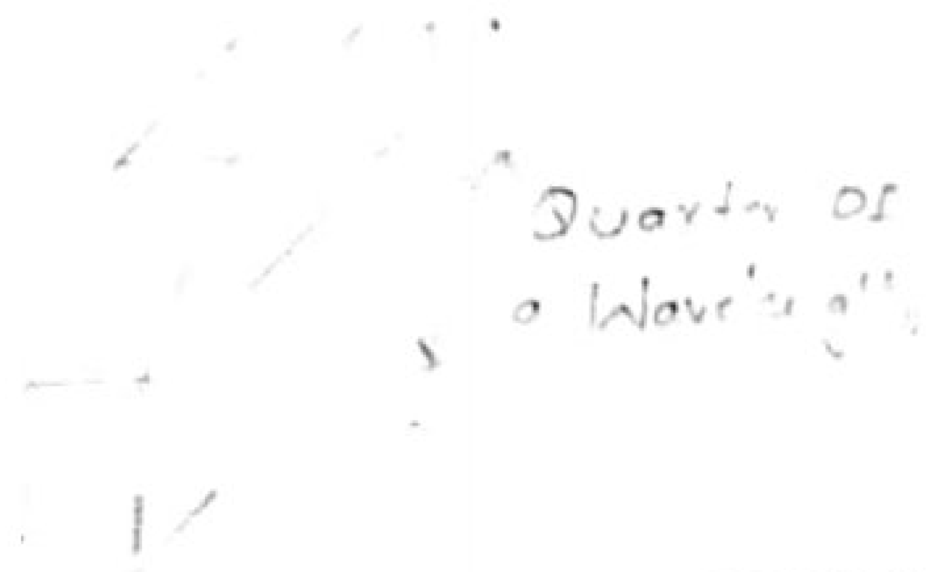


fig:- Waveguide Sharp H bend

Waveguide twists:-

→ There are also instances where the Waveguide may require twisting. This too can be accomplished.

→ A gradual twist in the waveguide is used to turn the Polarisation of the waveguide and hence the waveform.

→ Inorder to Prevent undue distortion on the waveform, a 90° twist should be undertaken over a distance greater than two wave-lengths of the frequency in use.

→ If a complete inversion is required, for example, for phasing requirements, the overall inversion (or) 180° twist should be undertaken over a four wavelength distance.

→ Waveguide bends and waveguide twists are very useful items to have when building a waveguide system.

→ Using waveguide E bends and waveguide H bends and their sharp bend counterparts allows the waveguide to be turned through the required angle to meet the mechanical constraints of the overall waveguide system.

→ Waveguide twists are also useful in many applications to ensure the polarisation is correct.

———— o ————

Transfered electron devices (TED's):-

TED's are two terminal semiconductor devices which are used to generate or amplify microwave signals. These are bulk devices having no junctions or gates as compared to eeo transistors which operate with either junction or gates. TED's are fabricate from Compound semi conductor such as GaAs, InP or with CdTe as against the fundamental semiconductor material Ge or Si.

Transistors operate with "warm" electrons whose energy is not much greater than thermal energy (0.026eV at room temperature) of electrons in the semi conductor. But TED's operate with "hot' electrons whose energy is very much greater than the thermal energy.

TED's have the -ve resistance property. (ie) the real part of their impedance is -ve over a range of frequency. In a +ve resistance, the current through the resistance and voltage across it are in phase. But in a -ve resistance the current and voltage are out of phase by 180°. The voltage drop across a -ve resistence is -ve and power of $(-I^2R)$ is generated by the power supply associated with -ve resistance. (ie) +ve resistence absorb power and -ve resistance generate power. So TED's used as oscillators.

Ridley - watkins - Hilsom (RWH) Theory :-

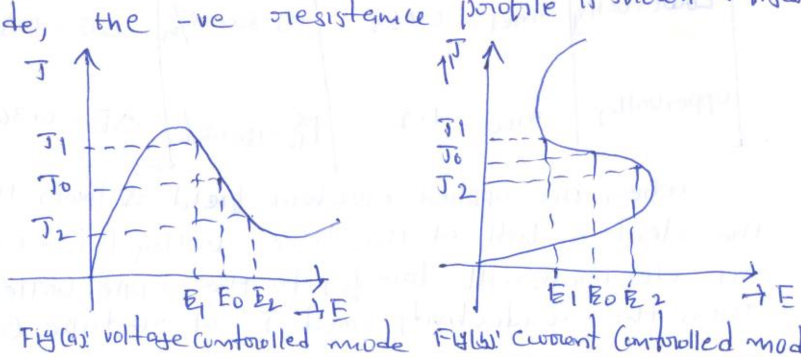
The fundamental concept of the RWH theory is the differential -ve resistance developed in a bulk solid state III-V compound by transfering electrons from high mobility energy band to low mobility energy band, which is explained as follows.

Differential -ve resistance :-

They are two modes of -ve resistance devices.

1. voltage controlled mode
2. current controlled mode.

In voltage controlled mode if an electric field E_0 is applied to the sample, the current density 'J_0' is generated. As the applied field is increased to E_2, the current density is decreased to 'J_2'. when the field is decreased to E_1, the current density is increased to J_1. These phenomena of the voltage controlled -ve resistance shown in fig(a). Similarly for the current controlled mode, the -ve resistance profile is shown in fig(b).



Fig(a): voltage controlled mode Fig(b): current controlled mode

Two valley model theory of RWH Theory:-

The basic mechanism to achieve -ve resistance in the n-type GaAs device is the transfer of electrons from lower conduction band (L-valley) to upper conduction band (U-valley), shown in fig bellow.

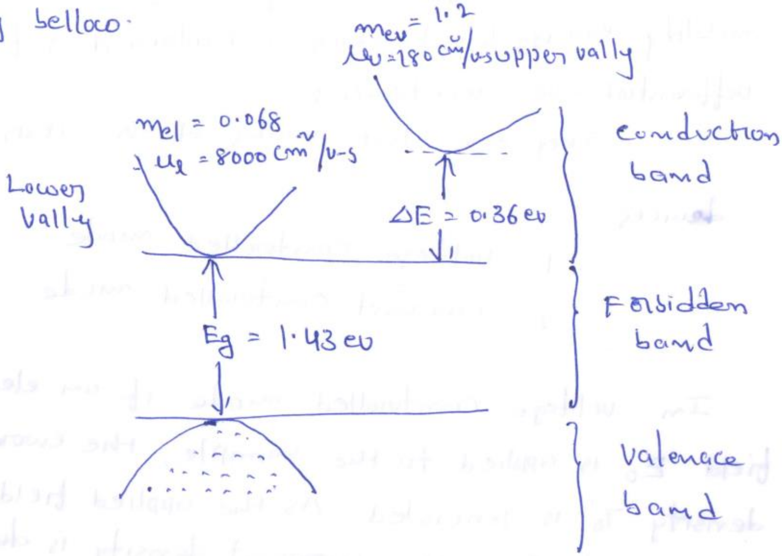


Table bellow shows data for two valleys in the n-type GaAs device.

valley	effective mass m_e	Mobility μ	Energy Separation
Lower vally	$m_{el} = 0.068$	$\mu_l = 8000\ cm^2/vs$	$\Delta E = 0.36\ ev$
upper valley	$m_{eu} = 1.2$	$\mu_u = 180\ cm^2/v\text{-}s$	$\Delta E = 0.36\ ev$.

when the applied electric field is lower than the electric field of the lower valley ($E < E_l$), no electrons will transfer to the upper valley. Then the conductivity and 'J' of n-type GaAs is

$$\sigma = e n_l \mu_l$$

$$J = \sigma E = e n_l \mu_l E \quad \text{--- (1)}$$

when applied field is higher than that of the lower valley and lower than that of upper valley ($E_l < E < E_u$), electrons will begin to transfer to the upper valley. Then the conductivity and 'J' is given by

$$\sigma = e[\mu_l n_l + \mu_u n_U]$$

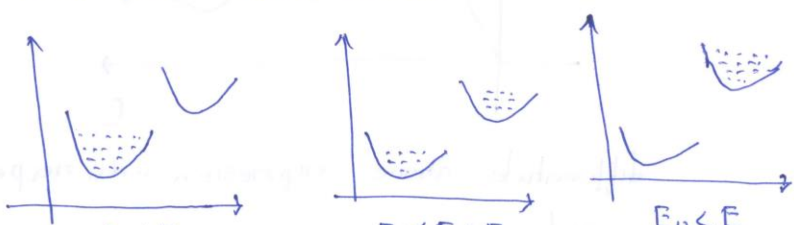
$$J = \sigma E = e[\mu_l n_l + \mu_U n_U] E \quad \text{--- (2)}$$

But when applied field is higher than that of the upper valley $[E > E_U]$, all the electrons will transfer to the upper valley, then conductivity and 'J' is given by

$$\sigma = e \mu_u n_U$$

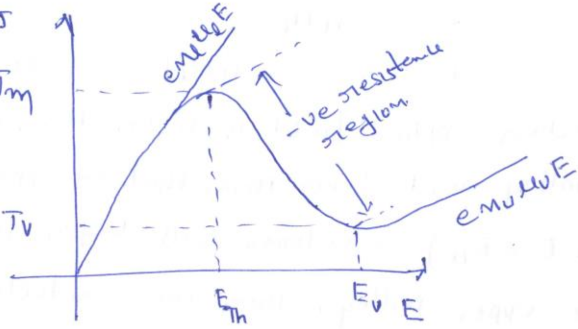
$$J = \sigma E = e \mu_U n_U E \quad \text{--- (3)}$$

The transfer of electrons for different electric field shown bellow



$E < E_l$ $\quad$ $E_l < E < E_U$ $\quad$ $E_U < E$

Fig bellow shows the current versus field characteristics of a two valley semiconductor.

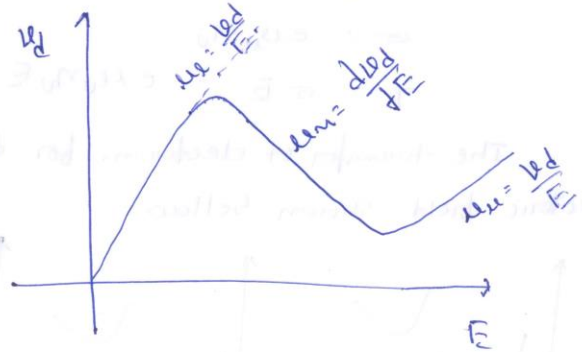


The current density in terms of drift velocity is given by

$$J = en\,v_d$$

where $v_d = \mu E$

Fig bellow shows the drift velocity versus electric field of a two valley semiconductor.



differentiate above expression with respect to E, we get

$$\frac{dJ}{dE} = qn\frac{dv_d}{dE}$$

The condition for -ve resistance region is $\frac{dJ}{dE} < 0$ (ie) $\frac{dv_d}{dE} = \mu_m < 0$, where μ_m is -ve mobility

Conditions to apply RWH theory to a semiconductor:-

The band structure of semiconductor must satisfy three conditions in order to -ve resistance.

1. The separation energy b/w the bottom of the lower valley and bottom of the upper valley must several times greater than the thermal energy (about 0.026 eV) at room temperature.

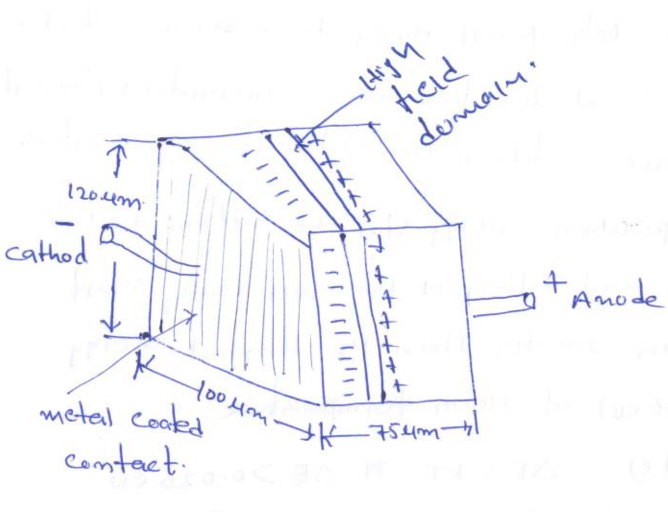
$$(i.e) \quad \Delta E > KT \text{ or } \Delta E > 0.026\ eV$$

2. The separation energy b/w the valleys must be smaller than the gap energy b/w the conduction band and valance band. (ie) $\Delta E < E_g$. otherwise semiconductor will breakdown and become highly conductive before the electrons to begin to transfer to the upper valley hole-electron pair formation is created.

3. Electrons in the lower valley must have high mobility, small effective mass; whereas in the upper valley must have low mobility, large effective mass.

Gunn diode and Gunn effect:-

A Gunn diode is one of the transfered electron devices, which is a form of diode used in high freq. applications. Its internal construction is different from other diodes and it consists only of N-doped GaAs semiconductor material. Figure bellow shows the schematic diagram of n-type GaAs diode.

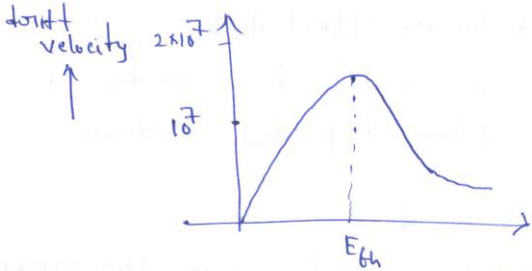


Gunn effect:-

Gunn effect means, the periodic fluctuations of current passing thorough the n-type GaAs sample when the applied field exceded a certain critical value (2-4 kv/cm). The Semiconductor devices which have this effect is known as Gunn effect devices.

The Scientist J.B Gunn also observed the following things.

From Gunn's observation the carrier drift velocity is linearly increased from zero to a max when the electric field is varied from zero to a threshold value. when the electric field is beyond the threshold value for the n-type GaAs, the drift velocity is decreased and the diode exhibit -ve resistance. This is shown bellow.



Gunn also found that, the period of oscillations was equal to the transit time of the electrons through the specimen

(ie) $\tau_0 = \tau = \frac{L}{v_d}$

freq of oscillations $f = \frac{1}{\tau_0} = \frac{v_d}{L}$

$v_d = fL$

Gunn also observed that the threshold electric field E_{th} varied with Length and type of material. For example for n-type GaAs of Length $L = 210\mu m$ and periodic fluctuation occured in the specimen voltage above 59V, then threshold field is

$$E_{th} = \frac{V_{th}}{L} = \frac{59}{210 \times 10^{-6} \times 10^{2}} = 2810 \text{ volt/cm}$$

This Gunn effect can be explained on the basis of two valley model theory of RWH theory. That means explain about two valley ~~chara~~ model theory of RWH theory.

Modes of operation of Gunn effect diodes:-

There are mainly two modes of operation of bulk negative differential resistance devices.

1. Gunn oscillation mode:-

This mode is defined in the region where the product of frequency multiplied by Length is about 10^7 cm/s and the product of doping multiplied by Length is greater than $10^{12}/cm^2$.

2. stable amplification mode:-

This mode is defined in the region where the product of frequency times Length is about 10^7 cm/s and the product of doping times Length is b/w 10^{11} and $10^{12}/cm^2$

Criterion for classifying the modes of operation:-

The Gunn effect diodes are basically made from an n-type GaAs, with the concentration of free electrons ranging from 10^{14} to 10^{17} per cubic centimeter at room temperature. Its typical dimensions are 150 X 150 um in cross section and 30um long.

The time rate of growth of space charge layers is given by

$$Q(x,t) = Q(x - v_d t, 0)\exp\left(\frac{t}{\gamma_d}\right) \quad \text{—①}$$

where $\gamma_d = \frac{\epsilon}{\sigma} = \frac{\epsilon}{e n_0 |u_n|}$

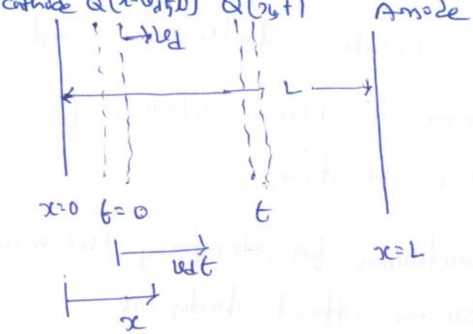
ϵ = semiconductor dielectric permittivity

n_0 = doping concentration

μ_n = -ve mobility

e = electron charge, σ = conductivity

Fig bellow clarifies the above equation



so the factor of max growth is given by

$$\text{growth factor} = \frac{Q(L, L/v_d)}{Q(0,0)} = \exp\left(\frac{L}{v_d t_d}\right)$$

$$= \exp\left(\frac{L n_0 e|\mu_n|}{\epsilon v_d}\right)$$

For a large space charge growth, this factor must be larger than unity.

That means $\dfrac{L n_0 e|\mu_n|}{\epsilon v_d} > 1$

$$L n_0 > \frac{\epsilon v_d}{e|\mu_n|}$$

Here for n-type GaAs the value $\dfrac{\epsilon v_d}{e|\mu_n|}$ is about $10^{12}\ /cm^2$.

This is the criterion for classifying the modes of operation for Gunn effect diodes.

Prob An n-type GaAs diode has the following parameter.

Electron drift velocity $v_d = 2.5 \times 10^5$ m/s

Negative electron mobility $|\mu_n| = 0.015 \, m^2/v\text{-}s$

Relative dielectric constant $\epsilon_r = 13.1$

Determine the criterion for classifying the modes of operation.

Sol The criterion for classifying the modes of operation for Gunn-effect diode is

$$n_o L > \frac{\epsilon v_d}{e|\mu_n|}$$

where $\epsilon = \epsilon_0 \epsilon_r$

$= 8.854 \times 10^{-12} \times 13.1$

$v_d = 2.5 \times 10^5$ m/s

$e = 1.6 \times 10^{-19}$ Col.

$|\mu_n| = 0.015 \, m^2/v\text{-}s$

$$\frac{\epsilon v_d}{e|\mu_n|} = \frac{8.854 \times 10^{-12} \times 13.1 \times 2.5 \times 10^5}{1.6 \times 10^{-19} \times 0.015}$$

$= 1.19 \times 10^{16} \, /m^2$

$= 1.19 \times 10^{12} \, /cm^2$

That mens $n_o L > 1.19 \times 10^{12} /cm^2$

So device is operated in Gunn oscillation mode.

Characteristics of Gunn diode:-

1. Gunn diode uses a 10-12 supply with typical bias current of 250mA.
2. The output power is 25mw to 250mw in X-band
3. efficiency is 2% to 12%.

Avalanche transit time devices:-

It is possible to make a microwave diode exhibit -ve resistance by having delay b/w voltage and current in an avalanche togather with transit time through the material. such device are called avalanche transit time devices.

There are three different modes of avalanche oscillator.

1. IMPATT: impact ionization avalanche transittime device.

2. TRAPATT:- Traped plasma avalanche trigged transit time device

3. BARITT:- Barrier injected transit time device.

IMPATT diode:-

IMPATT stands for impact ionization avalanche transit time device. In IMPATT diode the -ve resistence is proved by showing 180° phase diff b/w applied voltage and resulting current.

This 180° phase difference b/w voltage and resulting current provided by the combination of delay involved in generating avalanche current multiplication togather with delay due to transit time through the drift space.

~~Doping~~ Doping porofile:-

Figures (a), (b) and (c) bellow shows the doping porofile of varioos structure of IMPATT diode.

Fig (a): Aborupt p-n Junction

Doping porofile

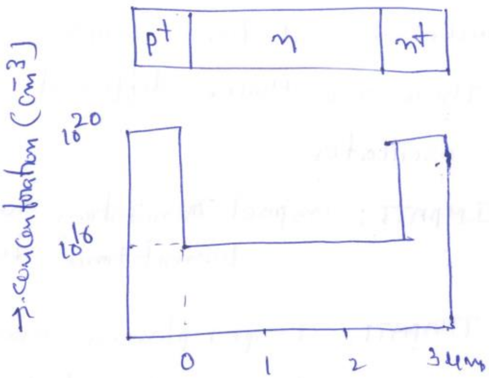


Fig (b): Lineerly graded PN Junction

Doping porofile

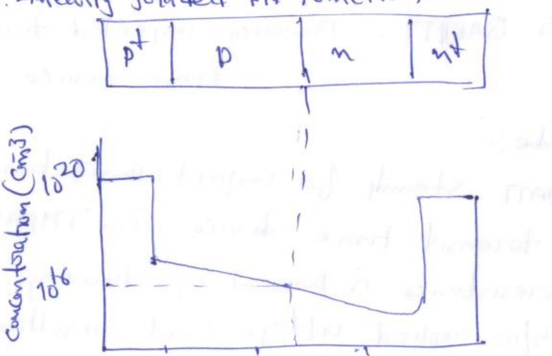
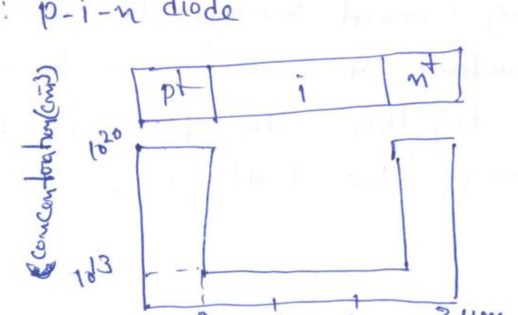


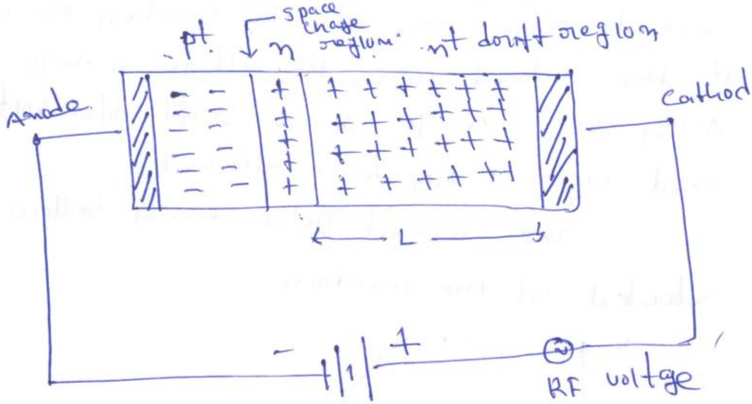
Fig (c): p-i-n diode

Doping porofile



working (operation) of IMPATT diode:-

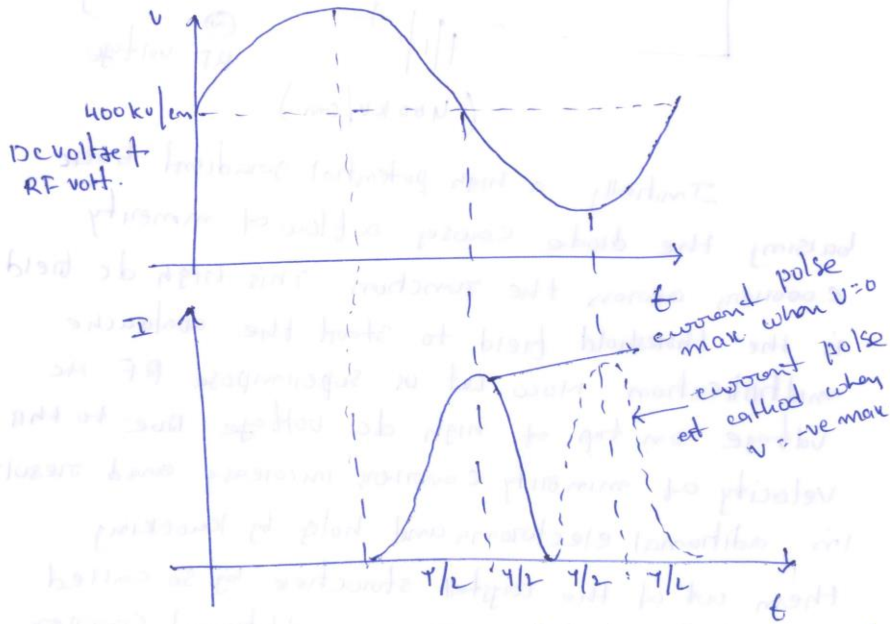
Figure bellow shows impatt diode with junction b/w p+ and n+ layers.



(400 kV/cm)

Initially a high potential gradient back basing the diode causes a flow of minority carriers across the junction. This high dc field is the threshold field to start the avalanche multiplication. Now let us superimpose RF Ac voltage on top of high dc voltage. Due to this velocity of minority carriers increases and result in aditional electrons and holes by knocking them out of the crystal structure by so called impact ionozation. These additional carriers inturn generate new carriers and this process continuous and is known as Avalanche multiplication. Since original dc field is threshold and so the voltage across the diode exceded threshold value during the RF +ve cycle and avalanche current multiplication taking place during this entire time. Since Avalanche multiplication is not instanious, this process infact takes a time such that the current pulse max at the Junction occurs at the instant when RF voltage is zero and going -ve. A 90° phase shift b/w voltage and current has been achieved.

The current pulse shown bellow is situated at the Junction.



The generated current pulse at the Junction moves towards the cathod due to applied reverse biase with a drift velocity V_d. The time taken by the pulse to reach the cathod depends on this velocity and on the length of the drift region. The length is adjusted such that time taken for current pulse to move from $V=0$ position to $V=-ve\ max$ of RF cycle exactly 90°. Hence voltage and current are 180° out of phase and so -ve resistance has been proved to exist. The frequency of oscillator or resonant frequency of IMPATT diode is given by.

$$f = \frac{1}{2\tau} = \frac{v_d}{2L}$$ where L is Length of the drift space.

Output power & efficiency of IMPATT diode:-

The max output power of diode is limited by semiconductor material and the max voltage that can be applied across diode is given by

$$V_m = E_m L \quad \text{--- (1)}$$

where L is the depletion Length and E_m is max electric field. The max voltage limited by the breakdown voltage and max current carried by the diode limited by the avalanche breakdown process.

∴ The max current is given by

$$I_m = J_m A = \sigma E_m \cdot A$$

$$= \frac{\epsilon_s}{\tau} \cdot E_m \cdot A$$

$$= \frac{v_d \epsilon_s E_m A}{L} \quad \text{--- (2)}$$

$\therefore$ upper limit of power input is given by

$$P_m = I_m V_m = E_m^2 \epsilon_s V_d A \quad \text{--- (3)}$$

The capacitance across the space charge region is defined as

$$C = \frac{\epsilon_s A}{L} \quad \text{--- (4)}$$

substitute eq (4) in (3) and apply

$$2fT = 1$$

(ie) $2f = \frac{1}{T} = \frac{V_d}{L}$

$$L = \frac{V_d}{2f}$$

$$P_m = E_m^2 V_d L \cdot C$$

$$= E_m^2 V_d \cdot \frac{V_d}{2f} \cdot C$$

$$= \frac{E_m^2 \cdot V_d}{2f \cdot \frac{2\pi f}{2\pi f C}} = \frac{E_m^2 V_d}{4\pi f^2 \times C}$$

(i.e) max power that can be applied to the mobile carriers decreases as $\frac{1}{f^2}$

The efficiency of IMPATT diode is given by $\eta = P_{ac}/P_{dc} = \left(\frac{V_a}{V_d}\right)\left(\frac{I_a}{I_d}\right)$

where V_a and I_a are ac voltage and current

V_d and I_d are dc voltage and current.

Theoritical efficiency of IMPATT diode is 30%. But practical efficiency is less than 30% and is 15% for Si, 23% for GaAs.

characteristics:-

1. Theoritical efficiency $\eta = 30\%$

   But practical efficiency is $< 30\%$

   and 15% for Si

   23% for GaAs

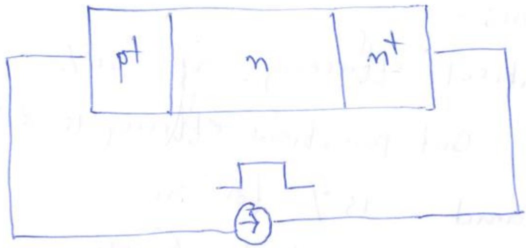
2. frequency: 1 — 300 GHz
3. Max o/p power from a single diode in X band is 5w.

Drawbacks:-

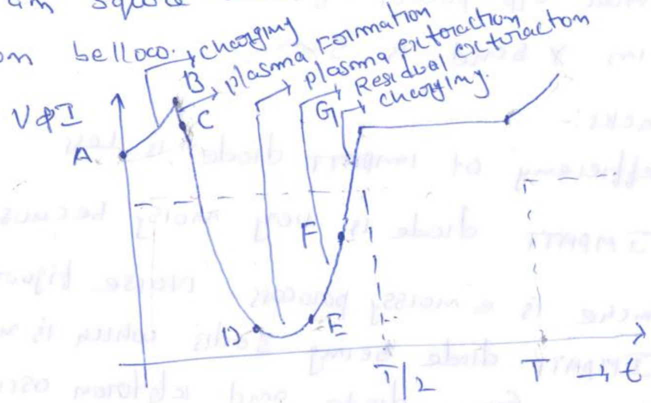
1. efficiency of IMPATT diode is less
2. IMPATT diode is very noisy because avalanche is a noisy process. Noise figure for IMPATT diode being 30dB which is not good as Gunn diode and klystron oscillator.

TRAPATT diode:-

~~Trap~~ TRAPATT diode stands for trapped plasma avalanche triggered transit diode. It is a high efficiency microwave generator capable of operating from several hundred MHz to several GHz. The configuration used for manifacturing TRAPATT diode is $p^+ n n^+$ and material used is Silicon. The TRAPATT diode is shown belloco.



A typical voltage - current waveforms for TRAPATT diode of an $p^+ n n^+$ operating with an squre wave current drive pulse is shown belloco.



The electric field is uniform at point A and its magnitude is large but less than the value required to breakdown. From point A the diode is linearly charges because of the generated minority carriers and certain field is reached say point B the electric field decoress to point C. During this time (B to C) field is sufficiently large for avalanche to continue and a dense plasma of electrons and holes are created. As these electrons and holes move to the ends of the depletion layer, the field further decreased to point D. A long time is required to remove the plasma because plasma charge is very large and at point E plasma is removed. Any residual charge in the depletion region removed later so voltge increases from E to F. At point F all the charge that was generated internally has been removed. From point F to G the diode is charged up like a capacitor. At point G the current goes to zero for half period and voltage remains constant at V_A until the current comes back (for next cycle).

The main advantage of Trapatt diode over Impatt diode is its efficiency. The efficiency of TRAPATT diode is 15% to 40%. The draw back of TRAPATT diode is its noise figure is 30dB. so it is very noisy compared to IMPATT diode.

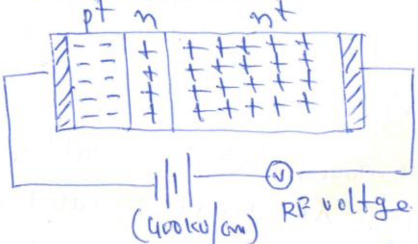
Compaorision b/w IMPATT and TRAPATT diodes:-

IMPATT diode | **TRAPATT diode**

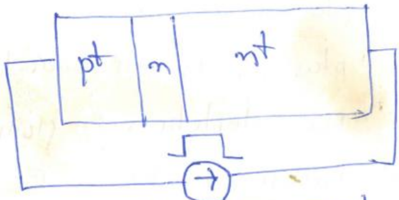
1. IMPATT stands for impact Ionization Avalanche transit time.

TRAPATT stands for Trapped plasma avalanche triggred transit diode.

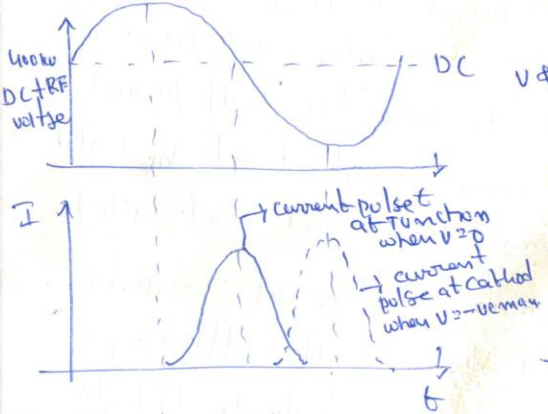
2. The configuration used for IMPATT diode is shown bellow



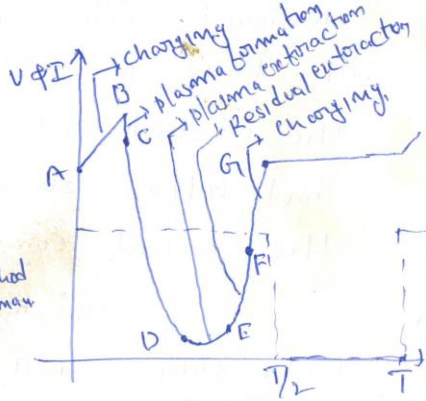
The configuration used for TRAPATT diode is shown bellow



3. The voltge and current wave forms shown bellow.



The voltge and current wave forms shown bellow.



4. Theoritical efficiency is 30%. practical efficiency is < 30%.

efficiency is large compared to IMPATT and its efficiency is 15 to 40%.

5. frequency of operation is 1 — 300 GHz

Its natural frequency a resonant frequency is limited to 10 GHz.

6. IMPATT diode is noisy and its Noise figure is 30 dB.

its Noise figure is greater than 30 dB

Unit-6 Measurements

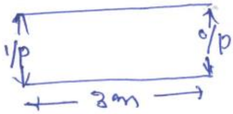
Expected Date	Description
Completion Date	

Low frequency measurements - Microwave measurements:-

1. At low frequency, it is convenient to measure voltage and current and use them to calculate power. But at microwave frequency, they are dificult to calculate because they vary with position in a transmission line, which is explained bellow.

Let a transmission line Length $l = 3m$

At low frequency



Let $f = 3KHz$

$\lambda = \frac{c}{f} = \frac{3\times10^8}{3\times10^3} = 10^5 m$

Here $\lambda >> l$

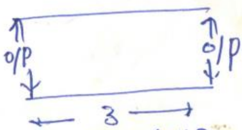
interms of time

$T = \frac{1}{f} = \frac{1}{3\times10^3} = 0.33 msec$

delay $\tau = \frac{l}{v} = \frac{3}{3\times10^8} = 10nsec$

so o/p changes instantaineously to input change. voltage and current at any point on the line is const at a perticular instent of time.

At high frequency.



Let $f = 1GHz$

$\lambda = \frac{3\times10^8}{3\times10^9} = 30 cm$

$\lambda < l$

interms of time

$T = \frac{1}{f} = \frac{1}{1GHz} = 1nsec$

delay $= \tau = \frac{3}{3\times10^8} = 10nsec$

$\tau > T$

So o/p changes not instantaneously to input change. voltage and current varies along the length of the line

2. At low frequency circuits use lumbed elements which can be identified and measured. But at mw frequencies circuit elements are distributed.

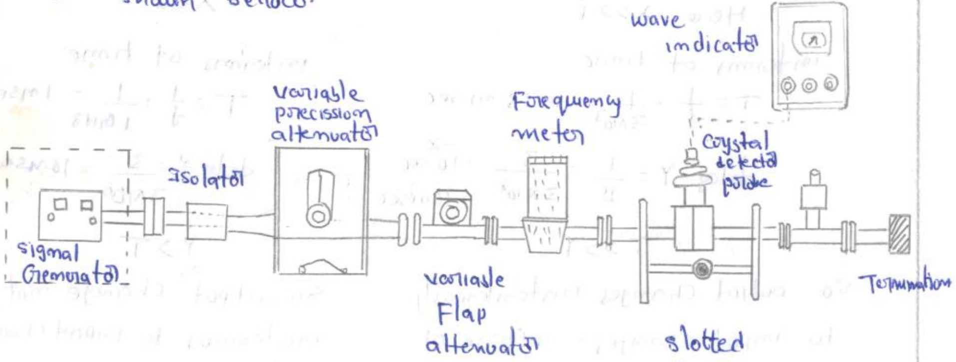
3. Unlike low frequency measurements, many quantities measured at μW frequencies are relative and it is not necessary to know their absolute values.

4. For power measurement it is usually sufficient to know the ratio of two powers rather than exact input or output powers.

5. At μW frequency we can measure the following parameters. 1. power 2. SWR 3. Attenuation 4. Frequency 5. phase 6. impedance 7. Insertion and reflection losses 8. Q-factor.

Microwave Bench-General Measurement Set up:-

The General Microwave bench set up for measurement of any parameter in microwaves is shown below.



Here signal generator is a microwave source whose output power is of the order of milliwatts. ~~They~~ It could be Gunn diode oscillator, a backward wave oscillator (or) reflex klystron oscillator.

Isolator is a two port microwave device, which has a property that it provides minimum attenuation in forward transmission and provides maximum attenuation in backward direction. Since it provides maximum attenuation in Backward direction, it prevents reflections if any (due to mismatch of Load and line) to reach the generator.

The precision attenuator can provide 0 to 50 dB attenuation above its insertion loss. A frequency meter is used for direct reading of frequency that consists of a single cylindrical cavity which can be adjusted to resonance to measure frequency and is slot coupled to the waveguide.

Slotted line consists of a slotted section, a travelling probe carriage and facility for attaching detecting instruments. The slot is made at the centre of the broad face of the waveguide parallel to the axis of waveguide. A small probe is inserted through the slot to sense the field strength of the standing wave pattern inside the waveguide. This probe is on a carriage plate which moves on the top surface of the waveguide. This probe is connected to connected to crystal detector so that the output from the detector is proportional to the square of the input voltage at that position of the probe. As the probe moves along the

the waveguide slot, it gives an output proportional to the standing wave inside the waveguide. Since the crystal diode is a square law device, the square root of the ratio of max output to min output gives the VSWR

$$(\text{i.e})\quad VSWR = \sqrt{\frac{V_{max}^2}{V_{min}^2}} = \frac{V_{max}}{V_{min}}$$

we can also find the position of V_{max} and V_{min} and from that calculate wavelength (λ_g) of the wave. Let y_{1min} and y_{2min} are two successive positions of V_{min} then

$$\frac{\lambda_g}{2} = y_{2min} - y_{1min}$$

$$\lambda_g = 2\left[y_{2min} - y_{1min}\right]$$

Attenuation Measurement:-

Microwave components and devices almost always provide some degree of attenuation. Attenuation is the ratio of input power to the output power and is normally expressed in dB.

$$(\text{i.e})\quad \text{Attenuation (in dB)} = 10\log\frac{P_{in}}{P_{out}}$$

The amount of attenuation of a device can be measured by two methods.

1. power ratio method
2. RF substitution method.

1. power ratio method:-

This method involves measuring input power and output power with and without the device whose attenuation is to be measured as shown in set up$_1$ and set up$_2$ shown in figures bellow. The powers are measured in each set up as P_1 and P_2. The ratio of powers $\frac{P_1}{P_2}$ expressed in dB gives the attenuation of that device.

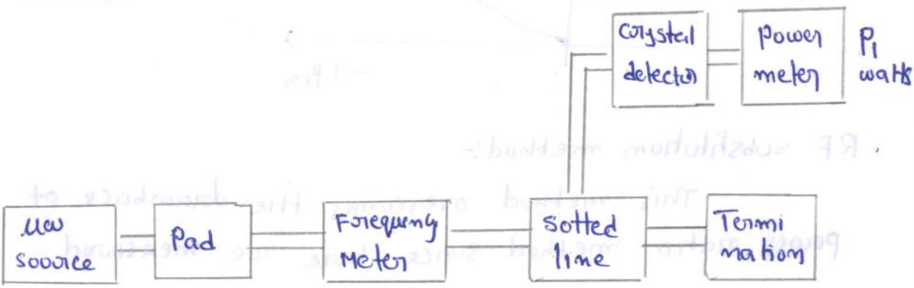


Fig:1. Set up 1, power ratio method

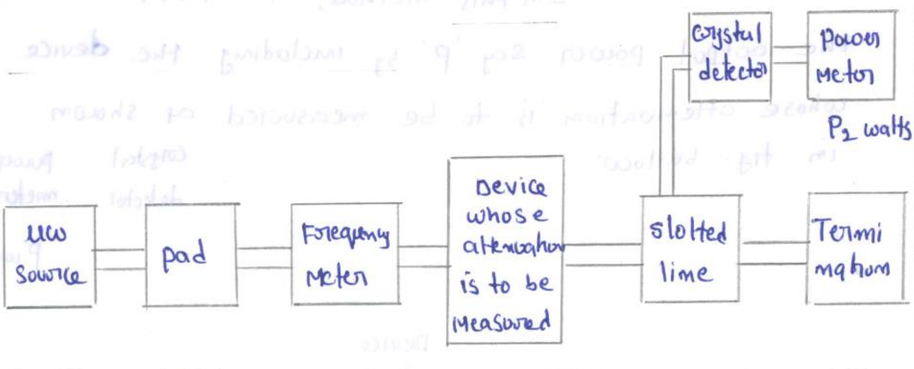
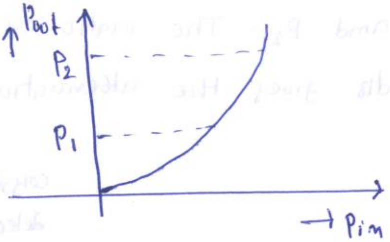


Fig 2: set up 2, power ratio method

$\therefore$ Attenuation (in dB) $= 10 \log \frac{P_1}{P_2}$

The drawback of this method is that the attenuation measured corresponds to two power positions on the power meter with a squre law crystal detector characteristics shown bellow. Due to non linear characteristics the two powers measured and the attenuation Calculated will not be accurate:



RF substitution method:-

This method overcomes the drawback of power ratio method since here we measured attenuation at a single power position.

In this method, in setup1 measure the output power say 'p' by including the device whose attenuation is to be measured as shown in fig. bellow.

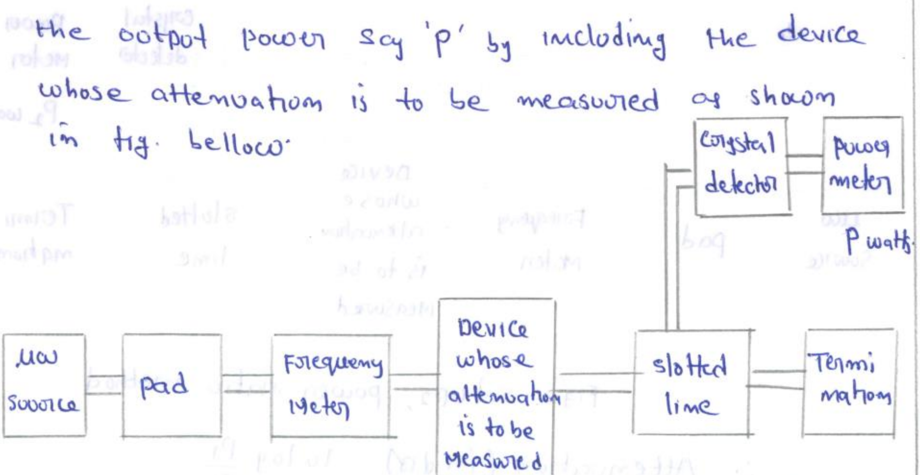


Fig.1: Setup1 RF substitution method.

In setup$_2$ this device is replaced by a precision attenuator which can be adjusted to obtain the same power 'P' as measured in setup$_1$. Under this condition the attenuation read on the precision attenuator would give attenuation of the device directly.

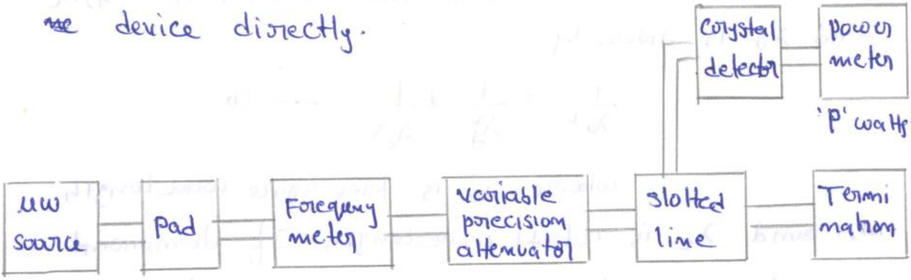


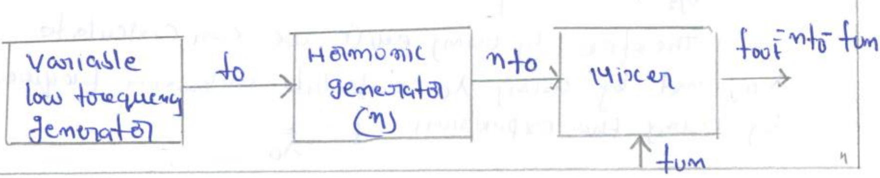
Fig 2: Setup$_2$, RF substitution method.

Frequency measurement:-

The frequency of the uw source can be measured by using any one of the following three methods.

1. Electronic method for frequency measurement:-

In this method unknown frequency is compared with the harmonics of a known lower frequency by use of a variable frequency generator, a harmonic generator and a mixer as shown in fig bellow.



Here foot and nto are known values and from that expression we can calculate microwave frequency of the signal.

Slotted line method:-

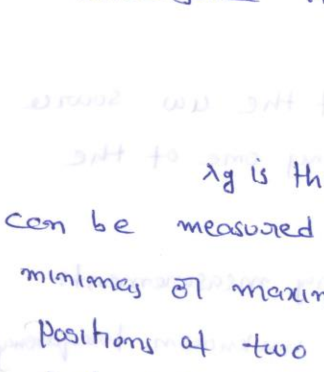
We know that the relation b/w λ_g, λ_c and λ_0 is given by

$$\frac{1}{\lambda_0^2} = \frac{1}{\lambda_g^2} + \frac{1}{\lambda_c^2} \quad \text{—— (1)}$$

where λ_0 is free space wavelength and λ_c is cut-off wave length. If dominant mode TE_{10} is propagated in a rectangular wave guide then $\lambda_c = 2a$

where 'a' is the broader dimension of Rectangular waveguide.

λ_g is the guide wavelength and which can be measured by finding positions of successive minimas or maximas. Let y_{1min} and y_{2min} are positions at two successive minimas as shown in fig. bellow

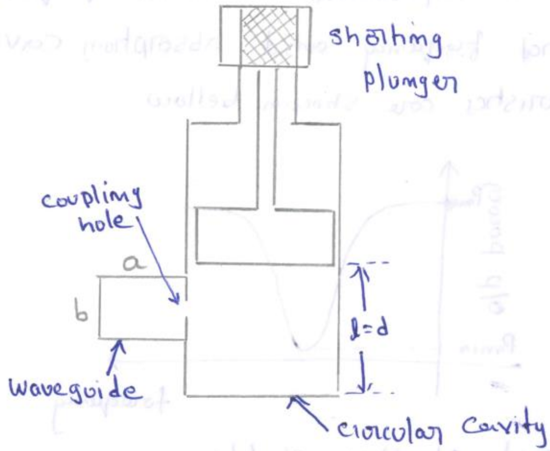


$$(i.e)\ \frac{\lambda_g}{2} = y_{2min} - y_{1min} \quad \text{—— (2)}$$

Therefore by using eq (1) we can calculate λ_0 and by using λ_0 calculate unknown frequency by using the expression $f = \frac{c}{\lambda_0}$.

3. wave meter (or) frequency meter method:-

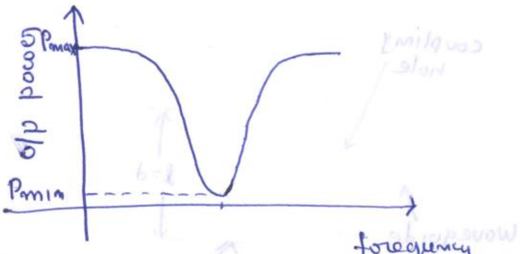
A wave meter is constructed of a cylindrical cavity resonator with a variable short circuit transmission. The shorting plunger is used to change the resonance frequency of the cavity by changing the cavity length. wave meter axis is so placed that it is perpendicular to broad wall of the waveguide as shown below.



Cavity wave meters are two types. transmission type and absorption type. In transmission type cavity signal transmitted only when cavity tuned to signal frequency and in absorption type cavity signal attenuated, when cavity tuned to signal frequency. The absorption type is preferred for laboratory frequency measurement. The resonant frequency of the cavity wavemeter is determined by the physical diamensions a, b, d and mode is deteromed by m, n and p as given by

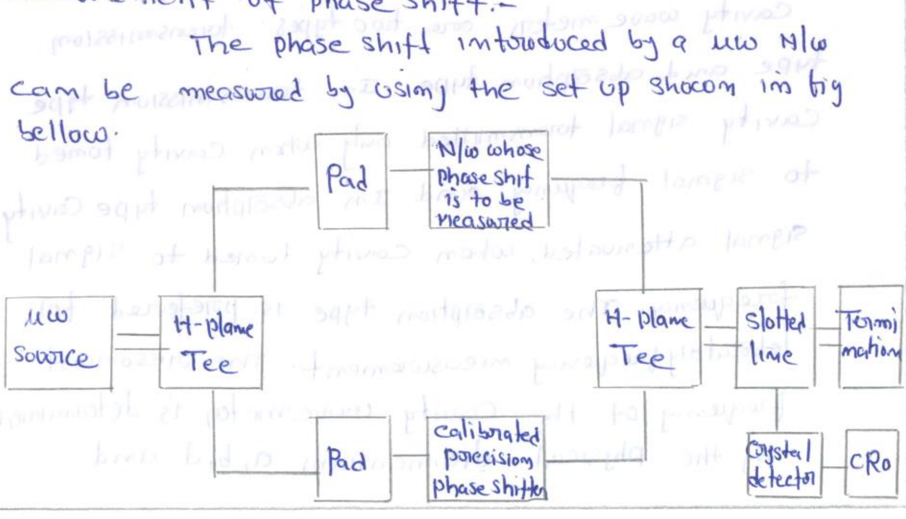
$$f_0 = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2+\left(\frac{p}{d}\right)^2}$$

So varying the cavity Length by movable short circuit, we can change the resonant frequency. while we are tuning whenever observe the min power in the power meter that ~~represent~~ resonant frequency is equll to signal frequency and absorption cavity charactoristics are shown bellow

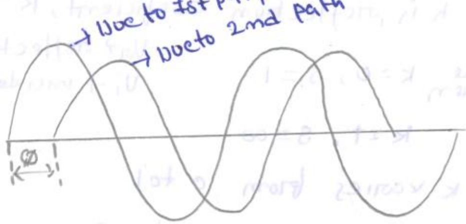


Measurement of phase shift:-

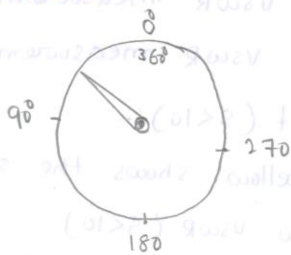
The phase shift introduced by a μw N/w can be measured by using the set up shown in fig bellow.



Here signal from uw source split up in to two equal parts using the H-plane Tee Junction, one going to the unknown network whose phase shift is to measure and other to the calibrated precission phase shifter. Now the standard phase shifter adjusted untill the two signals on the CRo are inphase as shown bellow and the ~~rotation~~ relative phists of the two networks are now equal.

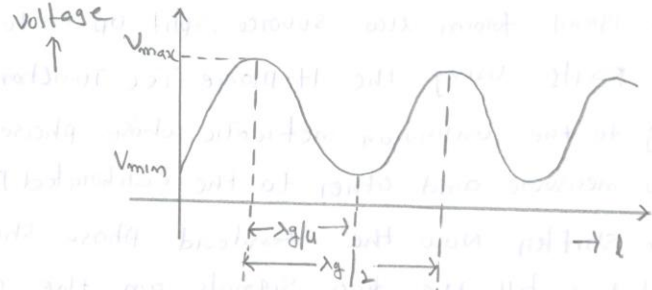


The reading on the precision phase shifter now gives the phase shift provided by the Network as shown bellow.



Measurement of voltage standing wave ratio (VSWR):-

The standing wave ratio is defined as the ratio of maximum to minimum voltage on a line having standing wave as shown bellow.



$$\therefore \ VSWR = S = \frac{V_{max}}{V_{min}} = \frac{1+|k|}{1-|k|}$$

where k is reflection coeficient, $K = \frac{V_r}{V_i}$

$V_r \rightarrow$ reflected voltage
$V_i \rightarrow$ incident voltage

~~where~~ when $k=0$, $S=1$

$k=1$, $S=\infty$

So k varies from 0 to 1

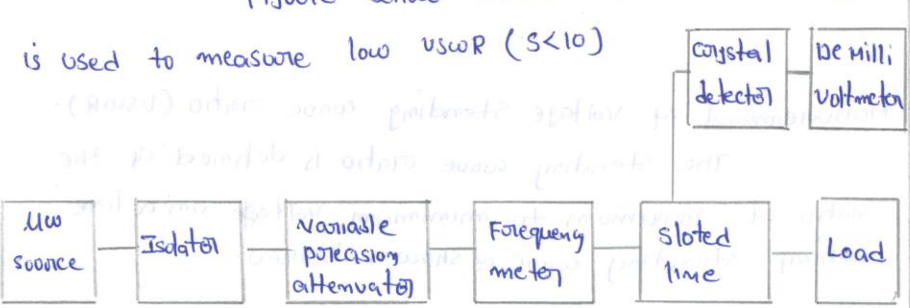
S varies from 1 to ∞

Depending on the value of VSWR, there are two types of VSWR measurements

1. Low VSWR measurement $(S<10)$
2. High VSWR measurement $(S>10)$

1. Low VSWR measurement $(S<10)$:-

Figure bellow shows the set-up which is used to measure low VSWR $(S<10)$



In this method of measurement, adjusting the attenuator to give an adequate reading on the voltmeter. The probe on the slotted section is moved to get max reading on the voltmeter (V_{max}). Next the probe on the slotted line adjusted to get min reading on the meter (V_{min}). The ratio of first reading to second reading (i.e $\frac{V_{max}}{V_{min}}$) gives the VSWR

The voltmeter itself calibrated in terms of VSWR. In this case the probe is moved to give max deflection on the ~~#~~ meter by adjusting attenuator. This full scale deflection (FSD) corresponds to a VSWR of 1. For example FSD of 10mV corresponds to VSWR of 1. Now the travelling probe is adjusted to get min reading on the meter. If min reading corresponds to 5mV, then

$$VSWR = \frac{10mV}{5mV} = 2$$

If min reading corresponds to 3.3 mV

$$\Rightarrow VSWR = 3$$

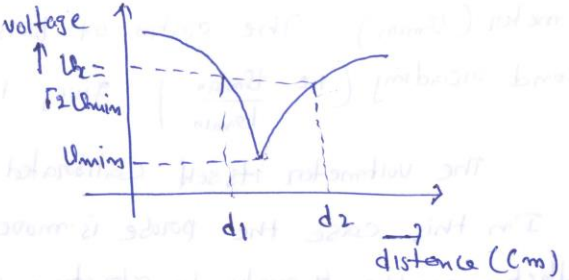
if $V_{min} = 2.5$, VSWR = 4,

if $V_{min} = 1mV$, VSWR = 10 etc.

2. High VSWR measurement ($S > 10$):-

The method which is used to measure high VSWR (ie) greater than 10 is called a double minimum method. In this method moving the probe measure the min power in the power meter.

Now the probe is moved to a point where the power is twice min. Let this position be denoted by 'd_1'. The probe is then moved to twice min power point on the other side of the min power. Let this position be d_2 and which is shown bellow.



we know that $P_{min} \propto V_{min}^2$

$$2P_{min} \propto V_x^2$$

$$\frac{1}{2} = \frac{V_{min}^2}{V_x^2}$$

$$V_x = \sqrt{2}\, V_{min}$$

If dominant TE_{10} mode is propagated

$\lambda_c = 2a$, 'a' is wider dramension.

If frequency is known, then

$$\lambda_0 = \frac{c}{f}$$

Then $\lambda g = \dfrac{\lambda_0}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}$

Then VSWR can be calculated by using the expression

$$VSWR = \frac{\lambda g}{\pi(d_2 - d_1)}$$

Impedance measurement:-

Impedance at µw frequencies can be measured using any ~~one~~ of the following two methods.

1. Using slotted line
2. Using Reflectometer.

Using slotted line:-

The impedance of unknown load can be measured by using slotted line in conjunction with the Smith Chart as follows.

1. First by using the set up-1 shown bellow determine standing wave ratio (by finding V_{max} and V_{min}) and with the value of 'S' draw the S-circle on the Smith chart.

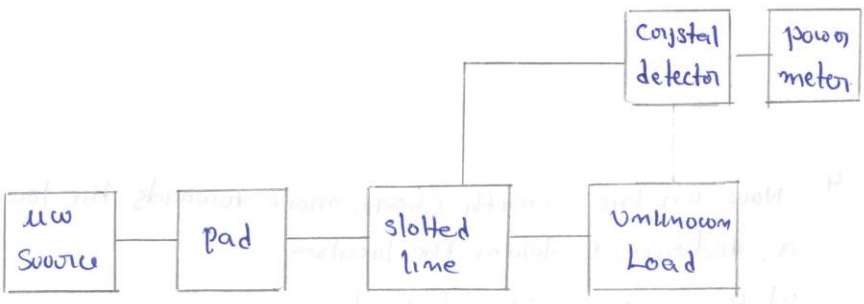


fig-1: ~~ste~~ set up-1

2. To determine Z_L, the Load replaced by a short circuit shown in set up-2 and note the locations of V_{min} on the scale and select any min ~~one~~ as Load reference point.

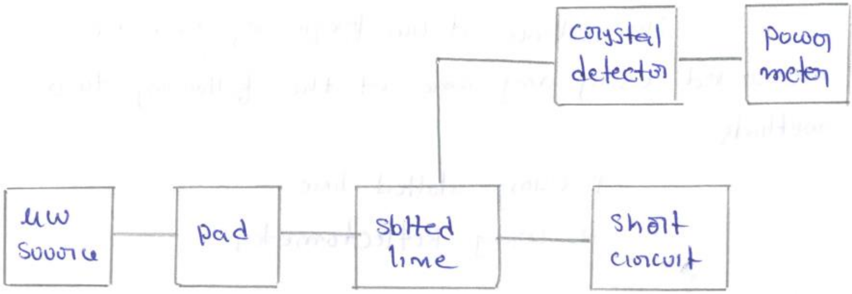
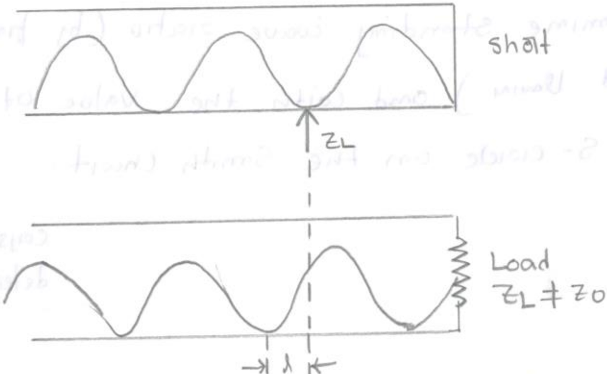
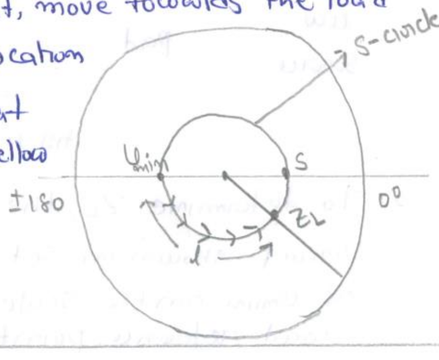


Fig-2: set up-2

3. Now with the load on the line, note the position of V_{min} and determine the shift (l) in mm as compared to short circuit case



4. Now on the smith chart, move towards the load a distance l from the location of V_{min}. Find Z_L at that point, which is shown bellow on a smith chart

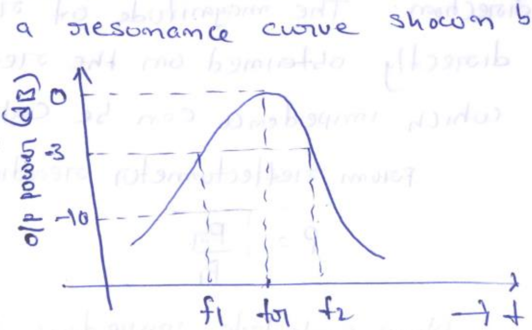


Measurement of 'Q' of a cavity resonator:-

There are several methods for measuring the 'Q' of a cavity resonator. Among that transmission method is the simplest and the setup for transmission method of measuring 'Q' is shown bellow.

Fig: Set up for measuring Q of a Cavity resonator.

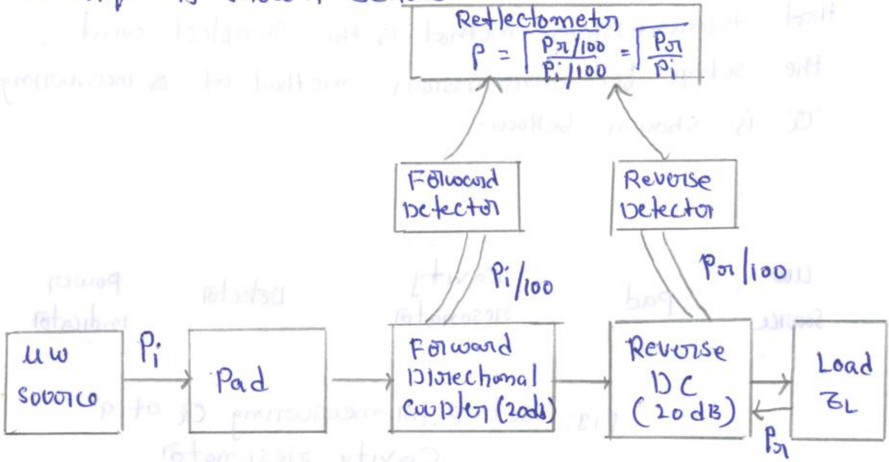
In this method the cavity resonator used as a transmission type device and output signal is measured as a function of the frequency results in a resonance curve shown bellow.



By varying the frequency of microwave source and keeping signal level constant, the output power is measured. Alternatively cavity can be tuned by keeping both signal level and frequency constant and output

2. Using Reflectometer:-

The typical set-up for reflectometer technique is shown bellow.



Here two directional couplers are used to sample the incident power P_i and reflected power P_r from Load. Both DC's are identical except their direction. The magnitude of reflection coef. directly obtained on the reflectometer from which impedence can be calculated.

From reflectometer reading we have

$$\rho = \sqrt{\frac{P_r}{P_i}}$$

Now calculate impedence by using the relation

$$\rho = \frac{Z_L - Z_0}{Z_L + Z_0}$$

where Z_0 is characteristic impedence and which is known value.

power is measured

From the resonance curve

Half power Bandwidth is given by

$$B.\omega = \omega_2 - \omega_1$$

$\therefore$ The Q of cavity resonator is

given by

$$Q = \frac{\omega_r}{\omega_2 - \omega_1}$$

where ω_r is resonance frequency.

From the expression, we can say that narrow the B.ω, Q of a system is high.

Measurement of power using Bolometer.

The following are various methods to measure power based on its level (i.e) low or high)

1. Measurement of low power (0.01mw-10mw) - Bolometer technique
2. Measurement of medium power (10mw-10w) calorimetric Technique
3. Measurement of high power (>10w) calorimetric watt meter.

Measurement of low microwave power (0.01mw - 10mw) using Bolometer technique:-

Bolometer technique is used to measure power (i.e) from 0.01mw to 10mw. Bolometer is a temperature sensitive device, whose resistance varies with temperature. These are two types Barretters and thermister. Barretters have +ve temperature coefficient (i.e resistance increases by increasing the temp) and thermistors have -ve temp. coefficient (i.e resistence decreases as temp. increases), which are shown bellow.

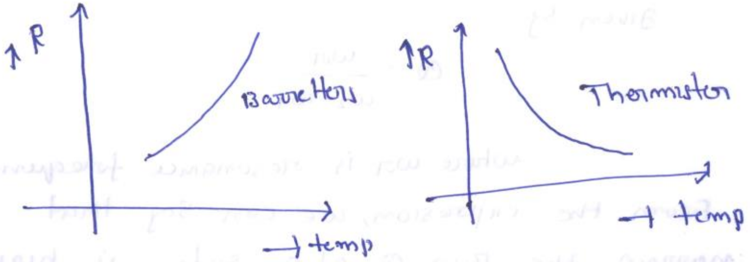
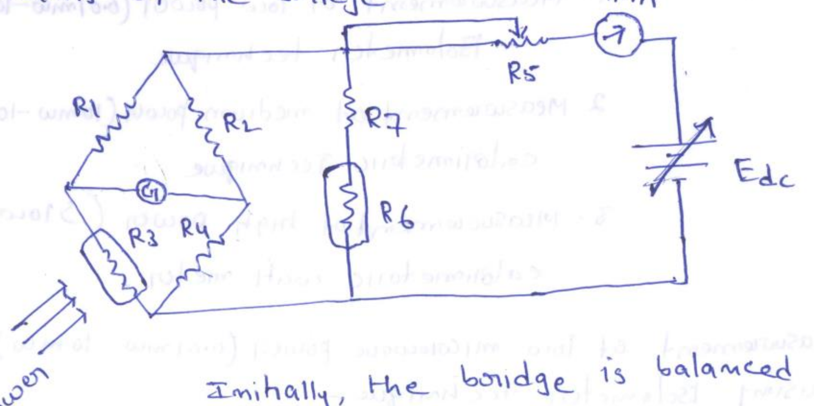


Figure bellow shows the circuit arrangement of a balanced bolometer bridge technique in which bolometer itself is used in one of the arms of the bridge.



Initially, the bridge is balanced by adjusting R5, which varies DC power applied to the bridge and the bolometer element is brought to a predetermined operating resistance.

before uw power is applied. Let the voltage of the battery be E_1 at the balance. The uw power is now applied and this power gets desipated in the bolometer. The bolometer heats up and it changes its resistence. Therefore the bridge becomes unbalanced. The applied dc power is is changed by changing voltage to E_2 to get back the bridge is balanced and this change in dc battery voltage $(E_1 \sim E_2)$ will be proportional to the uw power. Alternatevely, the detector G can be directly calibrated interms of microwave power so that when the bridge is unbalanced, the detector reads the uw power directly.

The errors in the above method must be avoided by providing some type of temperature compensation, because the bolometers are temp sensitive. The resistors R_6 and R_7 in the circuit arrangement shown in fig provide the required temp. compensation.

Measurement of Insertion loss:-

The insertion loss is defined as the difference in the power arriving at the terminating Load with and without the device in the circuit.

(i.e) insertion loss (dB) = $10 \log \frac{P_i}{P_o}$

where P_i – input signal power
P_o – output signal power

∵ if $\frac{P_i}{P_{or}} = \frac{P_i - P_{or}}{P_o} \cdot \frac{P_i}{P_i - P_{or}}$

Then insertion loss (in dB) $= 10\log \frac{P_i}{P_o}$

$$= 10\log \frac{P_i - P_{or}}{P_o} \cdot \frac{P_i}{P_i - P_{or}}$$

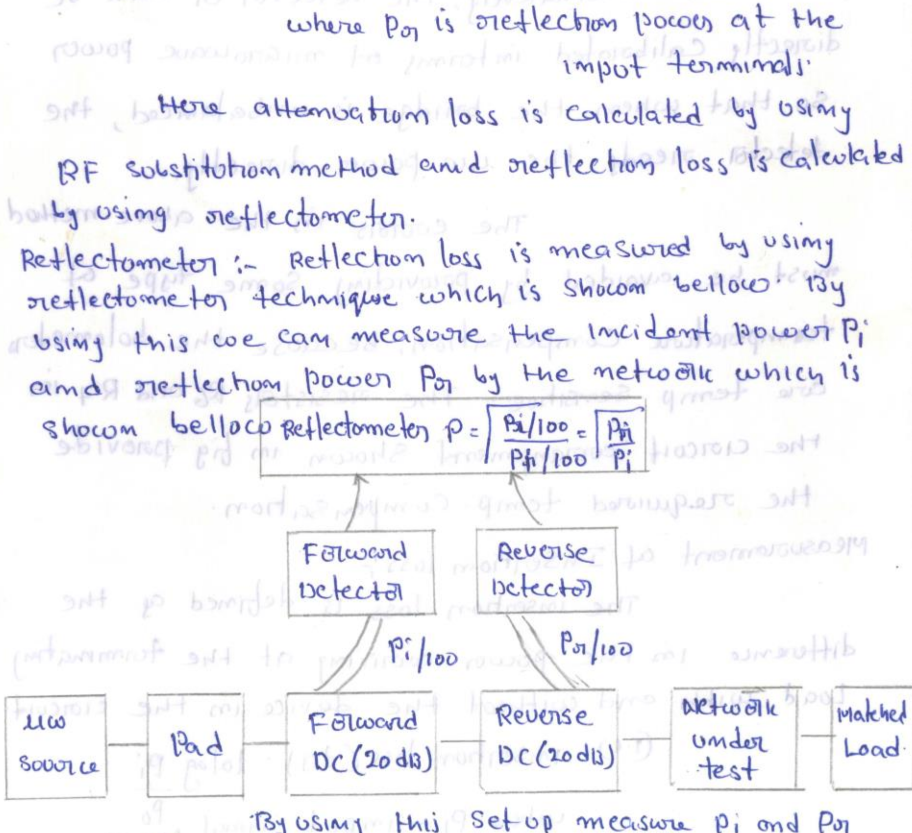
$$= 10\log \frac{P_i - P_{or}}{P_o} + 10\log \frac{P_i}{P_i - P_{or}}$$

$=$ Attenuation loss + Reflection loss.

where P_{or} is reflection power at the input terminals.

Here attenuation loss is calculated by using RF substitution method and reflection loss is calculated by using reflectometer.

Reflectometer :- Reflection loss is measured by using reflectometer technique which is shown bellow. By using this we can measure the incident power P_i and reflection power P_{or} by the network which is shown bellow



By using this set up measure P_i and P_{or}

Then reflection loss (dB) $= 10\log \frac{P_i}{P_i - P_{or}}$

∴ Insertion loss = Attenuation loss + reflection loss.

UNIT-4

HELIX TWTS

Significance of TWT :-

Travelling wave Tubes (TWTs) have gains of 40dB and above, with bandwidths more than an octave. A bandwidth of 1 octave is one in which the upper frequency is twice the lower frequency.

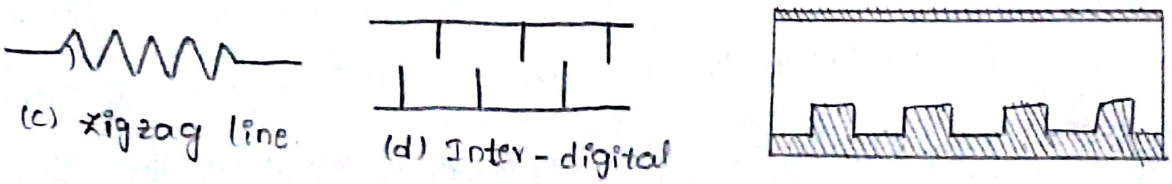
TWT is a broadband slow-wave device. Its operation is based on the interaction btw the travelling wave structure and the electron beam.

Types and Characteristics of slow-wave structures :-

The travelling-wave tubes (TWTs) are commonly employed where a high power is required. the ordinary resonators, which are used in Klystrons, cannot generate a large output, because the gain-bandwidth product is limited by the resonan circuit.

The phase velocity of a wave in ordinary waveguides is greater than the velocity of light in vaccum. In the operation of TWT, the electron beam should keep in step with the microwave signal.

(a) Helical line (b) Folded back line.



(c) zigzag line (d) Inter-digital line (e) Corrugated waveguide.

Helix :- Different types of slow-wave structures are shown in figure. A helix is the most commonly helix is also constructed by the use of a round wire that acts as a slow-rate structure.

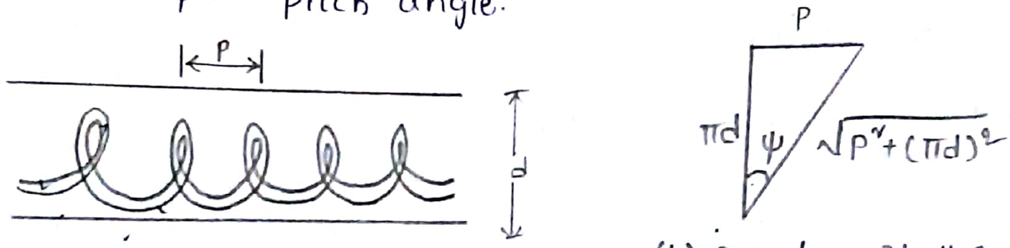
$$\frac{v_p}{c} = \frac{P}{\sqrt{P^2+(\pi d)^2}} = \sin\psi .$$

where, $c = 3\times10^8$ m/s is the velocity of light in free space.

P = helix pitch.

d = diameter of the helix.

ψ = pitch angle.



(a) Helical coil (b) one turn of the helix.

Mostly, the helix is surrounded by a dielectric filled cylinder. In the axial direction, the phase velocity can be given as

$$V_{pe} = \frac{P}{\sqrt{\mu\epsilon\,[P^2+(\pi d)^2]}}$$

If we consider the case of small pitch angle, the phase angle, the phase velocity along the coil the free space is given by

$$v_p \approx \frac{Pc}{\pi d} = \frac{\omega}{\beta} .$$

The ω-β (or Brillouin) diagram as shown in figure. is very useful in designing a helix slow-wave structure. once β is found, V_p Can be computed eq. Furthermore, the group velocity of the wave is merely the slope of the curve and is given by

$$v_g = \frac{\partial\omega}{\partial\beta} .$$

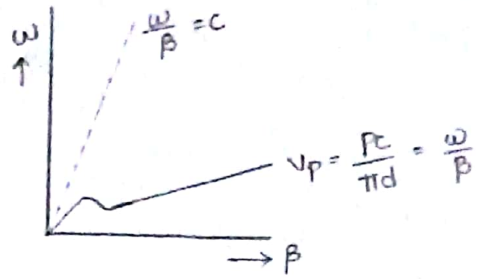


fig:- ω-β diagram for a helical structure

The helical periodic structure can be expanded as an infinite series of waves with a period 'L', all the same frequency but with different phase velocities, and is given by

$$V_{pn} = \frac{\omega}{\beta_n} = \frac{\omega}{\beta_0 + (2\pi n/L)}.$$

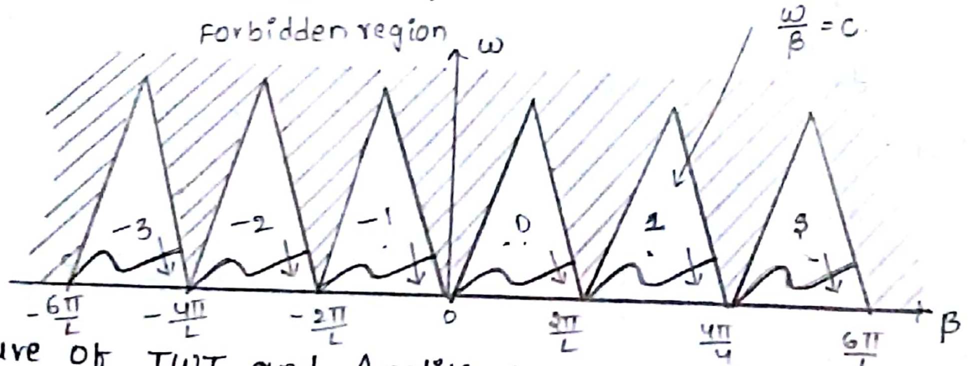
The group velocity that can be calculated from equation is

$$V_g = \left[\frac{d(\beta_0 + 2\pi n/L)}{d\omega}\right]^{-1} = \frac{\partial\omega}{\partial\beta_0}.$$

where β_0 = phase constant of the avg. electron velocity.

L = period of the helix.

n = any integer value.



Structure of TWT ~~and Amplification~~ process :-

The schematic diagram of a typical TWT is shown in figure. The TWT consists of an electron gun that is used to produce a narrow constant velocity electron beam. This electron beam is, in turn, passed through the centre of a long axial helix. Hence we use a magnetic field of high focusing capacity to avoid spreading and it will guide the wave through the centre of the helix.

A helix is a loosely wound, thin conducting helical wire that acts as a slow-wave structure. the signal to be amplified is applied to the end of the helix that is adjacent to the electron gun. the amplified signal appears at the output or the other end of the helix under appropriate conditions.

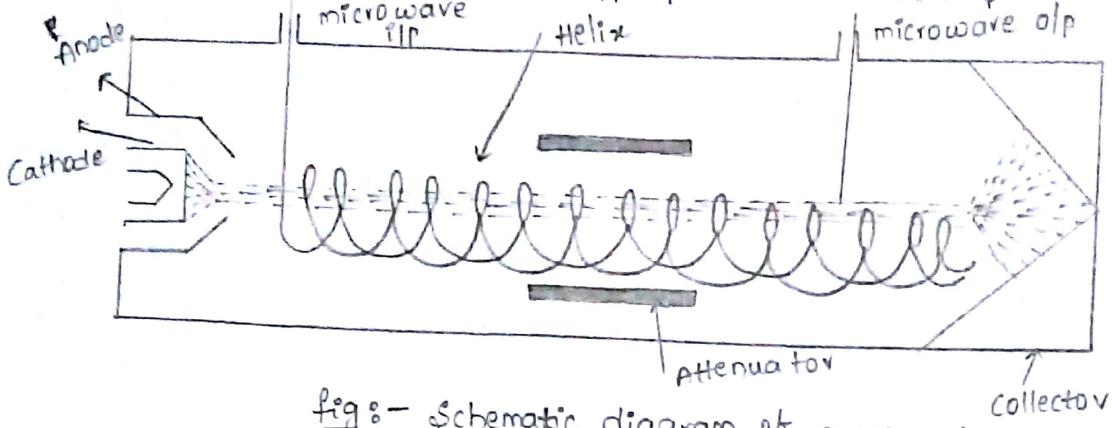


fig:- Schematic diagram of a Traveling-wave Tube.

Suppression of Oscillations:-

In order to prevent oscillations from being spontaneously generated in a traveling-wave tube, it is necessary to prevent internal feedback arising from reflections due to slight impedence mismatches at the output terminal.

It is necessary to prevent backward-wave oscillations from being generated in TWT. This situation is controlled by introducing an attenuator which is placed near the input end of the TWT. that absorbs any wave propagated along the helix.

Nature of the Four propagation Constants:-

By solving the electronic and circuit equations at the same time the wave modes of helix type travelling wave tube are determined. Thus, the values of the four propagation constants γ are given by

$$\gamma_1 = -\beta_e C \frac{\sqrt{3}}{2} + j\beta_e \left[1+\frac{C}{2}\right].$$

$$\gamma_2 = \beta_e C \frac{\sqrt{3}}{2} + j\beta_e \left(1+\frac{C}{2}\right].$$

$$\gamma_3 = j\beta_e (1-C).$$

$$\gamma_4 = -j\beta_e \left(1-\frac{C^3}{4}\right).$$

Derivation of Expression for four propagation constants of TWT:

From eqns, it can be observed that there are four different solutions for the propagation constants. It implies that there are four modes of travelling waves in the O-type travelling-wave tube.

$$(\gamma^2-\gamma_0^2)(j\beta_e-\gamma)^2 = -j\frac{\gamma^2\gamma_0 Z_0 \beta_e I_0}{2V_0}. \rightarrow (1)$$

It can be seen that the above equation is of fourth order in γ and therefore it has four roots. By numerical methods and digital computer, exact solutions can be obtained.

$$\gamma_0 = j\beta_e$$

Then eq (1) is reduced to

$$(\gamma-j\beta_e)^3(\gamma+j\beta_e) = 2C^3\beta_e^2\gamma^2. \rightarrow (2)$$

where, C is the travelling wave tube gain parameter and is given as

$$C = \left[\frac{I_0 Z_0}{4V_0}\right]^{1/3}. \rightarrow (3)$$

From eq (2), it can be observed that there are three travelling waves equivalent to $e^{-j\beta_e z}$ and one backward travelling wave which is equivalent to $e^{j\beta_e z}$. For the three forward travelling waves, the propagation constant is given by

$$\gamma = j\beta_e - \beta_e C\delta \rightarrow (4)$$

where it is assumed that $c\delta \ll 1$

Substitute of eq (4) in eq (2) results in

$$(-\beta_e c\delta)^3 (j2\beta_e - \beta_e c\delta) = 2c^3\beta_e^2(-\beta_e^2 - 2j\beta_e^2 c\delta + \beta_e^2 c^2\delta^2) \rightarrow (5)$$

Since $c\delta \ll 1$, eqn (5) is reduced to

$$\delta = (-j)^{1/3}.$$

From the theory of complex variables, the three roots of $(-j)$ can be plotted in figure.

$$\delta = (-j)^{1/3} = e^{-j((\pi/2 + 2n\pi)/3)} \qquad (\because n = 0, 1, 2)$$

The first root δ_1 at $n=0$ is

$$\delta_1 = e^{-j\pi/6} = \frac{\sqrt{3}}{2} - j\frac{1}{2}.$$

The second root δ_2 at $n=1$ is

$$\delta_2 = e^{-j5\pi/6} = -\frac{\sqrt{3}}{2} - j\frac{1}{2}.$$

The Third root δ_3 at $n=2$ is

$$\delta_3 = e^{-j3\pi/6} = j.$$

The fourth root δ_4 corresponding to the backward traveling wave can be obtained by setting

$$\gamma = -j\beta_e - \beta_e c\delta_4$$

$$\delta_4 = -j\frac{c^2}{4}.$$

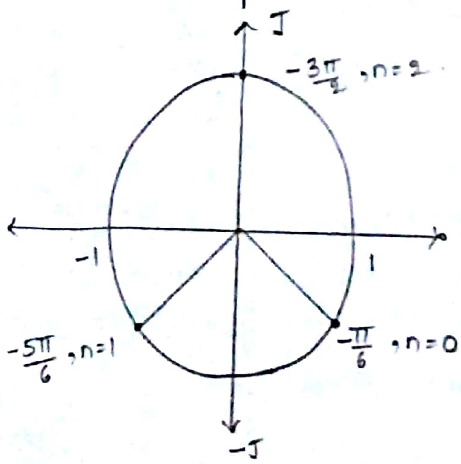


fig :- The roots of $(-j)$

Thus, the values of the four propagation constants γ are given by

$$\gamma_1 = -\beta_e c \frac{\sqrt{3}}{2} + j\beta_e \left(1+\frac{c}{2}\right).$$

$$\gamma_2 = \beta_e c \frac{\sqrt{3}}{2} + j\beta_e \left(1+\frac{c}{2}\right).$$

$$\gamma_3 = j\beta_e (1-c).$$

$$\gamma_4 = -j\beta_e \left(1-\frac{c^3}{4}\right).$$

The above four equations represent four different modes of wave propagation in the O-type helical travelling-wave tube.

M-type tubes :-

Crossed field tubes are referred to as m-type tubes, which deal with the propagation of waves in a magnetic field. In Crossed field tubes both static electric and magnetic fields are present and they are perpendicular to each other. The electron motion takes place in are where the fields are perpendicular to each other.

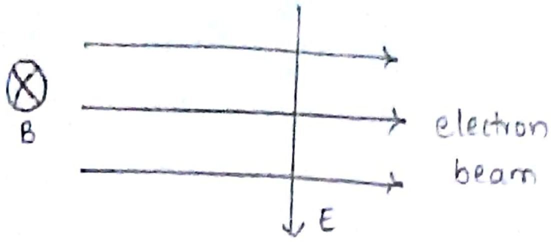
Crossed-field effects :-

If both electric and magnetic fields are present, motion of electrons depends on the Orientation of electric and magnetic fields.

(a) If electric and magnetic fields are in the same direction or the opposite direction, the magnetic field exects no force on electrons.

(b) If electric and magnetic fields perpendicular to each other, electron motion depends on both electric and magnetic fields, this type of field is called cross-field.

In crossed-field tubes, the electrons emitted by the cathode are accelerated by the electric field, and the motion of electrons is perpendicular to both fields as is indicated in figure.



(a) Cross-field tubes

(b) Linear Beam Tubes.

The presence of cross field interactions makes the electrons to give up some of its energy to the RF field. only those electrons which have given sufficient energy to the RF field can only be elegible to travel to the anode end.

Magnetrons :-

The magnetrons is a crossed field device, in which electric field and magnetic field are produced in a direction perpendicular to each other, in a way to cross each other therefore, the flow of electrons is perpendicular to both the fields. In magnetrons anode and cathode are concentric and cylindrical type structures.

Types of magnetrons :-

There are three basic types of magnetrons.

1. cyclotron - frequency magnetrons.
2. Negative - resistance (split-anode) magnetrons.
3. Cavity - type magnetrons.

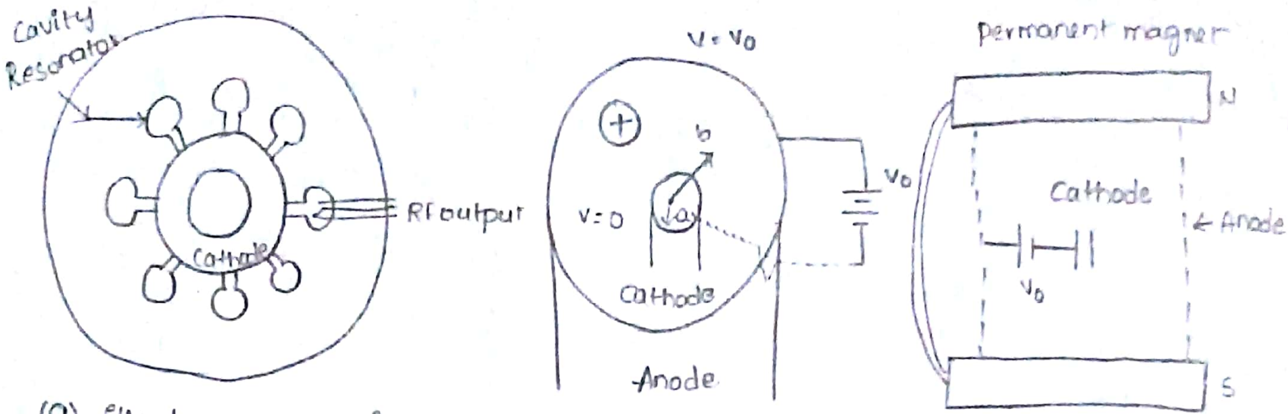
Cyclotron - frequency magnetrons :- Its principle of working is based on the synchronization btw orbiting electrons in a magnetic field and a resonant circuit that is tuned to the cyclotron frequency. In this magnetron the ac component of electric field and the oscillations of electrons are parallel to the field.

Negative - resistance magnetrons :- It uses the static negative resistance between two anode segments. In this operation when both segments are at the same potential. the magnetic field effects can only be sufficient to keep flow of electrons to reach anode.

Traveling - wave magnetrons :- These magnetrons provide oscillation of high peak power and peak power capability that is increased by about an order of magnitude to 100 kW. Since the efficiency is very low in the first two types. they are not dealt in this chapter. In general, travelling wave magnetrons uses cavity resonators.

8- Cavity cylindrical magnetron :-

Cavity magnetrons is high power microwave oscillator with high efficiency. The operating principle of this device is interaction of electrons with the perpendicularly oriented electric and magnetic fields. An 8- cavity cylindrical magnetron is shown in figure.



(a) Structure of a cavity magnetron.

(b) Cylindrical Configuration

(c) magnetic field.

The heated cathode is a source of electrons in a magnetron. the cavity magnetron consists of 8 cavity that are tightly coupled to each other.

$$\phi_v = \frac{2\pi n}{N}$$

$$n = 0, \pm 1, \pm 2, \ldots, \pm \left[\frac{N}{2} - 1\right], \pm \frac{N}{2}.$$

That is, N/2 mode of resonance can exist only in resonator systems that have an even number of resonators. If $n = N/2$, $\phi_v = \pi$. since the phase angle of π radians is in the N/2 mode, this mode of resonance is called the π-mode.

Electron trajectories at various magnetic field :- Comparing the magnitude of electric and magnetic fields, we can understand the trajectory of an electron coming from cathode, moving towards anode takes different path through the interaction space. electron trajectories at various magnetic fields V_0 are present.

(a) If $B = 0$, electrons emitted from the cathode move along the radical direction.

Scanned by CamScanner

Conversely, the cut-off voltage is given by

$$V_c = \frac{e}{8m} B^2 b^2 \left(1 - \frac{a^2}{b^2}\right)^2.$$

Hull cut-off voltage equation :-

A cavity cylindrical magnetron is the most commonly used magnetron, because for a cross-field device the electric and magnetic fields are perpendicular to each other and the path of the electrons in the presence of this cross-field is naturally parabolic. The eqn for the hull cut-off voltage is given by

$$V_c = \frac{e}{8m} B^2 b^2 \left(1 - \frac{a^2}{b^2}\right)^2.$$

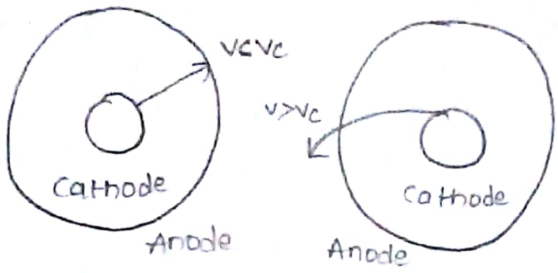
where, B = magnetic flux density.
a = Cathode radius.
b = anode radius.
e = charge of the electron.
m = mass of the electron.

Derivation of Hull Cut-off voltage equation :-

The Hull-cut-off condition is obtained, under the condition that there is no RF field, which in turn defines anode voltage is function of magnetic field.

Here, we will discuss the Hull cut-off voltage equation force acting on the electron is

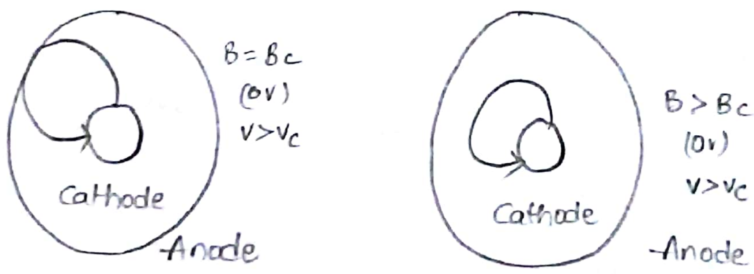
$$F = Bev$$



(a) No magnetic field (b) Small magnetic field

(b) when a small B is applied (at a perticular to radical electric field), electron trajectories bend and follow a curved path.

(c) The magnetic field required to return electrons to the cathode while just grazing the surface of the anode is called the critical magnetic field. (B_c) and is also known as the cut-off magnetic field. under this condition, the motion of electrons is shown in figure.



(a) magnetic field = Bc (b) magnetic field > Bc

(d) If the magnetic field is made larger than the critical field ($B>B_c$), the electrons travel with a greater velocity and may return to the cathode quite faster.

→ The eqn of the cut-off magnetic field is given by

$$B_c = \frac{(8V_0 m/e)^{1/2}}{b\left(1-\frac{a^2}{b^2}\right)}$$

In the direction of ϕ, the force component is given by

$$F\phi = eBv_p.$$

where, v_p = velocity in the direction of the radical distance ρ, from the center of the cathode cylinder.

Torque in direction of ϕ can be given as

$$T\phi = \rho F\phi = e.\rho.v_\rho.B. \rightarrow (1)$$

Angular momentum = angular velocity x moment of Inertia.

$$= \frac{d\phi}{dt} \times m\rho^2. \rightarrow (2)$$

Time rate of angular momentum = $\frac{d}{dt}\left[\frac{d\phi}{dt} \times m\rho^2\right]. \rightarrow (3)$

This gives the torque in ϕ direction. equating (3) and (1).

$$\frac{d}{dt}\left[\frac{d\phi}{dt} \times m\rho^2\right] = e.\rho\, v_\rho.B.$$

That is, $2m\rho \frac{d\phi}{dt} + m\rho^2 \frac{d^2\phi}{dt^2} = e.\rho.\, v_\rho.B \rightarrow (4)$

W.K.T

$$v_\rho = \frac{d\rho}{dt}.$$

Therefore eqn (4) becomes.

$$2m\rho \frac{d\phi}{dt} + m\rho^2 \frac{d^2\phi}{dt^2} = eB.\rho.\frac{d\rho}{dt}. \rightarrow (5)$$

Integrating eqn (5) with regard to 't' we will get

$$2m\rho.\phi + m\rho^2 \frac{d\phi}{dt} = e.B.\frac{\rho^2}{2}.$$

For a particular direction, $m.\rho.\phi$. can be considered a constant.

$$m\rho^2 \frac{d\phi}{dt} + C = e \cdot B \cdot \frac{\rho^2}{2} \rightarrow (6)$$

The value of C can be determined by applying boundary conditions.

$$0 + C = \frac{e \cdot B \cdot a^2}{2} \quad \text{(or)} \quad C = \frac{eBa^2}{2}$$

Substituting the above value of C in eq (6), we get

$$m\rho^2 \frac{d\phi}{dt} = \frac{eB}{2}(\rho^2 - a^2)$$

(or)

$$\frac{d\phi}{dt} = \frac{eB}{2m}\left(1 - \frac{a^2}{\rho^2}\right). \rightarrow (7)$$

When $\rho = a$ (i.e., at Cathode), $\frac{d\phi}{dt}$ approches 0.

When $\rho >> a$, $\frac{d\phi}{dt}$ approches $(\omega)_{max}$.

$$\left(\frac{d\phi}{dt}\right)_{max} = (\omega)_{max} = \frac{eB}{2m} = \frac{eB_c}{2m}. \rightarrow (8)$$

where, $B = B_c$ is the cut-off magnetic flux density.

We know that the potential energy of electron = Kinetic energy of electrons.

That is $\quad eV_0 = \frac{1}{2} mv^2$.

$$eV_0 = \frac{m}{2}(V_\rho^2 + V_\phi^2) \rightarrow (9)$$

where $V_\rho = \frac{d\rho}{dt}$ and $V_\phi = \rho \frac{d\phi}{dt}$.

Rewriting the equation (substituting V_ρ and V_ϕ) eqn(

$$eV_0 = \frac{m}{2}\left[\left(\frac{d\rho}{dt}\right)^2 + \rho^2\left(\frac{d\phi}{dt}\right)^2\right]$$

From eqn (7) and (8).

$$\left[\frac{d\phi}{dt}\right] = (\omega)_{max}\left(1-\frac{a^2}{\rho^2}\right)$$

$$eV_0 = \frac{m}{2}\left[\left(\frac{d\rho}{dt}\right)^2 + \rho^2(\omega)^2_{max}\left(1-\frac{a^2}{\rho^2}\right)^2\right].$$

→ At anode $\rho = b$, $\frac{d\rho}{dt} = 0$, substituting these boundary Conditions in the above equation.

$$\frac{m}{2}\left[b^2(\omega)^2_{max}\left(1-\frac{a^2}{b^2}\right)^2\right] = eV_0 \rightarrow (10)$$

Substituting eqn (8) in eqn (10) we get.

$$\frac{m}{2}b^2\left(\frac{eB_c}{2m}\right)^2 \times \left(1-\frac{a^2}{b^2}\right)^2 = eV_0.$$

$$\frac{e^2 B_c^2 b^2}{8m}\left(1-\frac{a^2}{b^2}\right)^2 = eV_0.$$

$$B_c = \frac{(8V_0 m/e)^{1/2}}{b\left(1-\frac{a^2}{b^2}\right)} \rightarrow (11)$$

i.e., for a given V_0, the electrons will not reach at anode, if $B > B_c$.

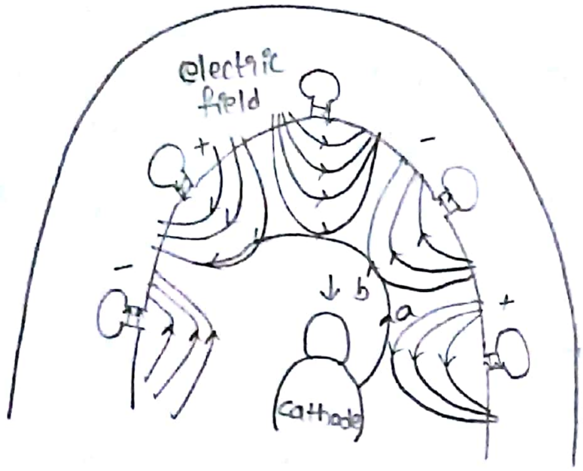
On the other hand, the cut-off voltage is given by

$$V_c = \frac{e}{8m}B^2 b^2\left(1-\frac{a^2}{b^2}\right)^2. \rightarrow (12)$$

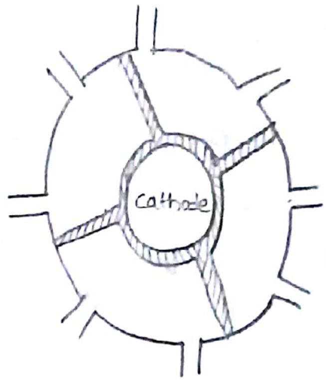
→ It can be observed that for a given B, the electrons will not reach at anode, if $V_0 < V_c$. eqn (12) Called the Hull cut-off voltage equation.

Modes of Resonance and π-mode operation :-

We have discussed the effect of electric and magnetic fields in the previous section when no RF field is applied. Let us assume RF oscillations are initiated and are maintained sustainably and assume that these oscillations are created by some noise which is transient in the magnetrons.



(a) magnetron Operation in π mode.



(b) electron cloud showing spokes

The electron 'a' that is entering the interaction space during the decelerating field gives some of it's energy to the RF field. Therefore, its velocity decreases and it spends more time in interaction space during its long Journey. In the same way, the electrons that are emitted a little later to be in the correct position move faster and try to catch up with electron "a".

The electron 'b' which is introduced during accelerated RF field takes energy from the oscillators. This results in increased velocity of electrons. Since the velocity is increased, the trajectory path of an the cathode early.

Separation of π-mode :-

Modes of Operation :- The resonant circuit that is used in cavity resonators acts similar to an LC tank ckt. If two resonant circuits are coupled, they produce two different resonant frequencies. In general, if resonant ckt are coupled together, they produce 'n' different and district resonant frequencies.

Strapping :- Keeping magnetron operations in the π mode is difficult; unless special means are employed. Strapping is one method that is used. Strapping means to connect alterate anode plates with two conducting rings of heavy gauge touching the anode's poles at the dots as shown in figure.

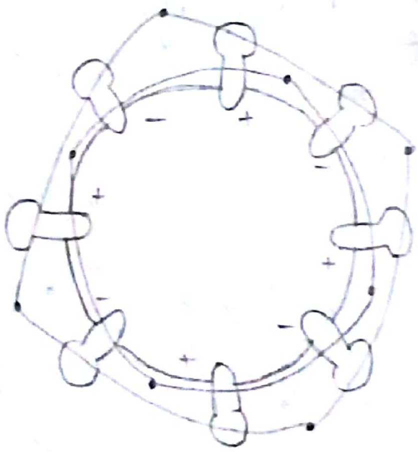


fig:- Strapping of magnetrons.

Disadvantages of Strapping :-

→ Strapping may cause power losses in the conducting rings.

→ Strapped resonators are very difficult.

→ As the number of cavities increase (16 or 32), strapping has no effect on mode Jumping.

A magnetron that needs no strapping is the rising sun magnetron and is shown in figure.

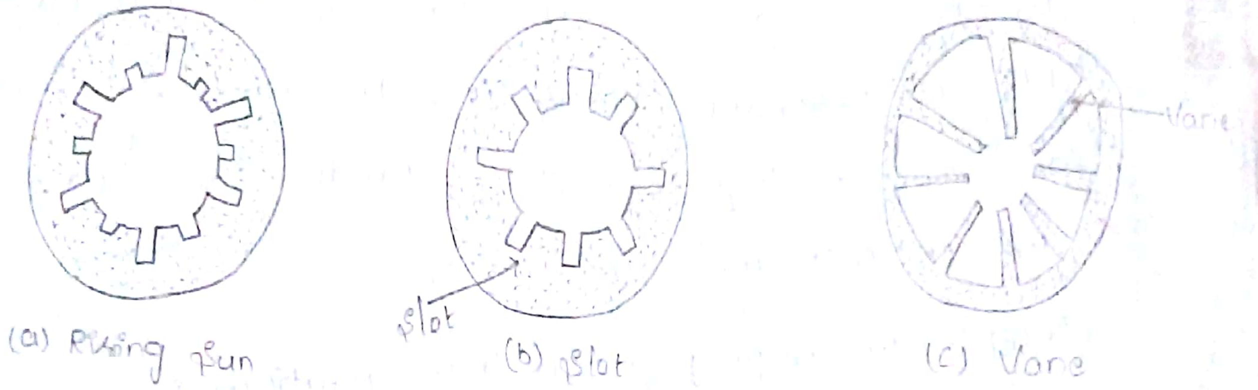


fig:- Traveling-wave magnetron resonators.

Frequency pushing and pulling :- Similar to reflex klystron, it is possible to change the resonant frequency of magne-trons by changing the anod voltage, which results in a change in the orbital velocity of electrons.

8-cavity cylindrical magnetron :-

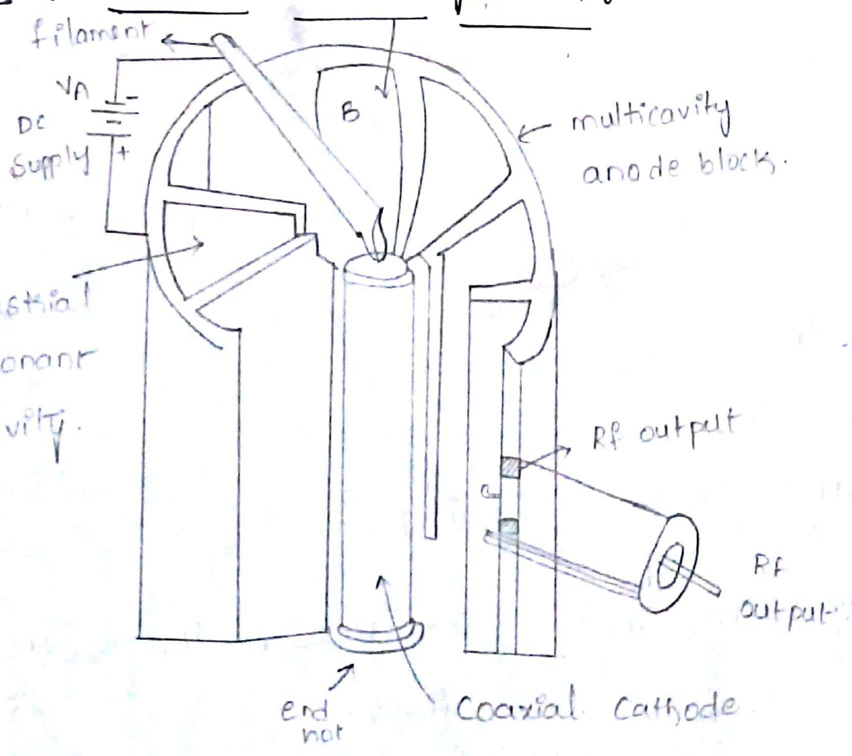


fig:- 8-cavity magnetron.

27/04/21 UNIT–II Circular Waveguides

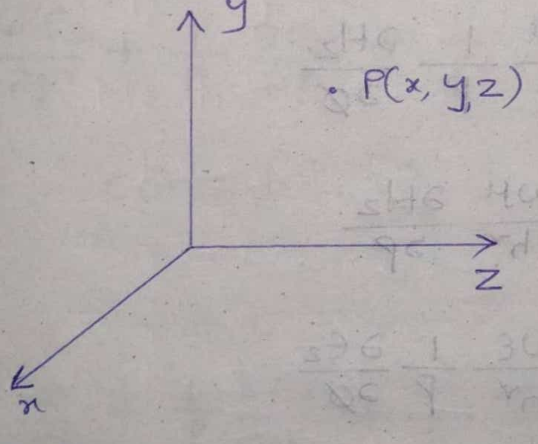
Why we have moved towards Circular waveguides from rectangular Waveguides ??

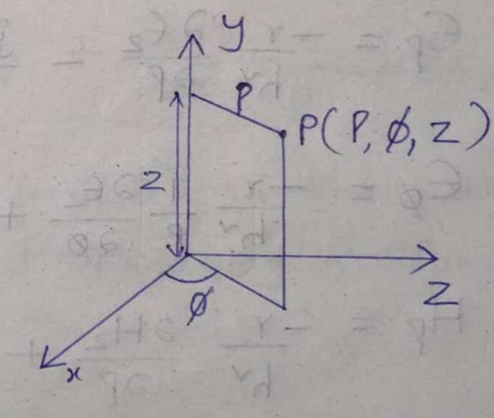
→ This is because of mode of Propagation. Different types of modes are possible through Circular Waveguides compared to Rectangular Waveguides.

→ The highest possible bandwidth allowing only a single mode to propagate with Circular Waveguides is only 1.360:1.

→ Rectangular Waveguides have a much larger bandwidth over which only a single mode can propagate.

Rectangular Waveguides	Circular Waveguides
* "Cartesian" Coordinate System is Used.	* "Cylindrical" Coordinate System is Used.





Rectangular Waveguides — Ranges:-

$x \rightarrow -\infty$ to $+\infty$

$y \rightarrow -\infty$ to $+\infty$

$z \rightarrow -\infty$ to $+\infty$

$dl = dx + dy + dz$

$ds = dy\,dz\,a_x$

$ds = dx\,dz\,a_y$

$ds = dx\,dy\,a_z$

$dv = dx\,dy\,dz$

Circular Waveguides — Ranges:-

$\rho \rightarrow 0$ to ∞

$\phi \rightarrow 0$ to 2π

$z \rightarrow -\infty$ to $+\infty$

$dl = d\rho\, a_\rho + \rho\, d\phi\, a_\phi + dz\, a_z$

$ds = \rho\, d\phi\, dz\, a_\rho$

$ds = d\rho\, dz\, a_\phi$

$ds = d\rho\, \rho\, d\phi\, a_z$

$dv = \rho\, d\rho\, d\phi\, dz$

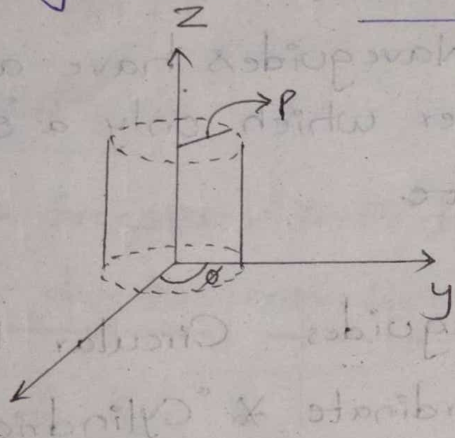
del operator:-

$$\nabla = \frac{\partial}{\partial \rho} a_\rho + \frac{1}{\rho}\frac{\partial}{\partial \phi} a_\phi + \frac{\partial}{\partial z} a_z \left(\frac{1}{m} \text{ or } m^{-1}\right)$$

Circular waveguide:-

A circular waveguide is a tubular conductor for transmitting a microwave signal.

Consider, a circular waveguide of inner radius 'ρ'. Suppose that it is varying over 'ϕ' which ranges from 0 to 2π.



Field expressions:-

$$E_\rho = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial \rho} - \frac{j\omega\mu}{h^2}\frac{1}{\rho}\frac{\partial H_z}{\partial \phi}$$

$$E_\phi = \frac{-\gamma}{h^2}\frac{1}{\rho}\frac{\partial E_z}{\partial \phi} + \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial \rho}$$

$$H_\rho = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial \rho} + \frac{j\omega\varepsilon}{h^2}\frac{1}{\rho}\frac{\partial E_z}{\partial \phi}$$

$$H_\phi = \frac{-\gamma}{h^2}\frac{1}{\rho}\frac{\partial H_z}{\partial \phi} - \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial \rho}$$

To obtain the above expression, consider the Maxwell's equations,

$$\nabla \times \bar{E} = -\dot{B}$$

$$\nabla \times \bar{H} = J + \dot{D}$$

Simplify them as you solved earlier in Rectangular waveguides.

Propagation of TM wave in a Circular Waveguide

TM wave:- A Wave whose magnetic field component is zero in the direction of propagation but with non-zero electric field component is referred to as "Transverse Magnetic wave".

i.e., $H_z = 0$ but $E_z \neq 0$

Consider, Helmholtz wave equations

$$\nabla^2 E_z = -\omega^2 \mu \varepsilon E_z \rightarrow (1)$$
$$\nabla^2 H_z = -\omega^2 \mu \varepsilon H_z \rightarrow (2)$$

For a TM wave, $H_z = 0$. Now, from eqn's - (1) & (2)

$$\nabla^2 E_z = -\omega^2 \mu \varepsilon E_z$$
$$\nabla^2 H_z = 0$$

Hence, our required wave equation is,

$$\boxed{\nabla^2 E_z = -\omega^2 \mu \varepsilon E_z} \rightarrow (3)$$

$$\Rightarrow \frac{\partial^2 E_z}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial E_z}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 E_z}{\partial \phi^2} + \frac{\partial^2 E_z}{\partial z^2} = -\omega^2 \mu \varepsilon E_z$$

Consider, $\frac{\partial}{\partial z} = -\gamma$ (indicating that wave is propagating in forward z-direction).

$$\Rightarrow \frac{\partial^2 E_z}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial E_z}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 E_z}{\partial \phi^2} + \gamma^2 E_z = -\omega^2 \mu \varepsilon E_z$$

$$\Rightarrow \frac{\partial^2 E_z}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial E_z}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 E_z}{\partial \phi^2} + \gamma^2 E_z + \omega^2 \mu \varepsilon E_z = 0$$

$$\Rightarrow \frac{\partial^2 E_z}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial E_z}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 E_z}{\partial \phi^2} + (\gamma^2 + \omega^2 \mu \varepsilon) E_z = 0$$

$$\Rightarrow \frac{\partial^2 E_z}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial E_z}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 E_z}{\partial \phi^2} + h^2 E_z = 0 \rightarrow (4)$$

Using variable- separable method

Let, $E_z = PQ$

Here, P is a pure function of ρ

Q is a pure function of 'ϕ'

from eqn-(4);

$$\frac{\partial^2 (PQ)}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial (PQ)}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 (PQ)}{\partial \phi^2} + h^2(PQ) = 0$$

$$\Rightarrow Q\frac{\partial^2 P}{\partial \rho^2} + \frac{Q}{\rho}\frac{\partial P}{\partial \rho} + \frac{P}{\rho^2}\frac{\partial^2 Q}{\partial \phi^2} + h^2(PQ) = 0$$

divide the entire expression with 'PQ'

$$\frac{1}{P}\frac{\partial^2 P}{\partial \rho^2} + \frac{1}{\rho P}\frac{\partial P}{\partial \rho} + \frac{1}{\rho^2 Q}\frac{\partial^2 Q}{\partial \phi^2} + h^2 = 0$$

multiply the entire expression with "ρ^2" (row squ

$$\frac{\rho^2}{P}\frac{\partial^2 P}{\partial \rho^2} + \frac{\rho}{P}\frac{\partial P}{\partial \rho} + \frac{1}{Q}\frac{\partial^2 Q}{\partial \phi^2} + h^2\rho^2 = 0 \longrightarrow (5)$$

Let, $\frac{1}{Q}\cdot\frac{\partial^2 Q}{\partial \phi^2} = -n^2$

Solution for 'Q' is given by,

$$\boxed{Q = A_n \cos n\phi + B_n \sin n\phi}$$ ($A_n, B_n \to$ constants)

Consider, $\frac{\rho^2}{P}\cdot\frac{\partial^2 P}{\partial \rho^2} + \frac{\rho}{P}\frac{\partial P}{\partial \rho} + (\rho^2 h^2 - n^2) = 0$

multiply the above equation with 'P'

$$\Rightarrow \rho^2\cdot\frac{\partial^2 P}{\partial \rho^2} + \rho\frac{\partial P}{\partial \rho} + \rho^2 P h^2 - n^2 P = 0$$

$\Downarrow$

$\left(x^2\frac{\partial^2 y}{\partial x^2} + x\frac{\partial y}{\partial x} + (x^2 - n^2)y = 0\right) \to$ (Bessel function)

Solution for Bessel function:-

$$y = c_n J_n(x)$$

$J_n(x) \rightarrow$ infinite no. of roots exist for the given Bessel function (i.e., acts as an oscillator)

$c_n \rightarrow$ constant value

$$\Rightarrow (\rho h)^2 \frac{\partial^2 P}{\partial^2 (\rho h)^2} + \rho h . \frac{\partial P}{\partial \rho} + \underbrace{(\rho h)^2 P - n^2 P}_{\rightarrow (\rho^2 h^2 - n^2) P} = 0$$

(Converted into Bessel function format)

$$\Rightarrow \boxed{P = c_n J_n(\rho h)} \rightarrow (6)$$

$$E_z = PQ$$

$$\Rightarrow E_z = c_n J_n(\rho h) \left[A_n \cos(n\phi) + B_n \sin(n\phi)\right]$$

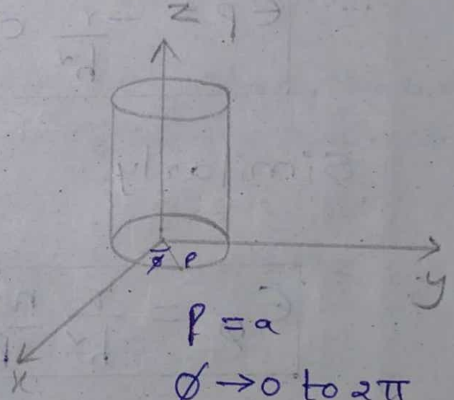
$$\Rightarrow E_z = c_n J_n(\rho h) \underbrace{\sqrt{A_n^2 + B_n^2}}_{\text{(magnitude format)}} \Big[\cos\underbrace{\left(n\phi + \tan^{-1}\left(\tfrac{B_n}{A_n}\right)\right)}_{n\phi'} \Big]$$

(Angle format)

$$\Rightarrow \boxed{E_z = c_0 J_n(\rho h) \cos(n\phi')} \rightarrow (7) \quad \left(\because c_n \sqrt{A_n^2 + B_n^2} = c_0\right)$$

Boundary conditions:-

$$E_z = 0$$



$\rho = a$

$\phi \rightarrow 0$ to 2π

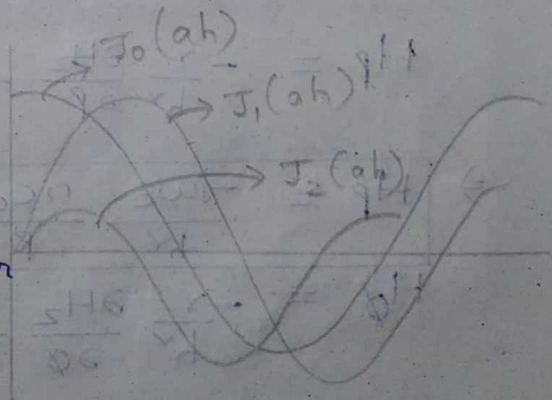
from (7):

$$0 = c_0 J_n(ah) \cos(n\phi')$$

$$\Rightarrow c_0 J_n(ah) = 0$$

$$\Rightarrow J_n(ah) = 0$$

$$\Rightarrow ah = P_{nm}$$



$P_{nm} \rightarrow$ sol^n for Bessel function

$$\Rightarrow \boxed{h = \frac{P_{nm}}{a}}$$

Now,

$$E_z = C_0 J_n\left(\rho \frac{P_{nm}}{a}\right)\cos(n\phi')$$

n\m	1	2	3
0	2.4	5.5	8.6
1	3.82	7.1	1.1
2	5.13	8.4	11.6
3	6.3	9.76	13.06

Field Expressions:-

$$E_\rho = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial \rho} - \frac{j\omega\mu}{h^2}\frac{1}{\rho}\frac{\partial H_z}{\partial \phi}$$

but $H_z = 0$

$$\Rightarrow E_\rho = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial \rho} - \frac{j\omega\mu}{h^2}\frac{1}{\rho}(0)$$

$$\Rightarrow E_\rho = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial \rho}$$

$$\Rightarrow E_\rho = \frac{-\gamma}{h^2}\frac{\partial}{\partial \rho}\left[C_0 J_n\left(\rho \frac{P_{nm}}{a}\right)\cos(n\phi')\right]$$

$$\Rightarrow E_\rho = \frac{-\gamma}{h^2} C_0\left(\frac{P_{nm}}{a}\right)J_n'\left(\rho \frac{P_{nm}}{a}\right)\cos(n\phi')$$

$$\therefore \boxed{E_\rho = \frac{-\gamma}{h^2} C_0\left(\frac{P_{nm}}{a}\right)J_n'\left(\rho \frac{P_{nm}}{a}\right)\cos(n\phi')}$$

Similarly,

$$\boxed{E_\phi = \frac{\gamma}{h^2}\frac{nC_0}{\rho} J_n\left(\rho \frac{P_{nm}}{a}\right)\sin(n\phi')}$$

$$H_\rho = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial \rho} + \frac{j\omega\varepsilon}{h^2}\frac{1}{\rho}\frac{\partial E_z}{\partial \phi} = \frac{j\omega\varepsilon}{h^2}\frac{1}{\rho}\frac{\partial E_z}{\partial \phi}$$

$$\Rightarrow \boxed{H_\rho = \frac{-j\omega\varepsilon}{h^2}\cdot\frac{nC_0}{\rho} J_n\left(\rho \frac{P_{nm}}{a}\right)\sin(n\phi')}$$

$$H_\phi = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial \phi}\cdot\frac{1}{\rho} - \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial \rho} = \frac{-j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial \rho}$$

$$\Rightarrow \boxed{H_\phi = \frac{-j\omega\varepsilon}{h^2} C_0 \frac{P_{nm}}{a} J_n'\left(\rho \frac{P_{nm}}{a}\right)\cos(n\phi')}$$

Propagation of TE Wave in Circular Waveguide:-

TE Wave:- A wave whose electric field component is zero in the direction of propagation but with non-zero magnetic field component is referred to as "Transverse Electric wave".

i.e., $E_z = 0$ and $H_z \neq 0$

Consider helmholtz wave equations

$$\nabla^2 E_z = -\omega^2 \mu \varepsilon E_z \rightarrow (1)$$

$$\nabla^2 H_z = -\omega^2 \mu \varepsilon H_z \rightarrow (2)$$

For a TE wave, $E_z = 0$. Now from eqn's-(1)&(2)

$$\nabla^2 E_z = 0$$

$$\nabla^2 H_z = -\omega^2 \mu \varepsilon H_z$$

Hence, our required wave equation is,

$$\boxed{\nabla^2 H_z = -\omega^2 \mu \varepsilon H_z} \rightarrow (3)$$

$$\Rightarrow \frac{\partial^2 H_z}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial H_z}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 H_z}{\partial \phi^2} + \frac{\partial^2 H_z}{\partial z^2} = -\omega^2 \mu \varepsilon H_z$$

Consider, $\frac{\partial}{\partial z} = -\gamma$ (indicating that the wave is in forward z-direction

$$\Rightarrow \frac{\partial^2 H_z}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial H_z}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 H_z}{\partial \phi^2} + \gamma^2 H_z = -\omega^2 \mu \varepsilon H_z$$

$$\Rightarrow \frac{\partial^2 H_z}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial H_z}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 H_z}{\partial \phi^2} + \gamma^2 H_z + \omega^2 \mu \varepsilon H_z = 0$$

$$\Rightarrow \frac{\partial^2 H_z}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial H_z}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 H_z}{\partial \phi^2} + (\gamma^2 + \omega^2 \mu \varepsilon) H_z = 0$$

$$\Rightarrow \frac{\partial^2 H_z}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial H_z}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 H_z}{\partial \phi^2} + h^2 H_z = 0 \rightarrow (4)$$

Using variable-separable method

Let, $H_z = PQ$

Here, P is a Pure function of 'P'

Q is a Pure function of 'Ø'

from eqⁿ-4

$$\frac{\partial^2(PQ)}{\partial P^2} + \frac{1}{P}\frac{\partial(PQ)}{\partial P} + \frac{1}{P^2}\frac{\partial^2(PQ)}{\partial \phi^2} + h^2(PQ) = 0$$

$$\Rightarrow Q\cdot\frac{\partial^2 P}{\partial P^2} + \frac{Q}{P}\frac{\partial P}{\partial P} + \frac{Q}{P^2}\frac{\partial^2 P}{\partial \phi^2} + h^2(PQ) = 0$$

Divide the entire expression with "PQ"

$$\frac{1}{P}\cdot\frac{\partial^2 P}{\partial P^2} + \frac{1}{PP}\frac{\partial P}{\partial P} + \frac{1}{P^2 Q}\cdot\frac{\partial^2 Q}{\partial \phi^2} + h^2 = 0$$

Multiply the entire expression with "P^2"

$$\frac{P^2}{P}\cdot\frac{\partial^2 P}{\partial P^2} + \frac{P}{P}\cdot\frac{\partial P}{\partial P} + \frac{1}{Q}\cdot\frac{\partial^2 Q}{\partial \phi^2} + P^2 h^2 = 0 \longrightarrow (5)$$

Let, $\frac{1}{Q}\cdot\frac{\partial^2 \phi}{\partial \phi^2} = -n^2$

Solution for 'Q' is given by,

$$\boxed{Q = A_n \cos(n\phi) + B_n \sin(n\phi)}$$

from (5);

$$\frac{P^2}{P}\cdot\frac{\partial^2 P}{\partial P^2} + \frac{P}{P}\cdot\frac{\partial P}{\partial P} + (P^2h^2 - n^2) = 0$$

multiply above expression with "P"

$$P^2\cdot\frac{\partial^2 P}{\partial P^2} + P\frac{\partial P}{\partial P} + (P^2h^2 - n^2)P = 0$$

$$\Rightarrow (Ph)^2\frac{\partial^2 P}{\partial (Ph)^2} + (Ph)\frac{\partial P}{\partial (Ph)} + (P^2h^2 - n^2)(P) = 0$$

$$x^2\frac{\partial^2 y}{\partial x^2} + x\frac{\partial y}{\partial x} + (x^2 - n^2)y = 0 \Rightarrow \text{(Bessel function)}$$

Solution:-

$y = c_n J_n(x)$

$\therefore \boxed{P = c_n J_n(\rho h)} \rightarrow ⑥$

We have, $H_z = PQ$

$\Rightarrow H_z = c_n J_n(\rho h)\left[A_n \cos(n\phi) + B_n \sin(n\phi)\right]$

$\Rightarrow H_z = c_n J_n(\rho h)\sqrt{A_n^2 + B_n^2}\left[\cos\left(n\phi + \tan^{-1}\left(\frac{B_n}{A_n}\right)\right)\right]$

$\Rightarrow \boxed{H_z = c_0 J_n(\rho h)\cos(n\phi')} \rightarrow ⑦$

Boundary Conditions:-

$\bar{E}$ may lie either in ρ-direction/ϕ-direction/z-direction. $E_z = 0$ from the definition of TE wave. $\bar{E}$ doesnot vary with ρ and hence $E_\rho = 0$. Therefore, $\bar{E}$ lies in the direction of ϕ.

Consider, $E_\phi = 0$, $\rho = a$ and ϕ ranges from 0 to 2π.

Substituting the boundary conditions in eqn-⑧ we get,

$$E_\phi = \frac{-\gamma}{h^2}\frac{1}{\rho}\frac{\partial E_z}{\partial \phi} + \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial \rho} \rightarrow ⑧$$

$$\Rightarrow 0 = \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial \rho} \qquad (\because E_z = 0)$$

$$\Rightarrow 0 = \frac{j\omega\mu}{h^2} \cdot \frac{\partial}{\partial \rho}\left[c_0 J_n(\rho h)\cos(n\phi')\right]$$

$$\Rightarrow 0 = \frac{j\omega\mu}{h^2} \cdot \frac{\partial}{\partial \rho}\left[c_0 J_n(ah)\cos(n\phi')\right]$$

$$\Rightarrow 0 = \frac{j\omega\mu}{h^2} \cdot c_0 h J_n'(ah)\underbrace{\cos(n\phi')}_{\neq 0}$$

Hence, $\frac{j\omega\mu}{h^2} c_0 h J_n'(ah) = 0$

$$\Rightarrow J_n'(ah) = 0$$ (J_n' acts as an oscillator)

$$\Rightarrow ah = P'_{nm}$$

$$\Rightarrow \boxed{h = \frac{P'_{nm}}{a}}$$

n \ m	1	2	3
0	3.8	7.01	10.1
1	1.8	5.8	8.5
2	4.20	8.07	9.9

$$\therefore \boxed{H_z = C_0 J_n\left(\rho \frac{P'_{nm}}{a}\right) \cos(n\phi')}$$

Field expressions:-

$$E_\rho = -\frac{\gamma}{h^2}\frac{\partial E_z}{\partial \rho} - \frac{j\omega\mu}{h^2}\frac{1}{\rho}\frac{\partial H_z}{\partial \phi}$$

$$= -\frac{j\omega\mu}{h^2}\cdot\frac{1}{\rho}\cdot\frac{\partial H_z}{\partial \phi}$$ ($\because E_z = 0$ for TE wave)

$$= -\frac{j\omega\mu}{h^2}\cdot\frac{1}{\rho}\frac{\partial}{\partial\phi}\left[C_0 J_n\left(\rho\frac{P'_{nm}}{a}\right)\cos(n\phi')\right]$$

$$\Rightarrow \boxed{E_\rho = \frac{j\omega\mu}{h^2}\frac{nC_0}{\rho} J_n\left(\rho\frac{P'_{nm}}{a}\right)\sin(n\phi')}$$

$$E_\phi = -\frac{\gamma}{h^2}\frac{1}{\rho}\cdot\frac{\partial E_z}{\partial \phi} + \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial \rho}$$

$$= \frac{j\omega\mu}{h^2}\cdot\frac{\partial H_z}{\partial \rho}$$

$$\boxed{E_\phi = \frac{j\omega\mu}{h^2} C_0 \frac{P'_{nm}}{a} J_n'\left(\rho\frac{P'_{nm}}{a}\right)\cos(n\phi')}$$

$$H_\rho = -\frac{\gamma}{h^2}\frac{\partial H_z}{\partial \rho} + \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial \phi}\cdot\frac{1}{\rho}$$

$$= -\frac{\gamma}{h^2}\frac{\partial H_z}{\partial \rho}$$

$$\Rightarrow \boxed{H_\rho = -\frac{\gamma}{h^2} C_0 \frac{P'_{nm}}{a} J_n'\left(\rho\frac{P'_{nm}}{a}\right)\cos(n\phi')}$$

$$H_\phi = -\frac{\gamma}{h^2}\frac{1}{\rho}\frac{\partial H_z}{\partial \phi} - \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial \rho} = -\frac{\gamma}{h^2}\frac{1}{\rho}\frac{\partial H_z}{\partial \phi}$$

$$\Rightarrow \boxed{H_\phi = \frac{\gamma}{h^2}\frac{nC_0}{\rho} J_n\left(\rho\frac{P'_{nm}}{a}\right)\sin(n\phi')}$$

Characteristics of Circular Waveguides:-

1. Cut-off frequency (f_c):-

It is defined as "the frequency at which the Propagation constant (γ) of a Circular waveguide becomes zero".

We know that

$$h^2 = \gamma^2 + \omega^2 \mu \varepsilon \rightarrow (1)$$

$$\Rightarrow \gamma^2 = h^2 - \omega^2 \mu \varepsilon$$

At f_c, $\gamma = 0$ and $\omega = \omega_c$

$$\Rightarrow 0 = h^2 - \omega_c^2 \mu \varepsilon$$

$$\Rightarrow \omega_c^2 \mu \varepsilon = h^2$$

$$\Rightarrow h = \omega_c \sqrt{\mu \varepsilon} \rightarrow (2)$$

for TE mode, $h = \frac{P'_{nm}}{a}$

for TM mode, $h = \frac{P_{nm}}{a}$

TE mode:-

$$h = \omega_c \sqrt{\mu \varepsilon} \quad \text{(from (2))}$$

$$\Rightarrow \frac{P'_{nm}}{a} = \frac{2\pi f_c}{c} \quad \left(\because c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}\right)$$

$$\Rightarrow f_c = \frac{P'_{nm}}{2\pi a} \cdot c$$

$$\therefore \boxed{f_c = \frac{P'_{nm}}{2\pi a} c}$$

TM mode:-

$$h = \omega_c \sqrt{\mu \varepsilon}$$

$$\Rightarrow \frac{P_{nm}}{a} = \frac{2\pi f_c}{c}$$

$$\Rightarrow f_c = \frac{P_{nm}}{2\pi a} \cdot c$$

$$\therefore \boxed{f_c = \frac{P_{nm}}{2\pi a} \cdot c}$$

② Cut-off Wavelength (λ_c):-

It is defined as "the wavelength at which the Propagation constant (γ) of a circular waveguide becomes zero".

$$\lambda_c = \frac{c}{f_c} = \frac{c}{\frac{P'_{nm}}{2\pi a}\cdot c} = \frac{2\pi a}{P'_{nm}} \quad \text{(TE mode)}$$

$$\lambda_c = \frac{c}{f_c} = \frac{c}{\frac{P_{nm}}{2\pi a}\cdot c} = \frac{2\pi a}{P_{nm}} \quad \text{(TM mode)}$$

③ Guided wavelength (λ_g):-

It is defined as "the distance travelled by a wave, to produce a phase shift of 360° (or) 2π radians".

$$\lambda_g = \frac{2\pi}{\beta}$$

$$= \frac{\lambda_0}{\sqrt{1-(\lambda_0/\lambda_c)^2}}$$

$$\therefore \boxed{\lambda_g = \frac{\lambda_0}{\sqrt{1-(\lambda_0/\lambda_c)^2}}}$$

④ Phase velocity (v_p):-

It is defined as "the velocity with which the phase of a wave changes".

$$V_p = \frac{\omega}{\beta} = \frac{c}{\sqrt{1-(\lambda_0/\lambda_c)^2}}$$

$$\therefore \boxed{V_p = \frac{c}{\sqrt{1-(\lambda_0/\lambda_c)^2}}}$$

⑤ Group velocity (v_g):-

It is defined as "the rate at which a wave propagates through a circular waveguide".

$$V_g = \frac{d\omega}{d\beta} = c\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}$$

$$\therefore \boxed{V_g = c\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}$$

⑥ Impedance (η):-

It is defined as "the ratio of strength of Electric field to magnetic field strength".

$$\boxed{\begin{aligned} \eta_{TE} &= \frac{\eta}{\sqrt{1-(\lambda_0/\lambda_c)^2}} \\ \eta_{TM} &= \eta\sqrt{1-(\lambda_0/\lambda_c)^2} \end{aligned}}$$

Dominant modes of Circular waveguide:-

→ If You consider, TE mode, TE_{11} is the dominant mode.

→ If You consider, TM mode TM_{01} is the dominant mode.

Cavity Resonator:-

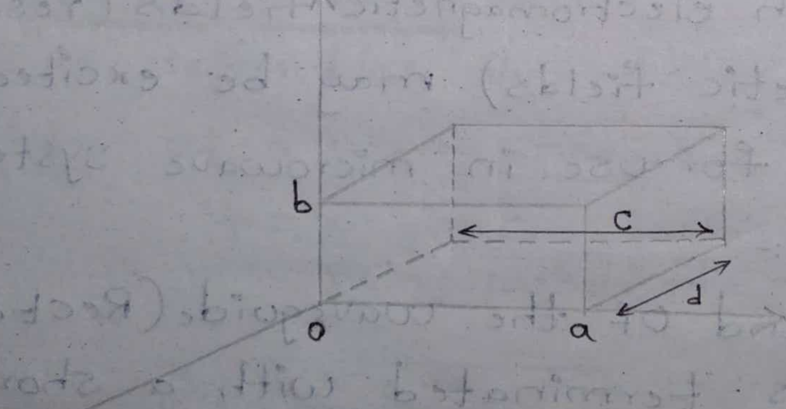
Definition:- An electronic device consisting of a space usually enclosed by metallic walls within which electromagnetic fields (resonant electromagnetic fields) may be excited and extracted for use in microwave systems.

Explanation:-

When one end of the waveguide (Rectangular/Circular) is terminated with a shorting plate and a wave is passed, there will be reflection of wave. When the other end is also terminated with another shorting plate, the reflected wave gets bounced back. This to-and-fro motion of the wave between the two shorting plates which are spaced at a distance of $\lambda g/2$, can produce standing waves inside the hollow space. So that it results in "Resonance Phenomenon". The hollow space is called "cavity" and this entire arrangement is known as "cavity Resonator".

Expression for Resonant frequency (f_o) in a Rectangular Waveguide:-

A cavity resonator is a useful microwave device. If we close off two ends of a rectangular waveguide with metallic walls, we have a rectangular cavity resonator. In this case, the wave propagating in the z-direction will bounce off the two walls resulting in a standing wave in the z-direction.



We know that

$$h^2 = \gamma^2 + \omega^2\mu\varepsilon \longrightarrow ①$$

$$\Rightarrow \omega^2\mu\varepsilon = h^2 - \gamma^2$$

$$\Rightarrow \omega^2\mu\varepsilon = h^2 + \beta^2$$

$\gamma = \alpha + j\beta$

$\gamma = j\beta$ if $\alpha = 0$

$h^2 = A^2 + B^2$ where $A = \left(\frac{m\pi}{a}\right)$, $B = \left(\frac{n\pi}{b}\right)$

$$\Rightarrow \omega^2\mu\varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \beta^2$$

$$\Rightarrow \omega = \frac{1}{\sqrt{\mu\varepsilon}}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \beta^2\right]^{1/2} \rightarrow ②$$

When the wave is oscillating between metallic walls of lengths 'a' and 'c' respectively, for a distance of 'd', resonance occurs in the hollow space. Therefore,

$$\omega = \omega_0 \, ; \quad \beta = \frac{2\pi}{\lambda} = \frac{p\pi}{d}$$

from ②;

$$\omega_0 = \frac{1}{\sqrt{\mu\varepsilon}}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2\right]^{1/2}$$

$$\Rightarrow 2\cancel{\pi} f_0 = \frac{1}{\sqrt{\mu\varepsilon}}\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{d}\right)^2\right]^{1/2} \cdot \cancel{\pi}$$

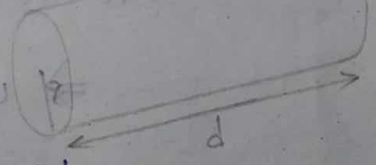
$$\Rightarrow f_0 = \frac{1/2}{\sqrt{\mu\varepsilon}}\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{d}\right)^2\right]^{1/2}$$

$$\Rightarrow f_0 = \frac{c}{2}\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{d}\right)^2\right]^{1/2} \quad \left(\because c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}\right)$$

$$\therefore \boxed{f_0 = \frac{c}{2}\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{d}\right)^2\right]^{1/2}}$$

<u>Expression for f_0 in Circular waveguide:-</u>

Consider a circular cavity resonator of cross-sectional area 'a' and length 'd'. If we close off the two ends of a circular waveguide with metallic walls, we have a circular resonator.



We know that

$$h^2 = \gamma^2 + \omega^2\mu\varepsilon \rightarrow ①$$

$$\Rightarrow \omega^2\mu\varepsilon = h^2 - \gamma^2$$

$$\Rightarrow \omega^2\mu\varepsilon = h^2 + \beta^2 \rightarrow (2) \quad (\because \gamma = \alpha + j\beta \Rightarrow \gamma = j\beta \text{ if } \alpha = 0)$$

$h = \frac{P'_{nm}}{a}$ (TE mode)

$h = \frac{P_{nm}}{a}$ (TM mode)

Case (i):- TE mode

from (2); $\omega^2\mu\varepsilon = \left(\frac{P'_{nm}}{a}\right)^2 + \beta^2$

$$\Rightarrow \omega = \frac{1}{\sqrt{\mu\varepsilon}}\left[\left(\frac{P'_{nm}}{a}\right)^2 + \beta^2\right]^{1/2}$$

At resonance, $\omega = \omega_0$ and $\beta = \frac{p\pi}{d}$

$$\Rightarrow \omega_0 = \frac{1}{\sqrt{\mu\varepsilon}}\left[\left(\frac{P'_{nm}}{a}\right)^2 + \left(\frac{p\pi}{d}\right)^2\right]^{1/2}$$

$$\Rightarrow 2\pi f_0 = \frac{1}{\sqrt{\mu\varepsilon}}\left[\left(\frac{P'_{nm}}{a}\right)^2 + \left(\frac{p\pi}{d}\right)^2\right]^{1/2}$$

$$\Rightarrow \boxed{f_0 = \frac{c}{2\pi}\left[\left(\frac{P'_{nm}}{a}\right)^2 + \left(\frac{p\pi}{d}\right)^2\right]^{1/2}}$$

Case (ii):- TM mode

from (2); $\omega^2\mu\varepsilon = \left(\frac{P_{nm}}{a}\right)^2 + \beta^2$

$$\Rightarrow \omega = \frac{1}{\sqrt{\mu\varepsilon}}\left[\left(\frac{P_{nm}}{a}\right)^2 + \beta^2\right]^{1/2}$$

At resonance, $\omega = \omega_0$ and $\beta = \frac{p\pi}{d}$

$$\Rightarrow \omega_0 = \frac{1}{\sqrt{\mu\varepsilon}}\left[\left(\frac{P_{nm}}{a}\right)^2 + \left(\frac{p\pi}{d}\right)^2\right]^{1/2}$$

$$\Rightarrow 2\pi f_0 = \frac{1}{\sqrt{\mu\varepsilon}}\left[\left(\frac{P_{nm}}{a}\right)^2 + \left(\frac{p\pi}{d}\right)^2\right]^{1/2}$$

$$\Rightarrow \boxed{f_0 = \frac{c}{2\pi}\left[\left(\frac{P_{nm}}{a}\right)^2 + \left(\frac{p\pi}{d}\right)^2\right]^{1/2}}$$

Analysis of modes:- (To determine whether TE_{mnp} / TM_{mnp} exists for a cavity resonator).

TM mode:- ($H_z = 0$; $E_z \neq 0$)

We know that

$$E_z = C_0 \sin\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y\, e^{-\gamma z} \longrightarrow ①$$

Here, $e^{-\gamma z} = e^{-(\alpha + j\beta)z} = e^{-j\beta z}$

Incase of a cavity resonator, the wave moves to-and-fro in between the hollow space.

Hence, $\boxed{e^{-\gamma z} = A^{+}e^{-j\beta z} + A^{-}e^{j\beta z}}$ indicating that the wave is propagating in forward z-direction as well as backward z-direction.

Differentiate $\left(A^{+}e^{-j\beta z} + A^{-}e^{j\beta z}\right)$ w.r.to 'z'

$$= A^{+}e^{-j\beta z}(-j\beta) + A^{-}e^{j\beta z}(j\beta)$$

$$= j\beta\left[-A^{+}e^{-j\beta z} + A^{-}e^{j\beta z}\right]$$

(Substituting the boundary condition z=0 and z=d)

$$= j\beta\left[-A^{+} + A^{-}\right]$$

now, $j\beta\left[-A^{+} + A^{-}\right] = 0$

$$-A^{+} = -A^{-}$$

$$\boxed{A^{+} = A^{-}}$$

$$\therefore A^{+}e^{-j\beta z} + A^{-}e^{+j\beta z} = A^{+}e^{-j\beta z} + A^{+}e^{j\beta z}$$

$$= 2\left(\frac{A^{+}e^{-j\beta z} + A^{+}e^{j\beta z}}{2}\right)$$

$$= \cos\beta z$$

$$= \cos\left(\frac{P\pi}{d}\right)z \quad \left(\text{if } \beta = \frac{P\pi}{d}\right)$$

from ①;

$$\boxed{E_z = C_0 \sin\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y \cos\left(\frac{P\pi}{d}\right)z}, \quad P = 0, 1, 2, \ldots$$

for $P=0$, E_z exists. Hence, for a cavity resonator TM_{mnp} exists.

TE mode:- ($E_z = 0$ and $H_z \neq 0$)

We know that

$$H_z = C_o \cos\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y \, e^{-\gamma z}$$

$$\Rightarrow H_z = C_o \cos\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y \left[A^+ e^{-j\beta z} + A^- e^{j\beta z}\right]$$

$$\Rightarrow H_z = C_o \cos\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y \left[A^+ e^{-j\beta z} - A^- e^{j\beta z}\right]$$

$$\Rightarrow H_z = C_o \cos\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y \left[\frac{A^+ e^{-j\beta z} - A^- e^{j\beta z}}{2j} \cdot 2j\right]$$

$$\Rightarrow H_z = C_o \cos\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y \sin\left(\frac{p\pi}{d}\right)z$$

$$\therefore \boxed{H_z = C_o \cos\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y \sin\left(\frac{p\pi}{d}\right)z}, \quad p = 0, 1, 2, \ldots$$

for $p=0$; $H_z = 0$ i.e., for a cavity resonator, TE_{mn0} doesnot exist.

Rectangular waveguides:-

TE mode $\rightarrow TE_{10}$

TM mode $\rightarrow TM_{11}$

Circular waveguides:-

TE mode $\rightarrow TE_{11}$

TM mode $\rightarrow TM_{01}$

Cavity Resonators:-

TE Mode $\rightarrow TE_{101}$ } Rectangular Cavity resonators

TM mode $\rightarrow TM_{110}$

TE Mode $\rightarrow TE_{111}$ } Circular cavity resonators

TM Mode $\rightarrow TM_{010}$

(Dominant modes)

Quality factor (Q) of a cavity resonator:-

→ Quality factor (Q) measures the frequency Selectivity of a resonant circuit (or) an anti-resonant circuit.

→ It is defined as "the ratio of maximum energy stored per cycle to the energy dissipated per cycle".

i.e.,
$$\boxed{Q = 2\pi \cdot \frac{\text{maximum energy stored per cycle}}{\text{energy dissipated per cycle}}}$$

$$\boxed{Q = \frac{\omega_0 W}{P}}$$

Here, ω_0 → Resonant frequency

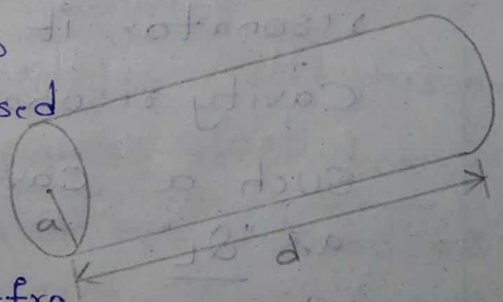
W → maximum energy stored

P → Average Powerloss

Note:- For an ideal cavity resonator, $Q = \infty$ since $P = 0$

How a cavity resonator works??

Consider a cavity resonator of length 'd' and cross-sectional area 'a' as shown below:



→ Assume that the two ends of the resonator are closed and a microwave signal is captured in it.

→ The signal moves to-and-fro inside the cavity, due which some energy (or) frequency is generated.

→ Hence, the principle of operation of a cavity resonator is, A signal which gets trapped, inside a cavity kind of space oscillates continuously and due to the resonance of that

Particular signal, some energy/frequency gets generated.

→ Whenever a signal captured within the cavity resonator starts generating the frequency, we need to consider the following three parameters:

$\boxed{W_0, W \text{ and } P}$

→ There are three conditions to be taken into consideration while obtaining the Quality factor (Q) of a cavity resonator.

(i) 'Q' of a Loaded cavity resonator (Q_L)

(ii) 'Q' of an unloaded cavity resonator (Q_0)

(iii) 'Q' due to external ohmic losses (Q_{ext}).

→ A cavity resonator is considered to be an unloaded cavity resonator, if at all it is empty i.e., contains no signal. The Quality factor of such a cavity resonator is referred to as "Q_0".

→ When a signal is passed into the cavity resonator, it is referred to as a Loaded cavity resonator. The Quality factor of such a cavity resonator is referred to as "Q_L".

→ When the signal starts resonating inside the cavity, some energy/frequency gets generated. At the same time, there might be a chance of some loss in power. The Quality factor of a cavity resonator in this cases, is referred to as "Q_{ext}".

from these ③ conditions, a relation is developed which is given by,

$$\boxed{\frac{1}{Q_L} = \frac{1}{Q_o} + \frac{1}{Q_{ext}}}$$

Quality factor of Rectangular Cavity Resonator:-

The Quality factor of a Rectangular Cavity Resonator is given by,

$$\boxed{Q = \frac{\omega^2 \mu^2 a^3}{6 R_s \pi^2}}$$

Here, $w\mu a \rightarrow$ volume of rectangular cavity resonator

$R_s \rightarrow$ Surface of the resonator

Quality factor of a circular cavity Resonator:-

The Quality factor of a Circular cavity Resonator is given by,

$$\boxed{Q = \frac{2.6178\, a \beta^2}{4\pi R_s \cdot V_c}\left[\frac{ac}{a+c}\right]}$$

Here, $V_c \rightarrow$ velocity of light

$\beta \rightarrow$ Phase constant

$R_s \rightarrow$ Surfac of the resonator

$a \rightarrow$ radius of the circular cavity resonator

Measurement of (Q) of a cavity resonator:-

→ Quality factor (Q) can also be defined as "the ratio of resonant frequency to the Bandwidth of the signal".

i.e., $\boxed{Q = \frac{\text{Resonant frequency}}{\text{Bandwidth of the signal}}}$

→ Whenever a microwave signal is transmitted through a cavity resonator, what matter all is, in what way, the cavity resonator/ the resonator circuit responds to that particular microwave signal frequency.

→ As a Quality factor measurement, we will consider a microwave source and will be transmitting different frequencies through the cavity resonator/ resonator ckt

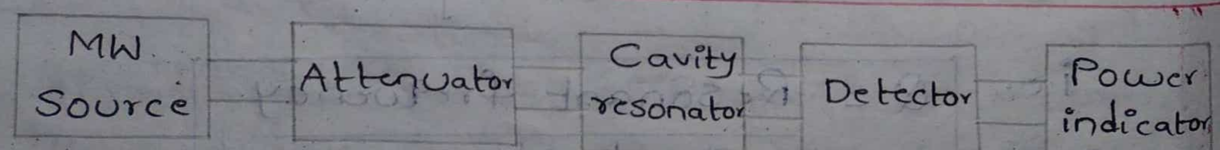
→ we will be observing how this resonant circuit responds to each and every transmitted frequency and we will be measuring the readings of the circuit through a power indicator.

→ There are several methods for measuring the 'Q' of cavity resonator.

- Transmission method
- Decrement method
- Impedance measurement method

→ Among these, "transmission method" is simplest

→ In this method, cavity resonator is used as a transmission device and the output signal is measured as a function of frequency resulting in the resonance curve.



Set UP for measurement of Q of a Cavity resonator Using transmission method

→ In the above set up, a microwave source, continuously generates microwave frequencies which are transmitted through Attenuator, Cavity resonator as well.

→ As it keeps on generating, several different microwave frequencies are generated. At what particular frequency, the circuit responds and the power altered from it will be obtained through power indicator.

→ Based on the power indicator measurements as well as frequency measurements, we will be a plotting a graph from which the resonant frequency and signal bandwidth is calculated.

→ From the setup above, the signal frequency of the microwave source is varied, keeping the signal level constant and then the outpower is measured.

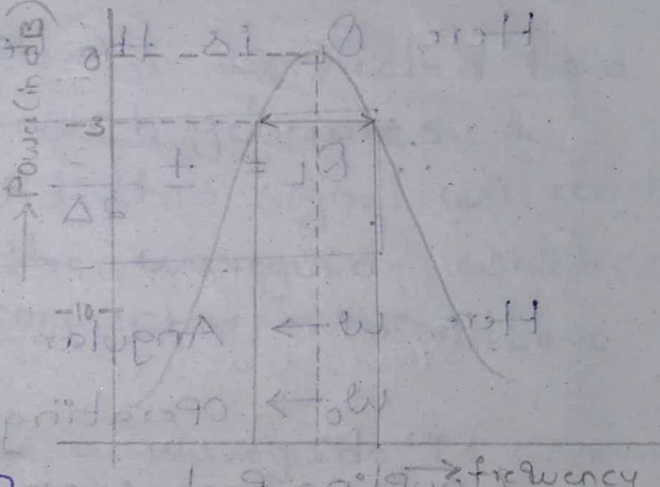


fig:- Resonance curve

→ As the process goes on, several different microwave frequencies are generated. At each frequency, the power generated is noted down from the power indicator. A graph is plotted between frequency and power (in dB).

→ As the frequency varies from 0 to few range, power also varies. At a particular frequency, maximum power is obtained and this frequency is known as "Resonant frequency".

→ The cavity resonator is tuned to this frequency and the signal level is again noted down to notice the difference.

→ Below 3dB line, we will consider two different point, from which the signal Bandwidth is obtained.

→ When the output is plotted, the resonance curve is obtained, from which we can notice the Half Power Bandwidth (HPBW) (2Δ) values.

$$\boxed{2\Delta = \pm \frac{1}{Q_L}}$$

Here, Q_L is the Loaded value

$$\boxed{\therefore Q_L = \pm \frac{1}{2\Delta} = \pm \frac{\omega}{2(\omega - \omega_0)}}$$

Here, $\omega \rightarrow$ Angular frequency

$\omega_0 \rightarrow$ Operating frequency

Coupling Probes and coupling loops:-

→ Coupling Probes and coupling loops are used as an antenna to transmit a signal into a Waveguide (or) to receive a signal from the Waveguide.

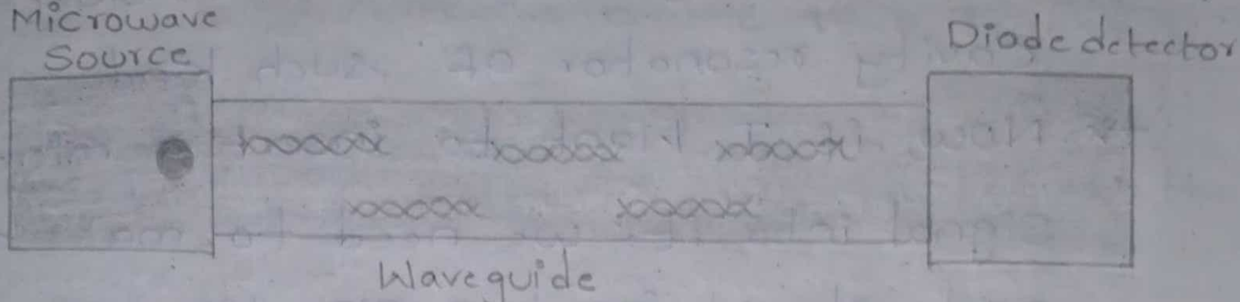
→ Both are a kind of wiring mechanism used for communication Purpose.

How a Waveguide is connected to a microwave Source:-

→ Waveguides are the Tx lines used for transmission of microwave signals that can be an electric field/ magnetic field/

electromagnetic field (TE wave, TM wave, TEM wave respectively).

→ These are in different shapes and are of different kinds.



→ To transmit a microwave signal through a Waveguide, we need a microwave source.

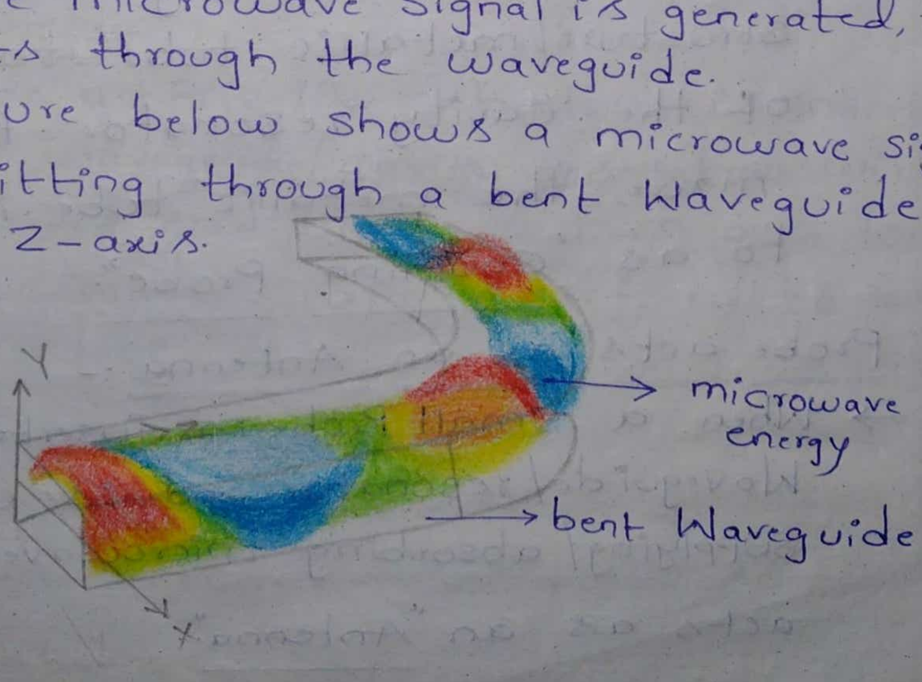
→ This microwave source can be an oscillator, a generator (or) any kind of device which continuously generates a microwave signal.

→ Always, a Waveguide is connected to a microwave source, which generates a microwave signal and this signal will reach the other end of the waveguide which is connected to a connector/a microwave junction/CRO.

→ Initially one end of a Waveguide is connected to a microwave source.

→ When the microwave signal is generated, it transmits through the waveguide.

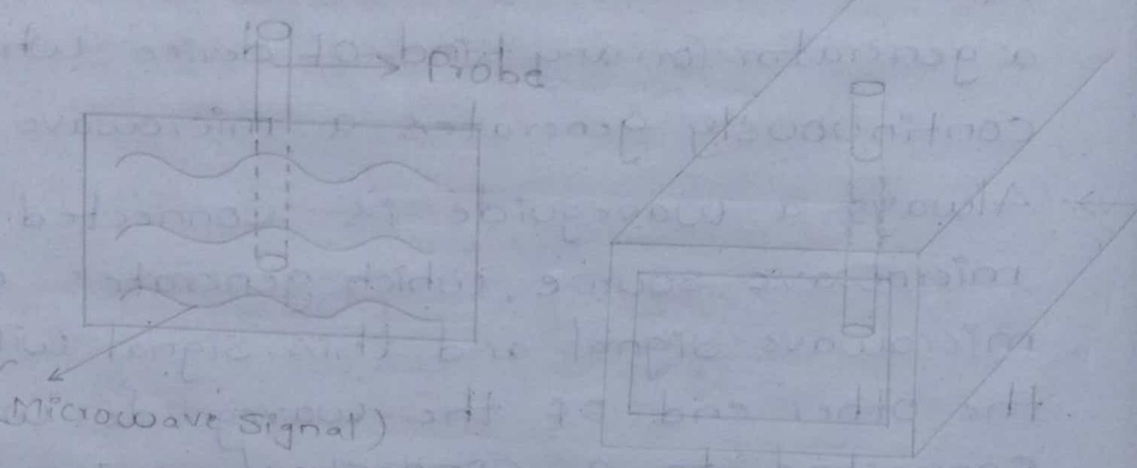
→ The figure below shows a microwave signal transmitting through a bent Waveguide along Z-axis.



Coupling Probes:-

→ A Waveguide closed at one end along with the other end also being closed behaves as a cavity resonator. Consider, a cavity resonator of such kind.

→ Now, if I want to insert a microwave signal into it, we need to make a hole and should insert a pipe like structure/ a metallic tube which is referred to as a Probe.



→ You can insert(transmit)/ extract(receive) a microwave signal through this hole.

→ As we are coupling the pipe like structure/metallic tube to the hole of the cavity resonator that has been made, the metallic tube is referred to as "Coupling Probe".

Probe acts as an Antenna :-

→ When a small probe is inserted into a Waveguide/resonator and is used for supplying/ absorbing microwave energy, it acts as an "Antenna".

* No signal in the Cavity-resonator → make a hole → Couple the Probe → insert the signal

In this Case, Antenna works as a Transmitter.

* Signal is Present → make a hole → Couple the Probe → extract the Signal

In this Case, Antenna Works as a Receiver.

→ In total, when a Probe is inserted/coupled into a Cavity resonator and is supplied with microwave energy, it act as either Transmitting Antenna / Receiving Antenna.

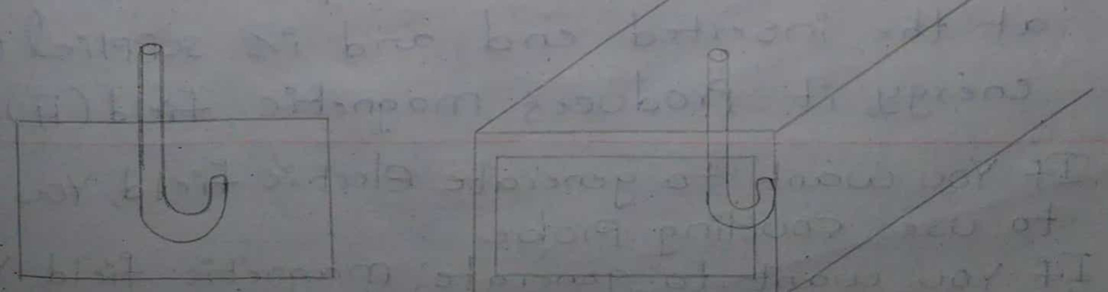
→ Always current flows into this Probe.

→ When you use a coupling Probe, it will set the Electric field ($\bar{E}$) associated with the Electromagnetic wave inside the waveguide.

→ In other words, if You want to generate electric field, You need to use a coupling Probe.

Coupling loops:-

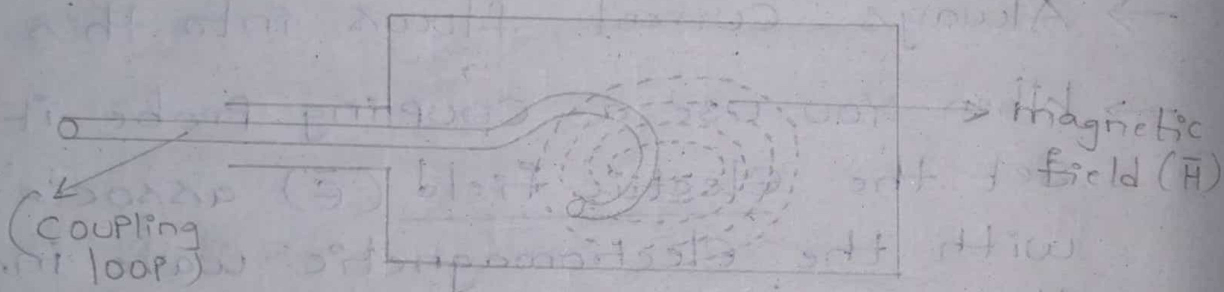
→ Whenever a Pipe like structure is inserted into a waveguide/cavity resonator with the inserted end having a turn (or) a loop (or) a circular shape is referred to as a coupling loop.



→ Another way of injecting energy into a waveguide is by setting up magnetic field ($\bar{H}$) in the waveguide.

→ This can be accomplished by inserting a small probe having a turn/a circular shape, a loop like structure at the inserted end, into a waveguide/a cavity resonator.

→ This will carry little amount of current into the waveguide. As a result, a magnetic field builds up around the loop and expands to fit in the waveguide.



→ The figure shows a coupling loop with a turn at the inserted end, being inserted into the waveguide. When a microwave signal is transmitted through it, the signal moves to-and-fro in a circular shape due to this coil like structure of the coupling loop. This results in the generation of magnetic field ($\bar{H}$).

→ In total, whenever an antenna has a circular shape/a coil like structure/a loop like structure at the inserted end and is supplied with energy, it produces magnetic field ($\bar{H}$).

If you want to generate electric field, you have to use coupling probe.
If you want to generate magnetic field, you have to use mostly coupling loop.

Coupling Coefficients:-

Coupling

- Critical Coupling
- Under Coupling
- Over Coupling

In general,

$$\boxed{\begin{aligned} Q_0 &= \frac{\omega_0 L}{R} \\ Q_{ext} &= \frac{Q_0}{K} \end{aligned}}$$

$K \rightarrow$ Coupling coefficient

We know that

$$\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_{ext}} \longrightarrow (1)$$

Case (i):- Critical Coupling $(K=1)$

from (1); $\dfrac{1}{Q_L} = \dfrac{1}{Q_0} + \dfrac{1}{Q_{ext}}$

$$\Rightarrow \frac{1}{Q_L} = \frac{R}{\omega_0 L} + \frac{KR}{\omega_0 L}$$

$$\Rightarrow \frac{1}{Q_L} = \frac{R}{\omega_0 L} + \frac{R}{\omega_0 L}$$

$$\Rightarrow \frac{1}{Q_L} = \frac{2R}{\omega_0 L}$$

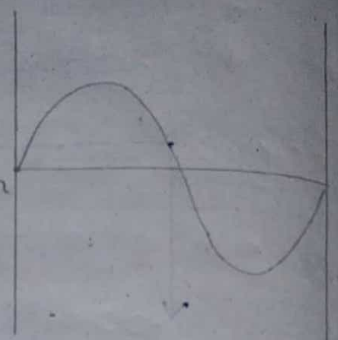
$$\Rightarrow Q_L = \frac{\omega_0 L}{2R}$$

$$\Rightarrow Q_L = \frac{Q_0}{2} = \frac{Q_{ext} \cdot K}{2} = \frac{Q_{ext}}{2}$$

$$\therefore \boxed{Q_L = \frac{Q_{ext}}{2}}$$

Case (ii):- Undercoupling ($K<1$)

In this case, the cavity is terminated at V_{min} and hence the i/p impedance is the reciprocal of VSWR.



i.e., $\boxed{K = \frac{1}{P}}$

from ①; $\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_{ext}}$

$$= \frac{R}{\omega_0 L} + \frac{RK}{\omega_0 L}$$

$$= \frac{R}{\omega_0 L} + \frac{R}{P\omega_0 L}$$

$$= \frac{R}{\omega_0 L}\left[1 + \frac{1}{P}\right]$$

$$= \frac{R}{\omega_0 L}\left[\frac{1+P}{P}\right]$$

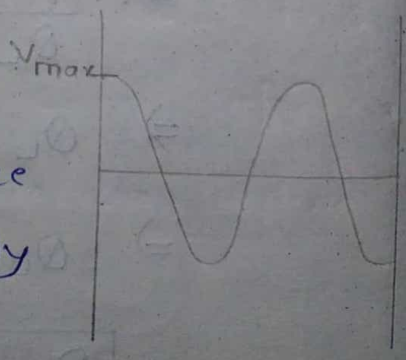
$$\Rightarrow Q_L = \frac{\omega_0 L}{R}\left[\frac{P}{1+P}\right]$$

$$\Rightarrow Q_L = Q_0\left[\frac{P}{1+P}\right]$$

$$\therefore \boxed{Q_L = Q_0\left[\frac{P}{1+P}\right]}$$

Case (iii):- Overcoupling ($K>1$)

In this case, the cavity is terminated at V_{max} and hence the i/p impedance is directly proportional to VSWR.



i.e., $\boxed{K = P}$

from ①; $\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_{ext}}$

$$= \frac{R}{\omega_0 L} + \frac{RP}{\omega_0 L}$$

$$= \frac{R}{\omega_0 L}[1+P]$$

$$\Rightarrow Q_L = \frac{\omega_0 L}{R(1+P)}$$

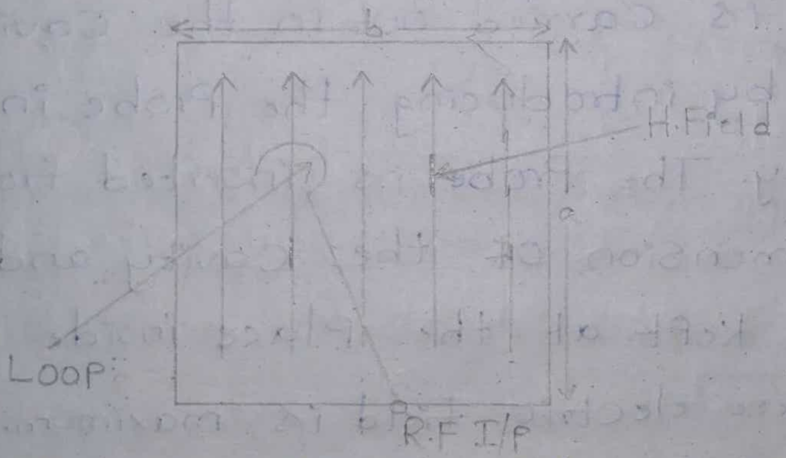
$$\Rightarrow Q_L = \frac{Q_0}{(1+P)}$$

$$\therefore \boxed{Q_L = \frac{Q_0}{1+P}}$$

Excitation techniques of Cavity Resonator:-

There are following methods of Cavity resonator excitation.

1. Loop Excitation

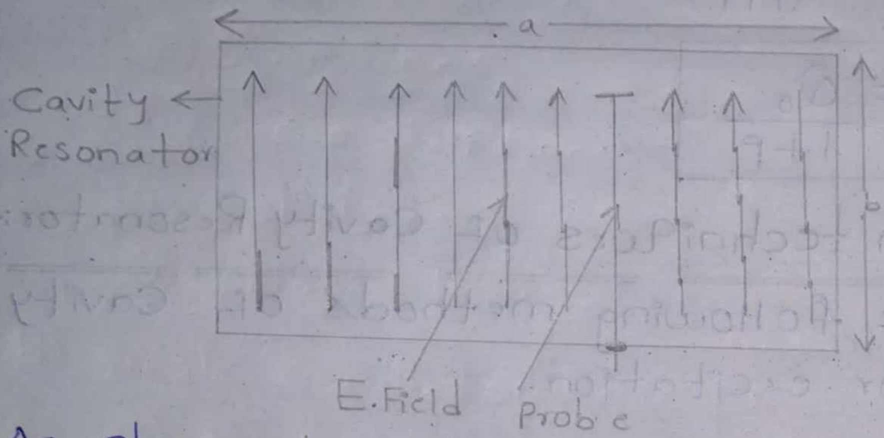


As shown in the given diagram, the loop excitation is carried out in the cavity resonator by introducing the loop inside the cavity. The loop is inserted from the narrow dimension of the cavity and it should be kept at the place inside the cavity where magnetic field is maximum.

When the R.F. signal is applied through the loop, the magnetic flux starts to expand and

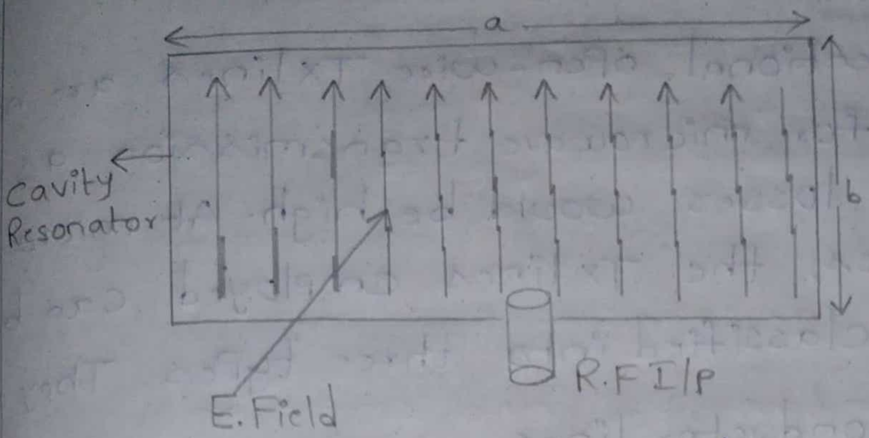
Collapse around the loop. This magnetic flux causes to induce the voltage in the walls of cavity resonator. As the induced emf is the microwave signal, therefore, the induced oscillation action inside the cavity resonator takes place.

2. Probe Excitation



As shown in the given diagram, the probe excitation is carried out in the cavity resonator by introducing the probe inside the cavity. The probe is inserted from the broad dimension of the cavity and it should be kept at the place inside the cavity where electric field is maximum. When the R.F. signal is applied through the probe, the electric field starts to expand and collapse around the probe, this electric field causes to excite the cavity resonator and the oscillation inside the resonator takes place.

3. Aperture Excitation



As shown in the given diagram, the aperture excitation is carried out by making the slot in the cavity resonator. In this case, we couple the E-field (or) H-field to the cavity with the help of a circular (or) rectangular waveguide. This field causes to excite the cavity resonator. If the coupling is carried out from the broad dimension of the cavity resonator, the operation will be TE mode. If we couple the input from the narrow dimension, the operation will be in TM mode.

Applications of cavity resonators:-

→ Tuned circuits

→ In ultra high frequency tubes

→ Klystron Amplifier

→ oscillators

→ cavity Magnetron

→ In Radars

Microstriplines:-

Introduction:-

The conventional open-wire Tx lines are not suitable for microwave transmission, as the radiation losses would be high. At microwave frequencies, the Tx lines employed can be broadly classified into three types. They are

a) Multi conductor lines
   (i) Co-axial lines
   (ii) Strip lines
   (iii) Micro strip lines
   (iv) slot lines
   (v) coplanar lines etc..

b) Single conductor lines
   (i) Rectangular Waveguides
   (ii) circular Waveguides
   (iii) elliptical Waveguides
   (iv) Single-ridged waveguides
   (v) Double-ridged waveguides, etc...

c) open boundary structures
   (i) Di-electric rods
   (ii) open Waveguides, etc...

a) Multi conductor lines

The TX lines which has more than one conductor are called as Multi-conductor lines.

Co-axial lines:-

→ This is mostly used for high frequency applications.

→ A coaxial line consists of an inner conductor with inner diameter d, and then a concentric cylindrical material, around it.

→ This is surrounded by an outer conductor, which is a concentric cylinder with an inner diameter D.

→ This structure is well understood by taking a look at the following figure.

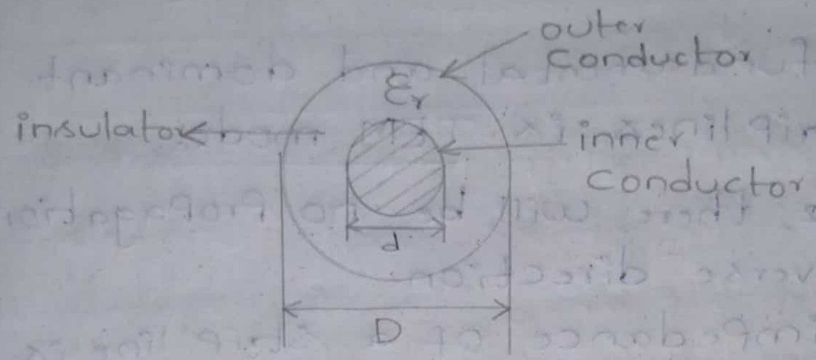


fig:- Cross-sectional view of a co-axial line

→ The fundamental and dominant mode in co-axial cables is TEM mode.

→ There is no cut-off frequency (f_c) in the co-axial cable. It passes all frequencies.

→ However, for higher frequencies, some higher order non-TEM mode starts propagating, causing a lot of attenuation.

Strip lines:-

→ These are the planar transmission lines, used at frequencies from 100MHz to 100GHz.

→ A strip line consists of a central thin conducting strip of width 'w' which is greater than its thickness 't'.

→ It is placed inside the low loss dielectric (ϵ_r) substrate of thickness b/2 between two wide group plates.

→ The width of the ground plates is five times greater than the spacing between the plates.

→ The thickness of metallic central conductor and the thickness of metallic ground planes are the same. The following figure shows the cross-sectional view of the stripline structure.

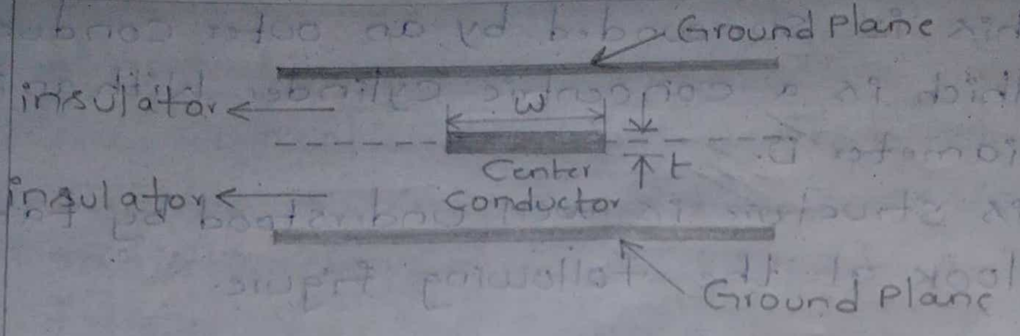


fig:- Stripline transmission line

→ The fundamental and dominant mode in strip lines is <u>TEM mode</u>.

→ For b/2, there will be no propagation in the transverse direction.

→ The impedance of a strip line is inversely proportional to the ratio of the width 'w' of the inner conductor to the distance 'b' between the ground planes.

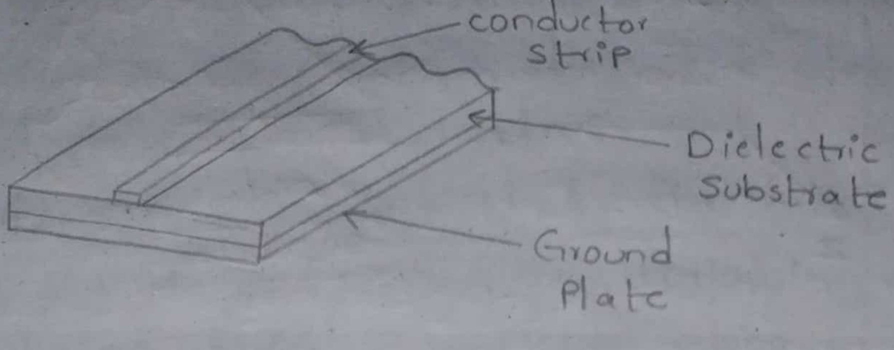
c) <u>Micro strip lines</u>

→ The strip line has a disadvantage that it is not accessible for adjustment and tuning.

→ This is avoided in microstrip lines, which allows mounting of active or passive devices, and also allows making minor adjustments after the circuit has been fabricated.

→ A microstrip line is an unsymmetrical parallel plate transmission line, having dielectric substrate which has a metallized ground on the bottom and a thin conducting strip on top with thickness 't' and width 'w'.

→ This can be understood by taking a look at the following figure, which shows a micro strip line.



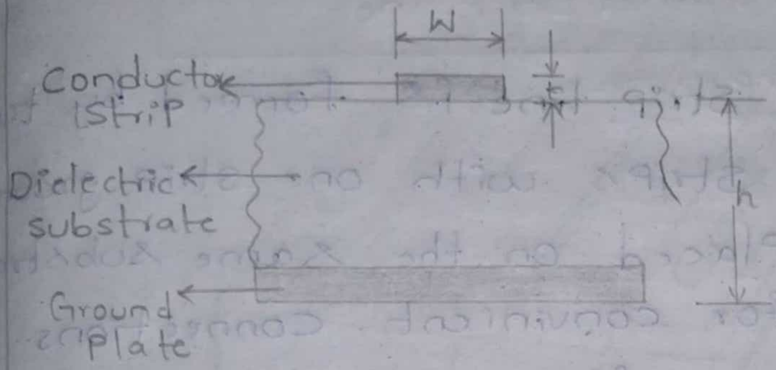


fig:- Microstrip line

→ The characteristic impedance of a micro strip is a function of the strip line width w, thickness 't' and the distance between the line and the ground plane h.

→ Microstrip lines are of many types such as embedded micro strip, inverted micro strips, suspended micro strip and slotted micro strip transmission lines.

→ In addition to these, some other TEM lines such as parallel strip lines and coplanar strip lines also have been used for microwave integrated circuits.

Other lines:-

→ A Parallel strip line is similar to a two conductor Tx line.

→ It can support quasi TEM mode.

→ The following figure explains this.

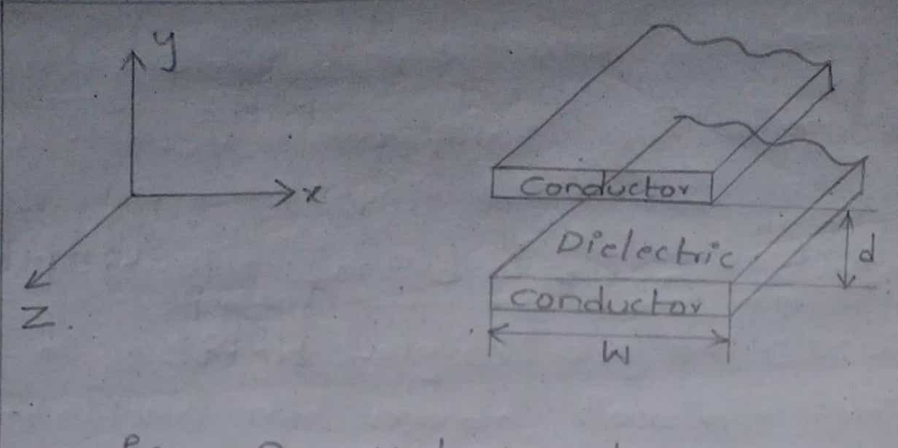


fig:- Parallel strip line

→ A Coplanar strip line is formed by two conducting strips with one strip grounded both being placed on the same substrate surface, for convinient connections. The following figure explains this.

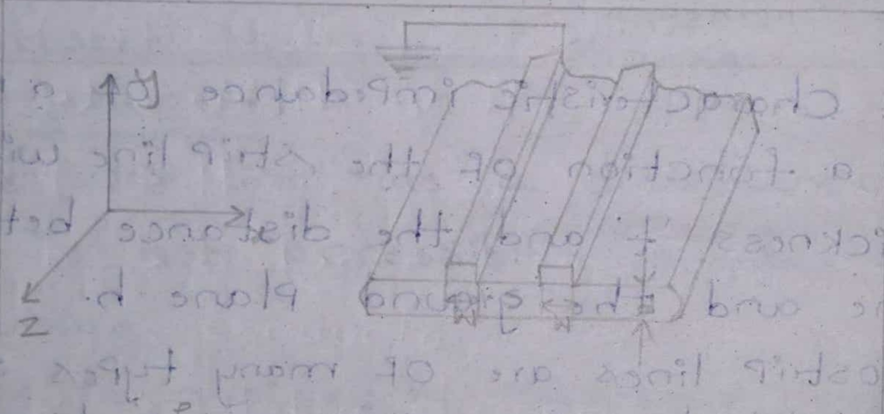


fig:- coplanar strip line

→ A Slot transmission line, consists of a slot or gap in a conducting coating on a dielectric substrate and this fabrication Process is identical to the micro strip lines. Following is its diagrammatical representation

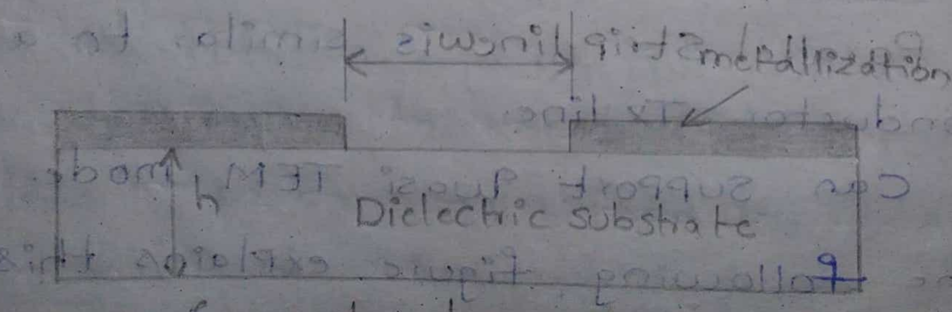


fig:- slot line

→ A coplanar waveguide consists of a strip of thin metallic film which is deposited on the surface of a dielectric slab.

→ This slab has two electrodes running adjacent and parallel to the strip on to the same surface. The following figure explains this.

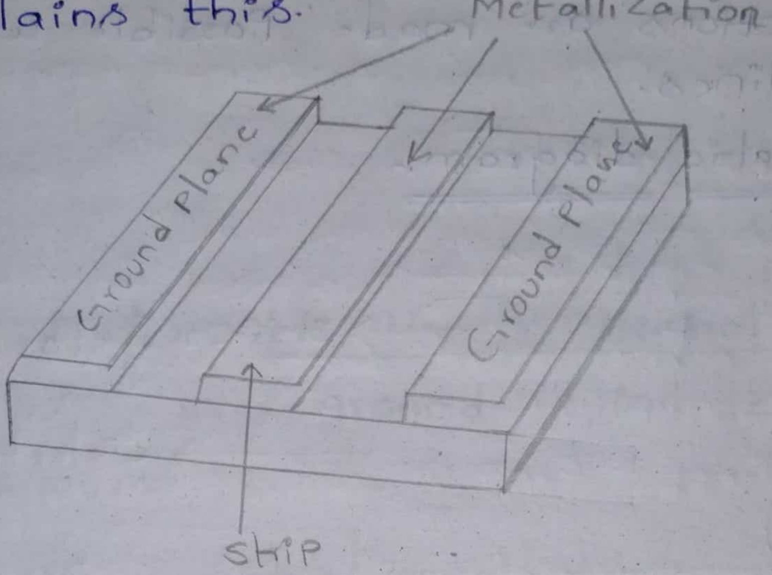


fig:- coplanar waveguide

All of these micro strip lines are used in microwave applications where the use of bulky and expensive to manufacture transmission lines will be a disadvantage.

Open Boundary structures:-

→ These can also be stated as "open electro-magnetic waveguides".

→ A waveguide that is not entirely enclosed in a metal shielding, can be considered as an open waveguide.

→ Free space is also considered as a kind of open waveguide.

→ An open waveguide may be defined as any physical device with longitudinal axial symmetry and unbounded cross-section, capable of guiding electromagnetic waves.

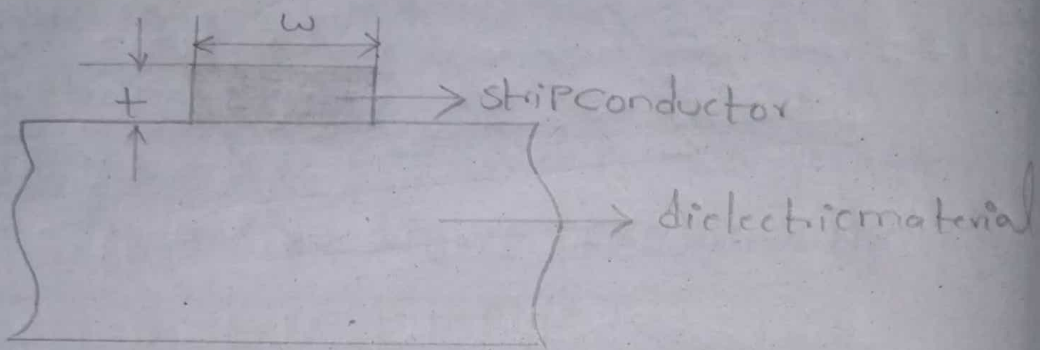
→ They possess a spectrum which is no longer discrete.

→ Micro strip lines and optical fibres are also examples of open Waveguides

Characteristic Impedance of Microstrip lines:-

For high speed Logical digital circuits, inter-connections are made possible with micro-strip lines.

Schematic diagram:-



Here, W → width of the strip conductor
t → thickness of the strip conductor
h → distance separating the strip conductor from the ground plane
ε_r → relative dielectric constant of the dielectric material.

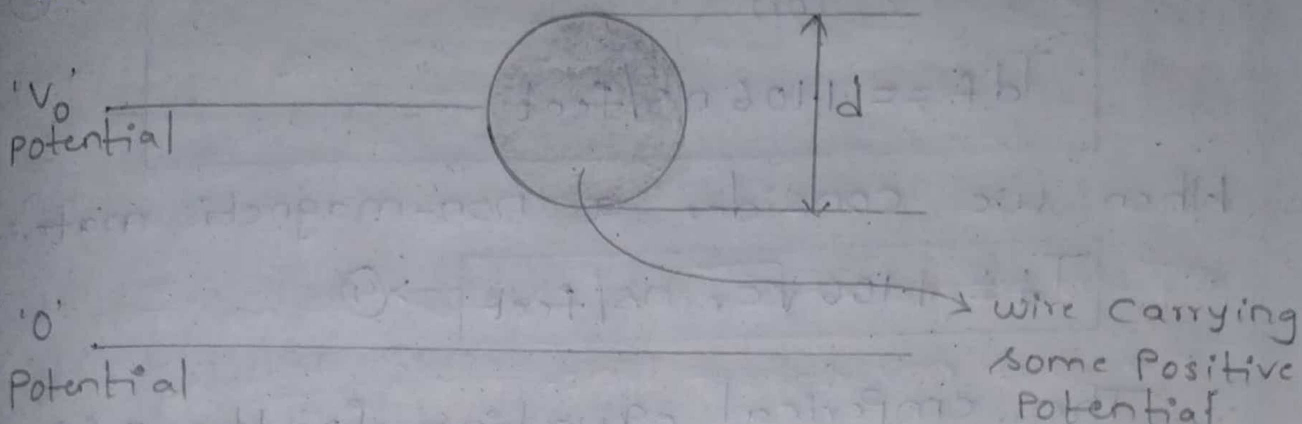
from the schematic diagram, we can write the characteristic impedance of Microstrip lines is a function of w, t, h and ε_r i.e.,

$$\boxed{Z_0(w, t, h, \varepsilon_r)}$$

It is very difficult to determine the value of Z_0. However, many methods were proposed to determine the value of Z_0, at accurate levels. One such among them is field Evaluation method. But this method is also too complicated. The alternate is, "Indirect method

In this method, we will determine the value of Z_0 of the given microstrip line by making a comparision with another microstrip line.

Schematic diagram:-



Here, d → diameter of the central conductor.

for a wire – over – ground Tx line, Z_0 is defined as follows:

$$\boxed{Z_0 = \frac{60}{\sqrt{\varepsilon_r}} \ln\left(\frac{4h}{d}\right)} \rightarrow (1) \quad \text{for } (h >> d)$$

To this equation, we shall make comparative modifications, to obtain equivalent attributes corresponding to the microstrip line.

Consider, the effective and equivalent values for the factors,

- ε_r → relative dielectric constant of the ambient medium
- d → diameter of the central conductor

Determination of effective dielectric constant:- (ε_{re})

To obtain ε_{re}, we have to first consider the propagation delay (T_d).

$$\boxed{T_d = \sqrt{\mu \varepsilon}} \rightarrow (2)$$

Here, μ → Permeability of the material

$\mu = 4\pi \times 10^{-7}$ H/m

ε → Permitivity of the material

$\varepsilon = 8.854 \times 10^{-12}$ F/m

In free space

$$\boxed{\begin{aligned} T_{df} &= \sqrt{\mu_0 \varepsilon_0} = \sqrt{(4\pi \times 10^{-7})(8.854 \times 10^{-12})} \\ &= 3.333\ ns/metre \\ &\quad (or) \\ T_{df} &= 1.106\ ns/feet \end{aligned}} \rightarrow (3)$$

When we consider a non-magnetic material,

$$\boxed{T_d = 1.106\sqrt{\varepsilon_r}\ ns/feet} \rightarrow (4)$$

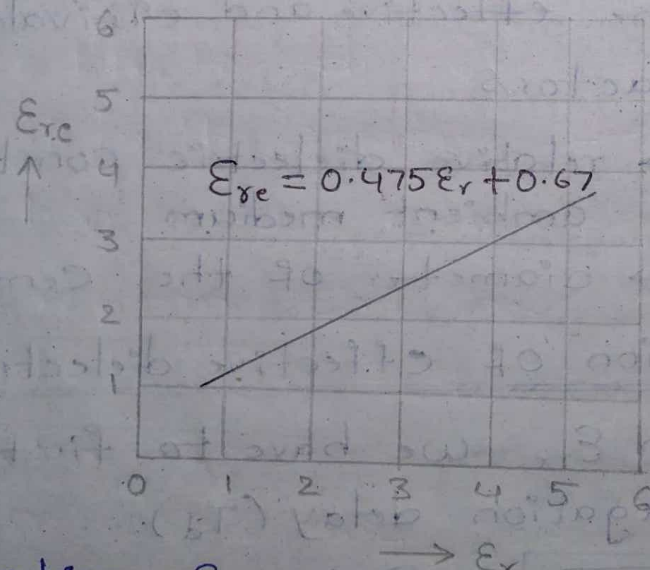
∴ The emperical equation for the effective dielectric constant is given by,

$$\boxed{\varepsilon_{eff} = \varepsilon_{re} = 0.475\varepsilon_r + 0.67} \rightarrow (5)$$

$\varepsilon_{re} \rightarrow$ relative dielectric constant

$\varepsilon_r \rightarrow$ effective dielectric constant

A graph showing the relation between ε_r and ε_{re} is as follows:



The equation, $\varepsilon_{re} = 0.475\varepsilon_r + 0.67$, helps us when we are shifting from circular cross-section having the wire over the ground to a microstrip line.

Transformation of a Rectangular conductor from an equivalent Circular Conductor:-

The emperical equation is given by,

$$\boxed{d = 0.67w\left(0.8 + \frac{t}{w}\right)} \rightarrow (6)$$

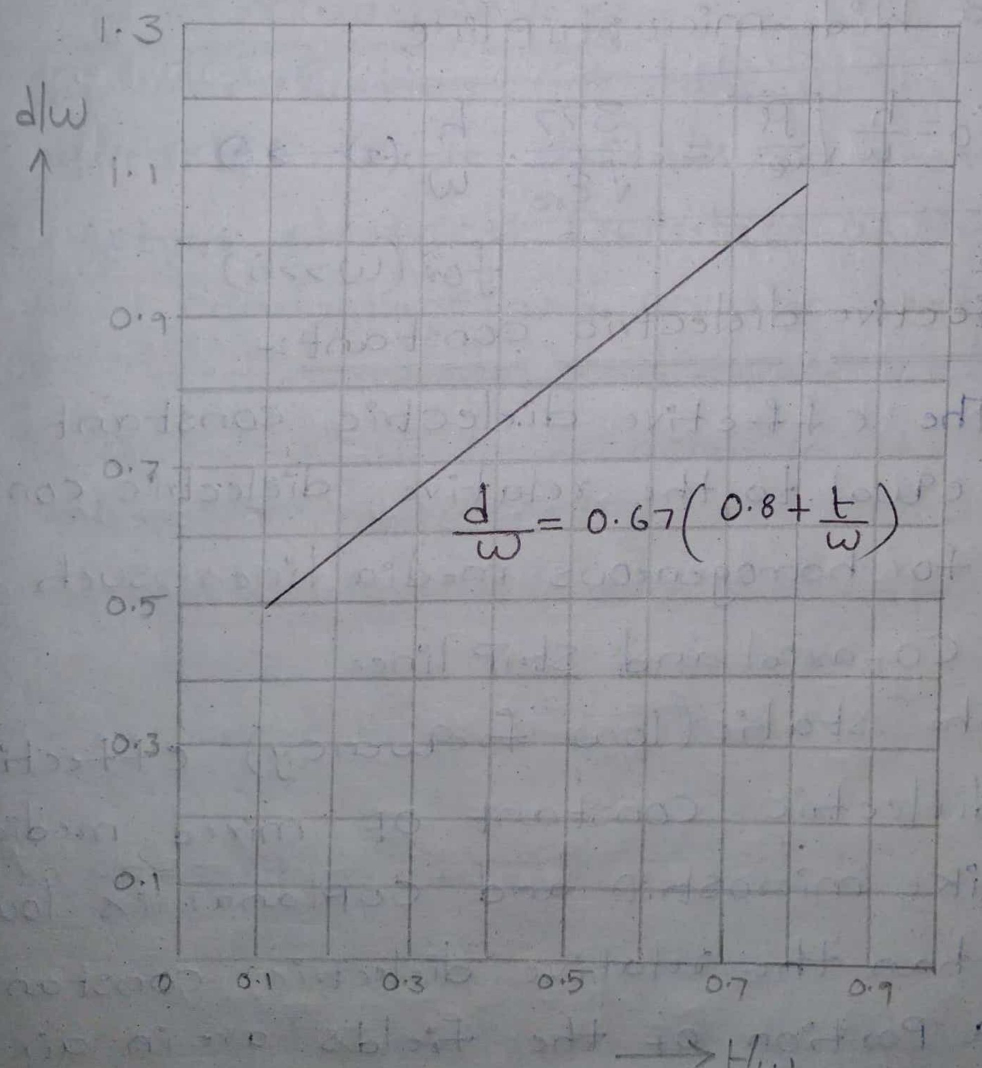
$$\Rightarrow \frac{d}{w} = 0.67\left(0.8 + \frac{t}{w}\right)$$

The ratio d/w Corresponds to a Circular conductor whereas the ratio t/w corresponds to a rectangular conductor.

$W \rightarrow$ width of the microstrip line

$t \rightarrow$ thickness " "

A graph showing the relation between the ratios d/w and t/w is as follows:



Substituting the above two emperical eqn's i.e, eqn's - ⑤ and ⑥ in equation - ① we get,

$$\boxed{Z_0 = \frac{87}{\sqrt{\varepsilon_r + 1.41}} \ln\left[\frac{5.98h}{0.8w + t}\right] (\Omega)} \rightarrow ⑦$$

for $(h < 0.8w)$

eqn - ⑦ represents the value of Z_0 for a "narrow microstrip line". Here, $\frac{t}{w} \simeq 0.1$.

According to the Performance Parameter,

Phase velocity $(v) = \frac{c}{\sqrt{\varepsilon_{re}}} = \frac{3\times10^8}{\sqrt{\varepsilon_{re}}}$ m/s $\rightarrow$ ⑧

For a Wide microstrip line,

$$\boxed{Z_0 = \frac{h}{w}\sqrt{\frac{\mu}{\varepsilon}} = \frac{377}{\sqrt{\varepsilon_{re}}} \cdot \frac{h}{w} (\Omega)} \rightarrow ⑨$$

for $(w >> h)$

Effective dielectric constant:-

→ The effective dielectric constant is equal to the relative dielectric constant for homogeneous media lines such as co-axial and strip line.

→ The static (low frequency) effective dielectric constant of mixed media lines like microstrip and coplanar is lower than the relative dielectric constant bcaz a portion of the fields are in air above the substrate.

→ This static effective dielectric constant is a function of the relative dielectric constant of the substrate, the strip width & even the strip thickness.

→ For microstrip as the frequency is increased, more and more of the field is confined to the substrate and hence the effective dielectric constant increases.

→ The effective dielectric constant given by Tx line includes all of these effects.

→ For coupled lines, the effective dielectric constant for both even and odd modes are given. They are different, even for the static (low frequency) case, and the effect of dispersion is different for each mode.

Losses in microstrip lines:-

* The attenuation constant of the dominant microstrip mode depends on geometric factors, electrical properties of the substrate and conductors and on the frequency.
* For a non-magnetic dielectric substrate, two types of losses occur in the dominant microstrip mode:
    1. Dielectric loss in the substrate
    2. ohmic skin loss in the strip conductor and the ground plane.
* The sum of these two losses may be expressed as losses per unit length interms of an attenuation factor.
* From ordinary Tx line theory, the power carried by a wave travelling in the positive z-direction is given by,

$$P = \frac{1}{2} V I^* = \frac{1}{2}\left(V_+ e^{-\alpha z} I_+ e^{-\alpha z}\right)$$

$$= \frac{1}{2} \frac{|V_+|^2}{Z_0} e^{-2\alpha z}$$

$$= P_0 e^{-2\alpha z} \rightarrow (1)$$

Where, $P_0 = \frac{|V_+|^2}{2Z_0}$ is the Power at $z=0$.

The attentuation constant can be expressed as,

$$\alpha = \frac{-\left(\frac{dP}{dz}\right)}{2P(z)}$$

$$\Rightarrow \boxed{\alpha = \alpha_d + \alpha_c} \rightarrow (2)$$

$\alpha_d \rightarrow$ dielectric attenuation constant

$\alpha_c \rightarrow$ ohmic attenuation constant

* The gradiant of Power in the z-direction in eq^n-(2) can be further expressed interms of the Powerloss Per unit length dissipated by the resistance and the Power loss Per unit length in the dielectric. i.e.,

$$\frac{-dP(z)}{dz} = \frac{-d}{dz}\left(\frac{1}{2} V I^*\right)$$

$$= \frac{1}{2}\left(-\frac{dV}{dz}\right) I^* + \frac{1}{2}\left(-\frac{dI^*}{dz}\right) V \rightarrow (3)$$

$$= \frac{1}{2}(RI) I^* + \frac{1}{2} \sigma V^* V$$

$$= \frac{1}{2}|I|^2 R + \frac{1}{2}|V^2| \sigma$$

$$= P_c + P_d$$

Where σ is the conductivity of the dielectric substrate board.

Substituting eqn-③ in eqn② results in,

$$\alpha_d = \frac{P_d}{2P(z)} \quad (NP/cm)$$

$$\alpha_c \simeq \frac{P_c}{2P(z)} \quad (NP/cm)$$

Dielectric losses:-

When the conductivity of a dielectric cannot be neglected, the electric and magnetic fields in the dielectric are no longer in time phase. In that case, the dielectric attenuation constant, as expressed is given by,

$$\boxed{\alpha_d = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}} \quad (NP/cm)}$$

Where σ is the conductivity of the dielectric substrate board in Ω/cm.

This dielectric constant can be expressed interms of dielectric loss tangent as:

$$\boxed{\tan\theta = \frac{\sigma}{\omega\epsilon}}$$

Then the dielectric attenuation constant is expressed by,

$$\boxed{\alpha_d = \frac{\omega}{2}\sqrt{\mu\epsilon}\,\tan\theta \quad (NP/cm)}$$

Q Factor of a microstrip line:-

→ Many microwave integrated circuits require very high quality resonant circuits.

→ The Quality factor (Q) of a microstrip line is very high, but it is limited by the radiation losses of the substrates and with low dielectric constant.

→ It is know that for uniform current distribution in the microstrip the ohmic attenuation constant of a wide microstrip line is given by,

$$\boxed{\alpha_c = \frac{8.686\, R_s}{Z_0 w} \quad (dB/cm)}$$

→ The characteristic impedance of a wide microstrip line, as shown below:

$$\boxed{Z_0 = \frac{h}{w}\sqrt{\frac{\mu}{\varepsilon}} = \frac{377}{\sqrt{\varepsilon_r}} \frac{h}{w} \quad (\Omega)}$$

→ The wavelength in the microstrip line is

$$\boxed{\lambda_g = \frac{30}{f\sqrt{\varepsilon_r}} \quad (cm)}$$

Where f is the frequency in GHz

→ Since Q_c is related to the conductor attenuation constant by,

$$\boxed{Q_c = \frac{27.3}{\alpha_c}}$$

Where α_c is in dB/λ_g, Q_c of a wide microstrip line is expressed as,

$$\boxed{Q_c = 39.5\left(\frac{h}{R_s}\right) f_{GHz}}$$

Where 'h' is measured in cm and R_s is expressed as,

$$R_S = \sqrt{\frac{\pi f \mu}{\sigma}} = 2\pi \sqrt{\frac{f_{GHz}}{\sigma}} \quad (\Omega/\text{square})$$

Finally, the Quality factor Q_c of a wide microstrip line is given by,

$$Q_c = 0.63h\sqrt{\sigma f_{GHz}}$$

Where σ is the conductivity of the dielectric substrate board in Ω/m.

For a copper strip

$\sigma = 5.8 \times 10^8$ $\mho/m$ and then Q_c becomes

$$Q_{cu} = 4780h\sqrt{f_{GHz}}$$

A Quality factor Q_d is related to the dielectric attenuation constant as shown below:

$$Q_d = \frac{27.3}{\alpha_d}$$

Here, α_d is in (dB/λ_g)

From the above equations, we can write

$$Q_d = \frac{\lambda_0}{\sqrt{\epsilon_{re}}\ \tan\theta} \simeq \frac{1}{\tan\theta}$$

Where λ_0 is the free-space wavelength in cm. Note that the Q_d for the dielectric attenuation constant of a microstrip line is approximately the reciprocal of the dielectric loss tangent θ and is relatively constant with frequency.

04/05/21 UNIT 5:- Waveguide components and Applications